competition

Leveraging collaborative competition in mathematics classrooms

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This article describes the implementation of a collaborative–competetative pedagogy in a Year 10 class and the productive motivational outcomes that followed.

When we teach mathematics, we are on a constant quest to motivate our students. Competition can be used as leverage for encouraging student engagement. However, an overly competitive environment presents its own challenges. Competing one-on-one can be motivating for those who win, but demotivating and detrimental for those who lose, sometimes leading to the development of work avoidance tendencies, anxiety, and decreased self-efficacy (Ryan & Deci, 2000). Collaborative learning may avoid these risks if teachers encourage students to work together in small groups to maximise their own and each other's learning.

There is ample empirical evidence from decades of research indicating that students in collaborative environments out-perform those in competitive environments in a variety of contexts (Johnson & Johnson, 2015; Vandercruysse & Elen, 2017). While collaborating on a range of problem-solving tasks, groups of students with shared goals, shared resources, and complementary roles among group members score higher in task achievement, motivation, and social variables than students working to outperform peers. In this way, students working in collaborative learning environments seemed to consistently attain greater educational successes than those in competitive ones.

However, these synthesised findings suggest students in collaborative settings tend to significantly outperform students in individual competitive situations (e.g., single students competing against other single students), not in all competitive situations. When comparing collaborative learning to collaboration with intergroup competition, although there are few examples, there were no real differences in student achievement and engagement, and researchers have called for more inquiry around combining collaboration with competition (Johnson & Johnson, 2015; Vandercruysse & Elen, 2017). Thus, collaborative competition, or collaborating students competing against other collaborating students, might be just as effective as collaborative learning in some contexts. Groups of students are still working together toward a common goal, but they are also enticed to persevere amidst challenge to outperform opposing groups. Further, the concern of disengagement from students on losing teams is less pertinant because the entire team shares the burden of defeat, not an individual. Yet, the idea of using competition in any capacity to motivate student engagement seems to have lost favour in education research, and collaborative learning alone has become widely recognised as a pedagogical practice that can promote learning.

Certainly collaborative learning can be productive, but also has its limitations. Simply encouraging students to work in groups does not automatically lead to productive engagement. In the context of learning mathematics, collaborators need a collective motivation to want to make meaning out of challenging ideas, by constructing and coordinating their different perspectives to problem-solve together (Sengupta-Irving & Agarwal, 2017). Without careful consideration of such motivators, group-based environments can result in fewer opportunities to learn, and more disruptive outcomes, such as: the exclusion of group member's point of view; lack of equal effort contributions; or general disinterest in achieving the learning goal (Johnson & Johnson, 2015). One approach to motivate students could involve incorporating competitive incentives to engage in productive collaboration. In situations where collaborative learning is ineffective for engagement, could a collaborative-competitive pedagogy give students the incentive to be more engaged during mathematics class?

Trying collaborative pedagogies

I explored these ideas in my Year 10 class when focusing on measurement and geometry. For the first few months I minimised direct instruction and encouraged collaborative work. My general pedagogical philosophy centered on students making their own meanings amongst challenging mathematics through exploration and productive struggle (Warshauer, 2014). Thus, after I made explicit the learning goals of the day, groups of students worked on problem sets containing exercises and activities which emphasised the proficiency strands of understanding, fluency, problem-solving, and reasoning (Australian Curriculum and Assessment Reporting Authority [ACARA], 2014). I circulated the room, answering questions and providing assistance where necessary. The final third of the class period was devoted to discussing the work of the day. Based on daily observation and weekly survey data, students were largely disinclined to work together. Students reported feeling bored and reluctant to persevere during group work. There were several instances of disruptive behavior, as well, including students arguing, sleeping, and even one physical confrontation during class. Additionally, class members were forthright in admitting they consistently "did not work hard while in this class" and were "unmotivated." To me, these data suggested that the way I had structured a collaborative learning environment was not motivating my students to engage productively during class. In an attempt to try something else, I considered incorporating some competition as a pedagogical alternative.

A pedagogical shift towards collaborative competition

Next, I implemented a collaborative-competitive pedagogy in my class. My philosophical stance on using exploratory productive struggle to build student understanding did not change; this shift simply replaced the purely collaborative work with collaborative-competitive team-based games. Collaborative competition has the potential to capitalise on the will many of us have to win, while being sensitive to potential downfalls of competition, such as anxiety or fear of failure. The desires for fun and success act as key motivators, exciting the student to meaningfully engage in the learning game and perform at a higher level. Also, teamwork fosters a collaborative learning setting since students can learn from one another and contribute to the group. Teams that win celebrate a sense of community with their teammates, and individuals on losing teams do not take it so personally since it was a group effort. Win or lose, students playing these games have numerous opportunities to engage with challenging content, build on team knowledge, and have fun learning mathematics.

