Pattern, the password to mathematics: Cracking the code with Year 2

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Making patterning the basis of mathematics investigations for an entire year produced unexpected results. A Year 2 class used the spatial tasks from the Pattern and Structure Mathematics Awareness Program, to develop their thinking mathematically skills.

Throughout last year, my Year 2 students worked with patterns for the first 20 minutes of each day. What did they do? What did it look like? How did this impact on their mathematical understanding? And was it really maths or were they just making pretty pictures? Inspired by Marilyn Burns and Joanne Mulligan, I decided to use spatial tasks from the Pattern and Structure Mathematics Awareness Program (PASMAP), with a specific emphasis on patterning, as the basis of all my class' maths investigations for the whole year. It was fun, it was exciting, but most significantly, it produced results that I was not expecting. My students started thinking mathematically, making observations and connections in deeply authentic ways, and demonstrated real passion and love for maths.

Initial inspiration

The idea that, 'Pattern is the password to mathematics' is not new. I first came across it in the *I Hate Mathematics Book* (Burns, 1975). This is the opening statement in the book and it is an idea that is developed consistently throughout it. I have no doubt that Pythagoras and Plato played with the same idea and probably thousands of other mathematicians over the years have as well. It seems simple and obvious but then, so many great ideas do. Pattern is indeed the password to mathematics.

However, it was not until I heard Joanne Mulligan speak about the PASMAP research work (Mulligan & Mitchelmore, 2016) that I finally realised the full consequence of this idea. If pattern really is the password to maths, then it should be at the heart of my teaching, not as a peripheral 'pretty idea' but as the central reference point from which learning will become relevant and meaningful.

The PASMAP program explores spatial relationships, structures and patterns that underlie mathematical concepts. Once I had found this resource, I knew that my teaching would never be the same again. I was going to take my 'professional hunch', that patterns were important, and I would have a framework that would give this idea some legs. My planning and teaching was going to be based on the PASMAP program.

The PASMAP resource is supported by an effective assessment tool, the *Pattern and Structure Assessment* (PASA) (Mulligan, Mitchelmore & Stephanou, 2015). I interviewed each of my students using the PASA interview tasks and used these data as the starting point for my launch into a year of inquiry.

Structure of the PASMAP program

The PASMAP tasks are presented in a clearly and purposefully organised structure. Each lesson begins with consideration of the previous learning of students and where to find useful entry points. The lessons then proceed through a sequence of steps aimed at developing a deep understanding of concepts and relationships:

1. **Modelling**: Students copy, model or describe a pattern or mathematical structure.
2. **Representing**: Students draw or construct the pattern while it is still visible.
3. **Visualising**: The pattern or structure is removed and students draw or represent it without being able to see it.
4. **Generalising**: An explicit understanding of the pattern or structure is explored and similar examples are considered.
5. **Sustaining**: Extension or reinforcement tasks are provided to consolidate the learning.

This model for learning is a particular strength of PASMAP. It resonates with the wisdom of professional practice and provides an effective scaffold to promote

student achievement. As a teacher, I really appreciate the way PASMAP supports hands-on activities with graphic and written documentation, providing data that is useful for formative and summative assessment.

The set-up

I quickly realised that to adopt PASMAP in my classroom would require a cognitive shift for my students. Adding this as an additional layer to my mathematical inquiries was going to be a significant extra for my students. Whilst I know that Year 2 children are young and adapt to new ideas quickly, I also wanted to support them to cope with the change by providing experience with pattern-making using physical materials as a daily activity. But where would I find the time do this?

I am not sure about your school, but the students I teach are not terribly punctual. When the bell goes in the morning, it can take up to 20 minutes before the whole class is present in the classroom. I decided to take this first 20 minutes as 'pattern time'. I put out a range of materials such as pattern blocks, 2D shapes, cubes, milk bottle lids and Cuisenaire rods on the floor and asked the students to make a pattern. It was fun, it was engaging, it was something you would not want to turn up late for and, as a result, my students started to arrive at school more promptly, and they also made some incredible patterns.

We made a different pattern every day—linear patterns; spiral patterns; 3D patterns; symmetrical patterns; tile patterns; patterns based on shape, colour and size; patterns based on numbers and letters; patterns made by individuals; and patterns made by groups.

The daily pattern-making sessions were often the springboard for our further mathematical inquiries. This time provided a great opportunity for students to model, represent and visualise patterns that we used as our tuning in to key mathematical concepts, leading us to find out more, to go further and to make generalisations.

