Igniting the joy of learning mathematics

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This paper discusses the importance of the joy of learning in mathematics and some ways that teachers can foster this in mathematics education.

Current state of mathematics in Singapore

Singapore has consistently been ranked in the top three countries for mathematics, according to the Trends in International Mathematics and Science Study (TIMSS) results for years 2007, 2011 and 2015. This achievement is nothing short of remarkable considering that Singapore is a small city-state with virtually no natural resources, and only human resources to rely on. In fact, Singapore has been climbing steadily to the top of the TIMSS rankings as evidenced by the average student mathematics achievement scores for TIMSS conducted in 2007, 2011 and 2015. In 2007, Singapore was ranked second and third internationally for Grades 4 and 8 respectively (Mullis et al., 2008) and four years later in 2011, Singapore inched further up the rankings to first and second for both grades respectively (Mullis et al., 2012). The most recent TIMSS results in 2015 was Singapore's best achievement thus far, as it attained the top ranking for both Grades 4 and 8 students (Mullis et al., 2016).

Such remarkable performances in mathematics at TIMSS bodes well for Singapore domestically and internationally as it shows that its education system is able to produce competent mathematics students relative to the rest of the world. This is evidence that Singapore's education system stands in good stead and has worked well for the country.

The top rankings in TIMSS are also reflected in the generally positive attitudes that students in Singapore have towards mathematics (Lim, 2010; Fan et al., 2005).

A survey questionnaire of 1215 Secondary One students from a range of high performing and non-high performing schools in Singapore, showed that lower secondary students held largely positive perceptions about learning mathematics and mathematics as a subject. Mathematics was perceived as interesting to 73% of the respondents while 74% felt they enjoyed doing mathematics. A good proportion, 77% and 61% of respondents respectively believed they could learn mathematics well and obtain reasonably satisfactory grades. When it came to beliefs about the usefulness of mathematics, an overwhelming majority of 91% agreed mathematics was useful and a corresponding 89% saw mathematics as important. More importantly, 84% of respondents did not feel that learning mathematics was a waste of time.

The above findings were corroborated by Lim (2010) in a survey study of 984 junior college students which utilised a five point Likert Scale, ranging from one point for

'strongly disagree' to five points for 'strongly agree'. Lim (2010) found that respondents generally indicated that they enjoyed mathematics (mean = 3.30), had confidence in their mathematical abilities (mean = 3.34) and perceived the importance of mathematics (mean = 3.49).

While all these findings from Lim (2010) and Fan et al. (2005) seem to suggest that Singaporean students generally have positive attitudes towards mathematics, both suggested that intrinsic motivation towards mathematics could be lacking in Singaporean students. In the study by Lim (2010), there was an apparent lack of intrinsic motivation in lower secondary students to study mathematics (mean = 2.82) prompting Lim (2010) to suggest that "more can be done to motivate students intrinsically" (p. 685). Fan et al. (2005) on the other hand, advocated more exposure to cognitively challenging problems in order to let students see the intrinsic value of mathematics as students appeared unwilling to attempt challenging problems. The finding by Lim (2010) that students were not intrinsically motivated to study mathematics, was somewhat paradoxical. How it is that Singaporean students on the one hand have generally positive attitudes towards mathematics, but on the other hand show a lack of intrinsic motivation?

This lack of intrinsic motivation in junior college and lower secondary students in the respective studies by Lim (2010) and Fan et al. (2005), might not come as a surprise when trends are observed across other studies in Singapore. Kong (2015) suggested that these studies pertaining to motivation seem to display a certain trend despite the varying focuses of those studies. Younger-aged students were found to have higher levels of intrinsic motivation compared with their older counterparts (Liu et al., 2009). As students move through the educational ladder, students' motivational regulations tend to be more controlled, as evidenced from the findings by Lim (2010). This trend is not surprising as the same trend of a decrease in intrinsic motivation and an increase in extrinsic motivation as students progressed through school is also evident in other countries (Anderman & Midgley, 1997).