The game of *Basketball*

The most popular collaborative-competitive structure with my students was the game of *Basketball*[1]. Incorporating *Basketball* into my daily lessons required very few changes to the structure of my class period. After I made the learning goals of the day explicit, students would assemble into their *Basketball* teams and the game would begin. All student work occurred within the framework of competitive games like *Basketball* and since each competition was team-based, there was ample occasion for collaboration among students. Similarly, we came together as a class during the final third of the period to discuss the details of what we had learned.

In *Basketball*, the goal was to score more points than the other teams over four quarters of play. To play *Basketball*, you first need to set up the court. All that is needed is a hoop, a ball, and three shooting lines on the floor: a one-point, two-point, and three-point line.

The rules of *Basketball* are presented in Figure 1. Teams earned points when they had possession of the ball, by correctly answering mathematical tasks and then making basketball shots. Teams could also earn points when they did not possess the ball, based on their opponents' mistakes or omissions. In other words, a team answering a question incorrectly or not at all could result in opposing teams earning points, if those opposing teams answered correctly. This caveat held all teams accountable at all times and increased motivation for continued student engagement. As teams engaged with the mathematical tasks in *Basketball*, I encouraged students to problem solve individually first, and then collaborate with their teammates to reach a solution. To encourage effort from all students, each member of each team was required to present a solution at least once, after which I chose presenters at random.

Consider Teams A, B, C, ... and assume Team A has possession of the ball.
A maths problem is presented to all teams for solving.
If Team A answers *correctly*...

- Team A earns 1 point, AND
- Team A earns 1 additional point for each opposing team who answered incorrectly, AND
- Team A can earn 1, 2, or 3 additional points by making a 1-, 2-, or 3-point basketball shot.

If Team A answers *incorrectly*...

- Team A earns 0 points, AND
- Opposing teams earn 1 point for answering correctly, AND
- Team A does not take a basketball shot.

Team B now has possession of the ball and the process repeats.
All teams possess the ball once per quarter.
The team with the most points by the end of four quarters wins.

Figure 1. The rules of *Basketball*.

1. *Basketball* is just one example of collaborative-competitive games that could occur in a classroom. Any team-based game facilitating collaborative-competition would suffice.

Selecting and sequencing mathematical tasks

Selecting and sequencing appropriate mathematical tasks is important for facilitating a collaborative-competitive game like *Basketball.* Suitable tasks vary in cognitive demand, ranging from lower-level tasks which students could solve by executing a rehearsed procedure, to higher-level tasks that required complex reasoning and non-algorithmic thinking (Stein & Smith, 1998) (see Table 1). There is a place for both lower-level and higher-level tasks in a collaborative-competitive pedagogy: to provide opportunities for students to develop understanding, fluency, problem-solving skills, and reasoning skills, and to create an encouraging space that allows for deep collaboration with teammates about mathematics content (ACARA, 2014).

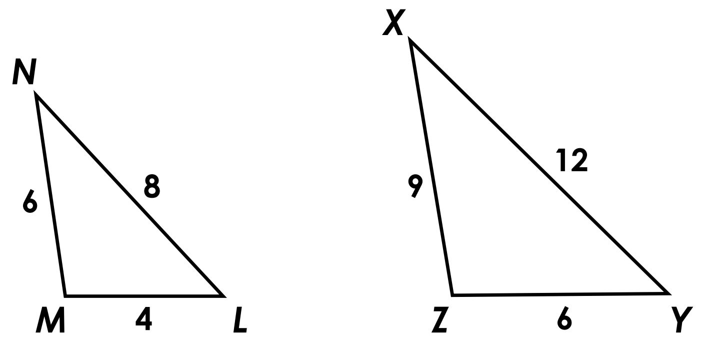
Table 1. Levels of cognitive demand in *Basketball* tasks (adapted from Stein & Smith, 1998).

Level	Type	Description
Lower-level demands	Memorisation	Involves either reproducing previously learned facts, rules, formulas, or definitions; has no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced.
	Fluency without understanding	Use of the procedure either is specifically called for or is evident from placement of the task; algorithmic; little ambiguity exists about what needs to be done and how to do it.
Higher-level demands	Fluency with understanding	Focuses students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas; students need to engage with the conceptual ideas that underlie the procedures to develop understanding and problem-solve successfully.
	Doing mathematics	Require complex reasoning and non-algorithmic thinking —a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example; require students to use exploratory problem-solving and understand the nature of mathematical concepts, processes, or relationships.