The tasks

Pattern making was not the end point of maths for the year. Instead, it became the springboard from which we launched into the PASMAP tasks. The beauty of PASMAP is that is covers every element of the Australian curriculum—space, measurement, data, probability, number and algebra.

One of the most enlightening elements of PASMAP is a diagram that appears at the start of the teacher's manual to describe a possible learning sequence. It isn't linear! While there are elements that naturally lead from one to the other, there is no expectation that learning follows a straight path from A to B to C. This was a breath of fresh air for any teacher from a traditional background who was seeking to embed inquiry into their classroom pedagogy. It was also a breath of fresh air for me, an experienced teacher with a strong leaning towards inquiry mathematics, who appreciated this as validation of my belief in the inter-connected nature of mathematical concepts.

The journey

The journey for my class began with an exploration of numeration, place value and how we represent the relationship between numbers. One significant task in this inquiry was to produce a large-scale number line, from 0 to 51. I rolled out a long roll of paper, about 5m long, and drew a blank number line. I told the students I wanted to make a number line from 0 to 50 but thought it would be quicker if we just counted up by 5s.

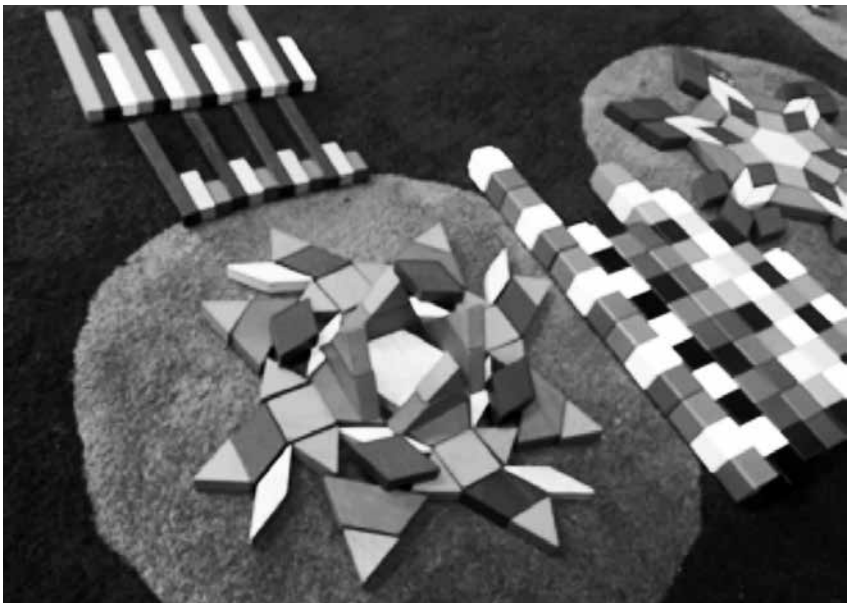


Figure 1. Examples of patterns that students constructed each day.

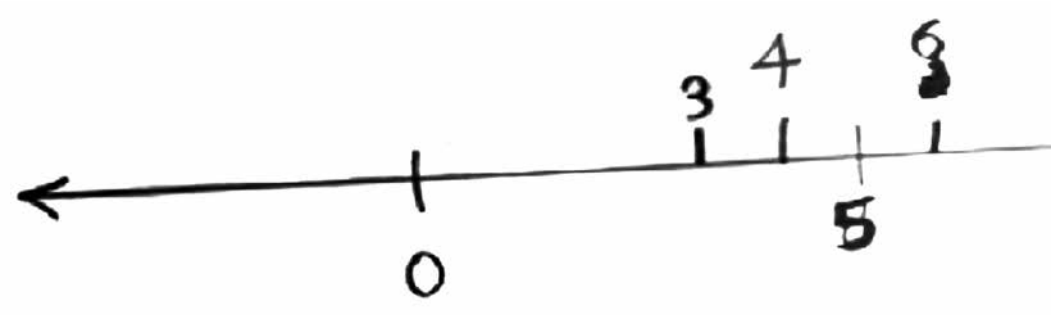


Figure 2. The start of the numberline.

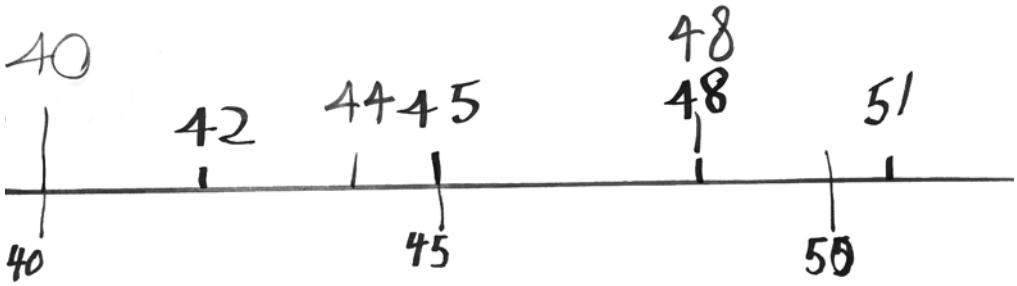


Figure 3. Looking for multiples and skip counting.