Intrinsic motivation

As seen above, there seems to be a lack of intrinsic motivation in Singaporean students. So what exactly is intrinsic motivation? Intrinsic motivation refers to a natural inclination "to seek out novelty and challenges, to extend and exercise one's capacity, to explore, and to learn" (Ryan & Deci, 2000, p. 70). The key ingredient behind intrinsic motivation is enjoying what students are doing (Middleton & Spanias, 1999), doing it because they really want to be involved in it and not because of any extrinsic reward (Schunk et al., 2008). This inherent motivation or inner drive is derived from students' love for learning (Middleton, 1993), which is an attribute or quality that comes from within a student and is not driven by external factors. This inner motivation propels students' behaviours and the capacity to control their own lives, which is also known as self-determination (Deci & Ryan, 1985). In this way, the learning goals are focused on comprehension and proficiency of mathematical concepts (Dweck, 1986) as opposed to some external rewards such as obtaining good grades or being recognised in a social setting (Ryan & Deci, 2000).

Importance of intrinsic motivation

What was rather poignant were the findings from a study carried out by Liu et al. (2009). In this study, Secondary Two students answered questionnaires that measured the extent to which they were motivated extrinsically, intrinsically and amotivated with regards to

project work. It was found that human motivation was indeed a complex phenomenon as students can be motivated both extrinsically and intrinsically at the same time to varying degrees. Furthermore, in terms of perceived greatest learning and most positive learning experience, findings showed that students benefited the most, when they approached learning with greater levels of intrinsic motivation and lower levels of extrinsic motivation. This illustrates that intrinsic motivation is important for students' learning.

In addition, studies have shown that intrinsic motivation is positively correlated with student achievement (Lim, 2010) implying that it has an important role to play in student achievement and learning. It is only logical therefore, that crucial steps be taken to direct attention towards motivating students intrinsically, especially at the secondary and junior college levels. Furthermore, the lack of enjoyment in solving challenging mathematics problems (31%) and the attitude of relying on provided answers and the unwillingness to venture and try problems on their own (24%) speaks volumes about the inner drive that students have towards mathematics (Fan et al., 2005).

Therefore, the findings by Lim (2010), Fan et al. (2005) and Liu et al. (2009) suggest that there is a lack of intrinsic motivation in Singaporean students, which reflects a hidden dimension to the seemingly picture-perfect portrayal of the mathematical competence of Singaporean students internationally. This lack of intrinsic motivation was affirmed by then Prime Minister of Singapore, Mr. Goh Chok Tong, in a speech back in 1997. In that speech, he expressed concerns over a lack of passion in Singaporean students for learning, even in the most academically inclined ones. He expressed his displeasure at the pragmatic learning attitude of "studying for the sake of obtaining good grades in examinations" and warned of knowledge stagnation (Goh, 1997). It was the desire to seek new knowledge even after the schooling years that Mr. Goh Chok Tong hoped Singaporean students would develop.

A more recent speech by the current Education Minister of Singapore, Mr. Ong Ye Kung, expressed that "an interest-driven choice will motivate students to want to learn their whole lives, to master their professions and their crafts, and to build deep expertise" (Ministry of Education, 2018). This was in the context of developing students for the future. This alludes to a new direction of developing students to find their own interests which means that intrinsic motivation will have a more central role to play in the education of the future.

It is undeniable that intrinsic motivation has an extremely important role to play in the education of students as it benefits students' learning more and is positively correlated with achievement. Moreover, developing intrinsic motivation in students has been an ongoing concern for Singapore since the late 1990s and will have a more central role to play in Singapore's education scene moving forward. A failure to instil intrinsic motivation can be detrimental to the joy of learning, which can have subsequent negative long-term effects on learning in general.

The joy of learning

Much has been discussed about intrinsic motivation and its importance but not about the joy of learning. So what exactly is the joy of learning? The joy of learning is an intrinsic motivation that pushes students to engage in exploration which leads to the discovery of their interests and passions (Ministry of Education, 2017). Ultimately, the joy of learning is about giving room for students to unearth their interests, cultivate their passions and simply enjoy learning. This concept was espoused by Singapore's former Education

Minister, Mr. Ng Chee Meng, at a Committee of Supply debate in 2017. It seems to clearly suggest that it is an important priority for Singapore's education system moving forward.