Mathematical tasks requiring lower-level cognitive demands were appropriate for the first quarter of *Basketball*, as students were still adjusting to working in their teams and competing against opponents. These tasks were often solved correctly and provoked initial engagement into the game. For instance, consider Problem One sampled from a *Basketball* game during a unit on similarity (see Figure 2). This lower-level task is suitable for students to encounter early in the game to reintroduce the concepts of similarity. Such a task reinforces the importance of congruent angles and proportional side lengths in similar polygons—topics which were explored together during prior class time. I found early successes on problems like this to be important for the continued engagement of each student playing the game. If teams answered mathematical tasks correctly, they earned the opportunity to accrue more points via *Basketball* shots. Also, if tasks were too difficult early on in the competition, students might disengage from the game because of doubts about whether their team could win. Thus, it was important to sequence tasks with lower-level cognitive demands early during gameplay to demonstrate that the game is enjoyable to play and to increase student/team confidence; this paves the way for more substantial mathematical learning via challenging tasks in the later *Basketball* quarters.

Problem One

$\Delta NLM \sim \Delta XYZ$



A. List all pairs of congruent angles.
B. Write the ratios of the corresponding sides in a statement of proportionality.
C. Find the scale factor of triangle ΔNLM to ΔXYZ.

Figure 2. A *Basketball* task suitable for early gameplay.

As the game unfolds and teams are more invested in the competition, tasks requiring higher-level cognitive demands become more appropriate because they provide opportunities for more conceptual learning. Such tasks are often ill-structured and encourage students to work harder together to try to overcome a problem's unveiling obstacles (Bass & Ball, 2015) and, in this context, win the game. Most of the more challenging tasks involve students making connections between their work and the concepts underlying their work. For example, consider Problem Two from the same Basketball game on similarity as Problem One (see Figure 3). The timing of this particular game was before the class had a chance to formally think about proving geometric similarity, and Problem Two was sequenced in the third quarter of the game.

Problem Two

Are any of these triangles similar? Explain your reasoning.

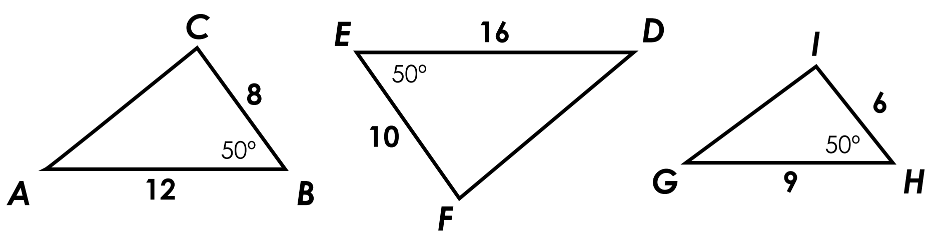


Figure 3. A *Basketball* task suitable for later gameplay.

By engaging with such a problem collaboratively, students can call on prior knowledge about the concepts of similarly while establishing connections to logic when describing their reasoning. If a teammate was confused or struggling on their own, peers would regularly share their understanding and help their teammates not only get the problem correct, but actually strive to master the concept. The following interchange is an excerpt from a student conversation observed while working on Problem Two:

Lisa: How do I know if two triangles are similar?
José: Similar just means the two shapes are the same, but different sizes. Like one has been perfectly shrunk from the other.
Lisa: How am I supposed to know that?
José: Look at the one we just did, they were similar. What is the same about them?
Lisa: Umm all the numbers.
José: What numbers?
Lisa: The angles.
José: True. What about the sides?
Lisa: No they are different because they've been shrunk. But the angles are still all the same.
José: Yeah, that's how you know that the shapes are similar. Look for those angles to be the same.

Naturally, students will construct different arguments and will do well to consider other points of view while reasoning about similarity. The timing and implementation of this type of task continues development of mathematical concepts—rather than keeping them stagnant—is an important feature of effective instruction (Norton & D'Ambrosio, 2008).

More challenging tasks, when selected and constructed to require higher cognitive demand, support high-level student learning outcomes and more sophisticated mathematical thinking. However, the level of these demands can decline if teachers shift the emphasis from mathematical understanding to correctness of the answer (Stein, Grover, & Henningsen, 1996). The collaborative-competitive gaming structure supports the maintenance of higher-level cognitive demands by employing the teacher as a facilitator alone during gameplay. Though I answered student clarifying questions about challenging tasks, the competitive nature of *Basketball* limits teacher involvement with teams because it would be unfair to help one team over another. In this way, students working on Problem Two were left to discuss the underlying ideas of proof themselves and with their teammates, further supporting mathematical concept development (Warshauer, 2014). Facilitating productive struggle is not easy, but gradually incorporating group-worthy tasks within collaborative competitions can motivate students to do so through both the collaboration of team work and the will to win the game.