So, where should we put the tick marks? And where should we put 0? And where should we put 50? We knew we had to make the marks evenly spaced so after trying a few unsatisfactory alternatives (a hand, a pen, a shoe, a small diary) we finally settled on a library book that we could use to mark out our number line. We knew how to count by 5s so marking it out was pretty straightforward. We did this with a red marker.

"Oh no!" I said. "I didn't want us to count by 5s. I meant count by 3s! Where are we going to mark the counting by 3s pattern?"

After some negotiation we were able to agree where to put the 3s numbers. We did this with a blue marker. Once we had marked in the counting by 3s pattern, it wasn't too hard to convince them to find where the 4s pattern would go. We did that one in green. This is how our number line started—see Figure 2.

As we progressed along the number line we noticed something.

"Hey! Some of these numbers have two colours!"

"Really?" I said. "I wonder why that is?"

Quick as a flash, one bright young lad said, "Well obviously twelve is going to be green *and* blue because three times four is 12."

Great observation for an eight year old.

And even though we had started off simply constructing a number line, we had just explored common multiples, factors, numeration, place value, addition, number patterns and standard units of measurement.

This was to be our experience throughout the year. We used the PASMAP book as our gateway into exploration of fractions, measurement, geometry, algebra, the calendar, symmetry, proportional reasoning, multiplication, division and a whole range of other mathematical ideas.

Experiences that came out of our 'play' with patterns each morning became the backbone of an enriched understanding of, and a palpable enthusiasm for, mathematics.

The results

The PASMAP program also provides an assessment tool, the Patten and Structure Assessment (PASA). This is a one-on-one interview with each student that reveals much about their understanding of mathematical concepts in terms of spatial awareness. There are 16 assessment items in PASA. It requires students to construct, draw and complete patterns, operate with money, construct a clock face and a ruler, use spatial relationships to interpret a graph and demonstrate their knowledge of the relationships inherent in place value and the 100s chart. There are three different assessments, one for each year level.

I used the PASA tool to assess my students twice during the year—initially in Term 2, to establish a baseline, and subsequently in Term 4, to see if progress had been made.