However, the joy of learning does not simply involve intrinsic motivation and the engagement in exploration. It also involves the expression of human emotions that is a result of sustained ongoing effort from a student during the exploration process (Rantala & Määttä, 2012). These emotions can either quickly fade away or be enduring. According to Varila and Viholainen (2000), the joy of learning can either be active or passive. An active joy of learning is the result of effort from individual students, while the passive joy of learning implies a state of satisfaction with the current situation. In this paper, the joy of learning will be defined as an inner drive or intrinsic motivation to engage in exploration where positive emotions are expressed in the process. These positive emotions have the potential to evoke curiosity, which is a form of intrinsic motivation (Ryan & Deci, 2000). Such curiosity motivates further exploration and this has the potential to become a self-perpetuating cycle. It must be emphasised that intrinsic motivation or inner drive is fueled by positive emotions, which the author believes plays a substantial role in this cycle.

Importance of the joy of learning

Fostering the joy of learning is crucial to learning in a few ways. For one, there is greater tenacity and perseverance during times of failure (Lepper, 1988) which makes students mentally stronger, an important life skill. As motivation comes from within and not from external sources, failures and setbacks are not the determining factors of success. There is also greater sophistication in the cognitive processing and understanding skills in students, more creativity exhibited and a higher propensity to take risks (Lepper, 1988) as students are motivated by their love for learning. All these lead to more comprehensive and productive performances and learning strategies (Lepper, 1988), which are beneficial in the long run. Once students perceive that they are competent and capable of performing well in mathematics, there is a greater tendency to view mathematics more highly (Midgley et al., 1989) and the joy of learning therefore becomes evident.

Objectives

The aim of this paper is therefore to expound on the importance of the joy of learning in mathematics in the context of the Singaporean education system. As seen above,an extensive exposition on the current state of mathematics learning in Singapore was provided and evidence of a lack of intrinsic motivation was highlighted despite Singapore's commendable mathematics rankings in TIMSS over the last decade. This paper will also recommend some ways that teachers can encourage or foster the joy of learning in mathematics before concluding with some future directions.

Fostering the joy of learning in mathematics

Fostering the joy of learning in mathematics can be achieved only if teachers first have the mindset that students are the centre of learning. This notion of student-centred learning positions students right at the heart of classroom instruction and learning (Zain et al., 2012) where students are expected to actively engage in learning and are expected to be proactive and independent (Lea et al., 2003). In order to encourage such active learning, it is important that teachers create opportunities for students to share their views and not sit and listen to long speeches (Rantala & Määttä, 2012).

However, for such sharing opportunities to be created, an environment where freedom of speech within an appropriate classroom context first needs to be fostered. This freedom entails giving students the flexibility of choice to respond to the challenge of a task and does not imply allowing students to do as they please (Rantala & Määttä, 2012). It is about creating an environment where students can make choices safely without fear of repression. The onus is on the teacher to set the boundaries within which students can make these choices. It is therefore vital that a teacher exercises authority but not become authoritarian in managing such an environment of freedom, since fostering the joy of learning is inadvertently linked with freedom. With this as a starting point, teachers can begin to structure activities or instruction around students in order to foster the joy of learning in mathematics.

Another way to foster the joy of learning is by posing challenges which go a long way in motivating students intrinsically (Ryan & Deci, 2000). These challenges should have objectives that are moderately difficult to meet and designed such that success is not assured. An example would be to use open-ended and ill-defined problems which aid students in discovering crucial and interesting knowledge that inadvertently encourages comprehension of the mathematical concept (Middleton & Spanias, 1999). Consider the following challenge in Figure 1.

Consider the following quadrilateral, *ABCD*. Can you find a point on the diagram such that it is:

(1) Equal distance from the vertices *B* and *C*.

(2) Equal distance from the segments *AD* and *CD*.