Learning from a collaborative-competitive intervention

Using team-based competition paid huge dividends for the motivation of my Year 10 class. Test scores, behaviour measures, and attendance all improved in a statistically significant way, and students were more curious and vocal about mathematics than ever before (see DiNapoli, 2011, for full data analyses). These students put forth more effort prior to gameplay to make sense of the day's learning goals so they could perform better during the games that followed. While playing, if teammates could not properly scaffold their peers' learning, teams did not hesitate to reach out to me for assistance – not just for answers, but for understanding. Teams asked questions like "[We] found different scale factors, but we think we are both right. Is that possible?" Such questions sparked fruitful in-game explorations and thoughtful class-wide discourse about perspective regarding geometric dilations and reductions. Additionally, via an anonymous survey, no students mentioned feeling demotivated from losing; instead, they simply looked forward to the next collaborative-competitive game.

Classroom games are not a new idea, but structuring classrooms in a collaborative-competitive way (see Figure 4) is a fresh take on old pedagogies and could help struggling students reach their full potential. Incorporating these games into a learning program focused on student productive struggle and exploration seemed to help students learn how to collectively persevere to make sense of mathematics (Sengupta-Irving & Agarwal, 2017). The most important thing about using such a pedagogical method is the selection, sequencing, and use of appropriate tasks in an engaging, team-based, competitive environment. In my classroom, students were engaging with more challenging content in productive ways than they ever had before. The collaborative-competitive structure drew students in to learn more mathematics together.

- Mathematical tasks should help students review, explore, and extend their understanding.
  - Tasks in early rounds should require lower-level cognitive demands to invite engagement and support confidence.
  - Tasks in middle and later rounds should require higher-level cognitive demands to invite collaboration, productive struggle, and support the development of mathematical concepts.
- Teams should earn points in multiple ways, such as by solving mathematics problems and from opponents being accountable for their mistakes.
- Games should blend mathematics content and team-based sport/activity to increase engagement.
- The students themselves can establish their own agreed upon rules to improve gameplay and support autonomy.

Figure 4. Key features of a collaborative-competitive game.

Previously I was hesitant about using competition in the classroom because it might facilitate strictly performance-based goals and could lead to students trying to memorise procedures or cheat just to win the games. However, my experience with my Year 10 class convinced me that performance and mastery goals can coexist: students aimed to perform well and win, but they also wanted to develop their mathematical understanding through teamwork. It is important to remember that using competition in the classroom might not be for everyone. Research does suggest, however, that combining cooperation and competition in certain contexts can be useful in motivating students and helping them learn mathematics (Johnson & Johnson, 2015; Vandercruysse & Elen, 2017). In my classroom, students were initially reluctant to engage with any mathematics, but I found they enjoyed the competitive atmosphere of team-based gaming and consistently engaged with mathematical tasks requiring both low- and high-level cognitive demands to help them learn mathematics. I invite other teachers to explore the benefits of collaborative competition in their classrooms as they search for effective pedagogies for motivating their mathematics students. This example of teacher research suggests that leveraging collaborative competition could be a useful motivator.

References

Australian Curriculum and Assessment Reporting Authority (ACARA). (2014). *Australian Curriculum: Mathematics.* Retrieved from https://www.australiancurriculum.edu.au/f-10-curriculum/mathematics/

Bass, H. & Ball, D. L. (2015). Beyond "you can do it!": Developing mathematical perseverance in elementary school. In N. Alpert (Ed.), *Mathematics instruction for perseverance: The collected papers.* Chicago, IL: Spencer Foundation. Article Available Online [http://www.spencer.org/collected-papers-april-2015].

DiNapoli, J. (2011). *The effect of competitive teaching strategies on student motivation in secondary mathematics classrooms.* Unpublished master's thesis, Millersville University, Millersville, PA, USA.

Johnson, D. W., & Johnson, R. T. (2015). Theoretical approaches to cooperative learning. In R. Gilles (Ed.), *Collaborative learning: Developments in research and practice* (pp. 17–46). New York: Nova.

Norton, A. & D'Ambrosio, B. S. (2008). ZPC and ZPD: Zones of teaching and learning. *Journal for Research in Mathematics Education, 39*(3), 220–246.

Ryan, R. M. & Deci, E. L. (2000). Self-determination theory and the facilitation of intrinsic motivation, social development, and well-being. *American Psychologist, 55*, 68–78.

Sengupta-Irving, T. & Agarwal, P. (2017). Conceptualizing perseverance in problem solving as collective enterprise. *Mathematical Thinking and Learning, 19*(2), 115–138.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455–488.

Stein, M. K. & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School, 3*(4), 268–275.

Warshauer, H. K. (2014). Productive struggle in middle school mathematics classrooms. *Journal of Mathematics Teacher Education, 18*(4), 375–400.

Vandercruysse, S. & Elen, J. (2017). Towards a game-based learning instructional design model focusing on integration. In P. Wouters & H. van Oostendorp (Eds.) *Instructional techniques to facilitate learning and motivation of serious games* (pp. 17–35). Switzerland: Springer.