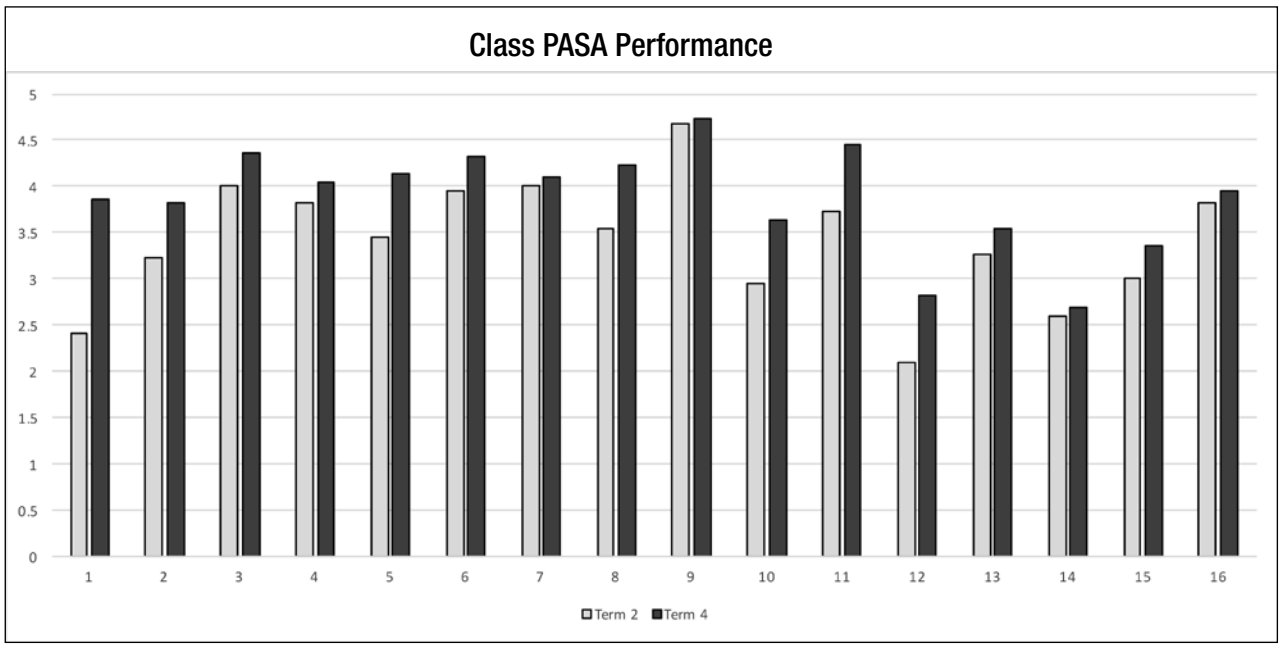


Figure 4. Class results for all PASA assessment items at beginning and end of year (n = 23).

My class improved on every item over the seven months between assessments (see Figure 4).

The items assessed:

Item 1: Partitioning lengths in thirds
Item 2: Cyclic patterns
Item 3: Triangular array
Item 4: Partitioning money
Item 5: Ten frames
Item 6: Skip counting
Item 7: Completing a pattern
Item 8: Rectangular array
Item 9: Hundred chart
Item 10: The clock face
Item 11: Completing a grid
Item 12: Comparing triangles
Item 13: Continuing a pattern that grows
Item 14: Making a ruler
Item 15: Bar chart
Item 16: Comparing capacities

As well as being able to demonstrate improved academic results on testing, PASMAP had other impacts on my class.

Professional learning of peers

During the year, the teachers of Year 1 noticed the learning in our classroom. They became interested in our patterns and asked if they could bring their classes to observe a pattern session. Once they had had a look, they were interested in having me come to their rooms to help run similar sessions for their classes.

And then I had a great idea. Instead of me going into the Year 1 classes to work with the younger children, I sent my students. Each week, a group of five or six students from my class spent some time working in Year 1 as the 'experts'—guiding and directing the learning about patterns. The Year 2 students also acted as 'examiners' in an assessment task where they assessed the Year 1 students on their ability to create, describe and extend patterns. It was magical seeing the students sharing their understandings with each other. The 'assessment' was a real celebration of learning.

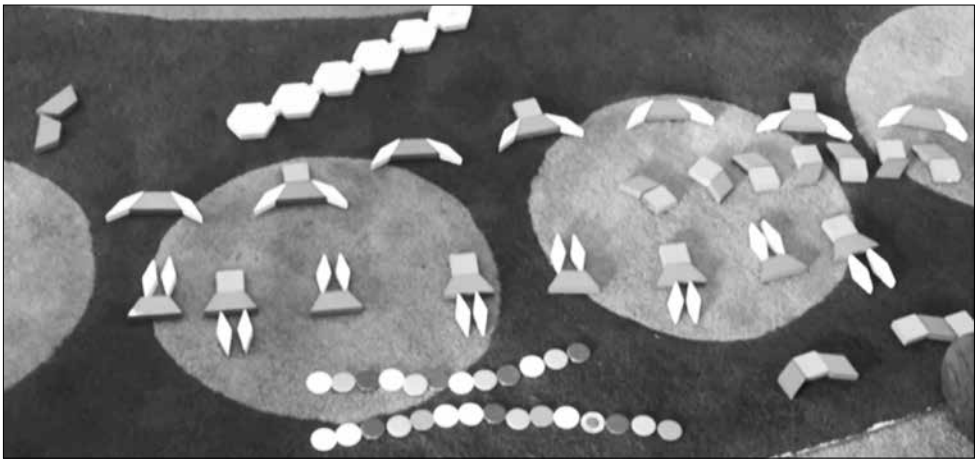


Figure 5. Examples of the complex patterns we were constructing.

Figure 6. Constructing complex patterns in the sandpit using footprints.