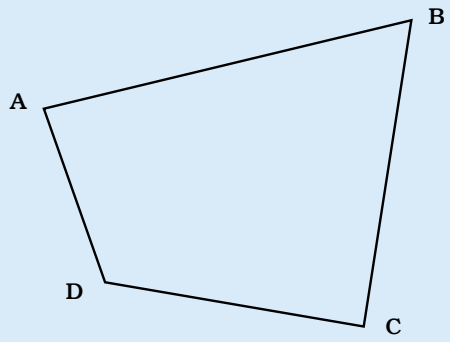

Figure 1. Challenging task.

When given after students learn how to construct an angle and perpendicular bisector, such a task can enable students to discover the usefulness of drawing both bisectors while strengthening their conceptual understanding of each bisector. Achieving the objectives of such challenges can be emotionally satisfying (Ryan & Deci, 2000) and could become a catalyst to intrinsically motivate students to discover more on their own accord. Such challenges provide stimulation, and being able to match activities with students' interests could further spur a greater level of intrinsic motivation (Middleton, 1993).

In addition to posing challenges, generating interesting contexts within these challenges can also help stimulate students' imaginative power and highlight the usefulness of mathematics in a variety of situations (Bransford et al., 1988). Interesting contexts have the potential to evoke curiosity and therefore activate intrinsic motivation since these experiences are original, new and can be surprising (Brophy, 2004). In fact, incorporating creativity and even fantasy into those interesting contexts, can spur students' imagination further and allow them to create their own content using their imaginative powers (Lepper & Hodell, 1989). All these could evoke emotional reactions which are powerful in intrinsically motivating students (Ryan & Deci, 2000). Therefore, being able to stimulate students thinking and evoke emotional responses through challenges is an important way to encourage students to experience the joy of learning.

With reference to the challenging task in Figure 1, an interesting context can be added as seen in Figure 2 below.

Debbie's room is in a rather odd shape as shown in Figure 1. One day, she said to her mum, "I want to put a table with memorabilia of my favourite singers from Korean Boy Band, 'Big Bang', somewhere in my room. Can I do that please?"

With raised eyebrows and a quizzical look, her mum turned to her and said "Why dear? Won't that take up a lot of space in your room?"

"I don't think so Mum. It should only take up a small space as I only have a few soft toys and figurines." Debbie replied with a tinge of heightened expectation.

"Are you sure?" Her mum looked at her with a look of scepticism.

"Yes Mum, I promise." Debbie replied.

"Okay dear, go ahead please." her mum replied.

"Thank you mum! You are the best!" Debbie replied happily.

Debbie desperately needs your help to find a suitable position to put the table in her room. However, she wants the table to be placed:

(1) Equal distance from the corners B and C.

(2) Equal distance from wall AD and wall CD.

Figure 2. Challenging task with interesting context.

By adding an interesting context to the task in Figure 1, curiosity and interest can be invoked in students to want to explore possible solutions to this task. It might serve as a way to induce emotional reactions as the context of the task relates to something students are familiar with. This could give students a taste of the joy of learning in mathematics.

Moving forward

Fostering the joy of learning is a complex process. It involves imbuing intrinsic motivation in students through presenting challenges that stimulate thinking and evoke emotional responses. These challenges need to be situated within a proper classroom context where exploration and play are structured to enable curiosity and interest to be evoked appropriately. Such a classroom context could involve providing opportunities for students to engage in exploration and also establishing the classroom culture where freedom of speech situated within predetermined boundaries, is encouraged.

Moving forward, I believe that the provision of such thought-provoking challenges is one way to foster intrinsic motivation which drives students to further exploration. Cognisant of the importance of the role the joy of learning plays in shaping a student's attitude towards mathematics, the Ministry of Education (2013) in Singapore has explicitly spelt out the importance of learning experiences in shaping students' attitudes towards mathematics and the processes and skills required for the learning of each topic. Through the provision of carefully constructed learning experiences such as thought-provoking challenges, it is hoped that students would develop an inquisitive mindset, which would drive them to explore further. This is exactly what the task in Figure 2 sets out to accomplish and it is my hope that more of such tasks can be crafted and used in mathematics classrooms to spur on the joy of learning.

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