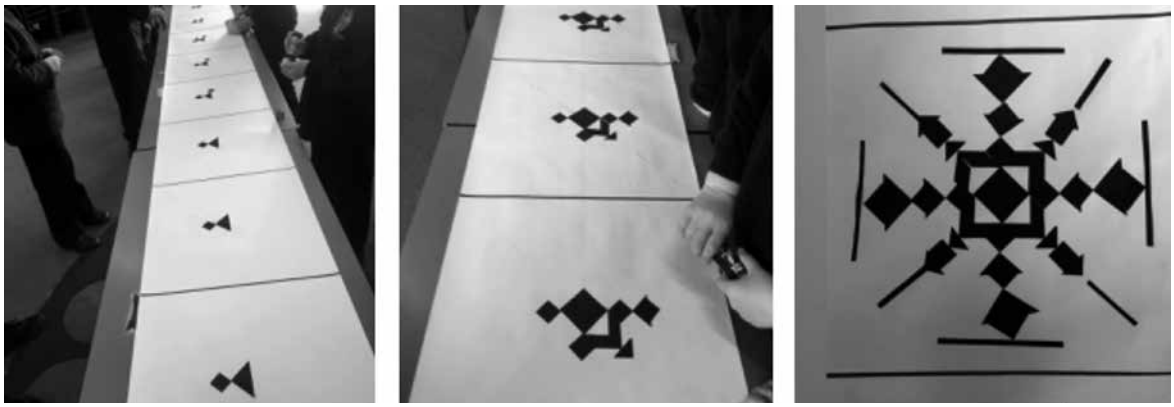
Complex repeating patterns—an example of one of our favourite maths activities during the year

We had many memorable inquiries throughout the year. I tried to follow the structure presented in the PASMAP resource—modelling, representing, visualising, generalising and sustaining. This structure help us develop strong and deep mathematical understanding.

Our inquiry into complex repeating patterns was an extended investigation that started with modelling and representing complex patterns using counters, cubes, shapes and pattern blocks.

We then took the complex patterning into the sandpit, making a series of footprint patterns by visualising and generalising from our previous experience with lateral patterns. Working in pairs, students imagined and then made complex arrangements of footprints.

This ultimately led us to the place where we could construct a complex pattern as a class. A long roll of paper was divided into 24 sections. I drew in the diagonals and vertical and horizontal axes to provide general reference points. Each student was then given a choice of black shapes to glue into each square, building up a communal pattern that was symmetrical. The students were sustaining their learning about patterns by identifying, completing and extending an unseen pattern.



Figures 7, 8, 9. Building up our class pattern.

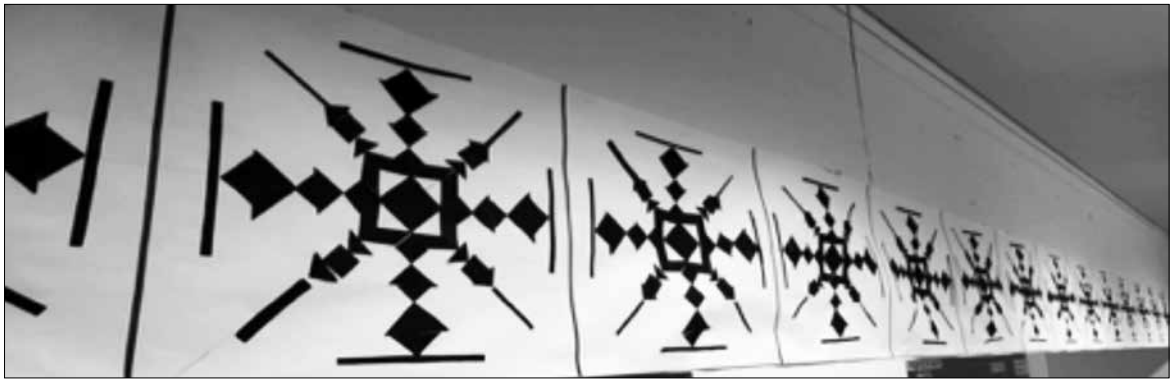


Figure 10. The finished pattern hanging as a frieze around the classroom.

And now for a new year

At my school, we start each year with a brief conversation with the parents of each student. The parents bring with them a completed "Getting to Know Your Child" form with important information for the class teacher. One of the questions on the form is, "What is your favourite subject at school?" Normally, students will reply with PE or Art or Lunchtime.

This year, the Year 3 teachers started to see a change. Several students were nominating Maths as their favourite subject. Some examples are provided here from the students' comments:

"Maths."

"My favourite subject is maths."

"I love maths."

After hearing this response repeated many times, the Year 3 teachers became suspicious and began to ask, "Were you in Mr Ferrington's class last year?"

References

Burns, M. (1975). *The I hate mathematics book.* Boston: Little, Brown and Company

Mulligan, J. T., & Mitchelmore, M. C., (2016). *Pattern and structure mathematics awareness program. Books 1 and 2*. Melbourne: ACER Press.

Mulligan, J. T., Mitchelmore, M. C., & Stephanou, A. (2015). *Pattern and structure assessment (PASA): An assessment program for early mathematics (Years F-2) teacher guide.* Melbourne: ACER Press.