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Characteristics of Students' Mathematical Understanding in Solving Multiple Representation Task based on Solo Taxonomy

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ABSTRACT

This articles discusses the characteristics of students' mathematical understanding in solving multiple representation tasks. Qualitative explorative methods were used to clarify the characteristics of mathematical understanding. Data were obtained by assigning multiple representation tasks to and interviewing 25 students. It is concluded that there are two characteristics of mathematical understanding in solving multiple representation tasks: flexibility and compartmentalization. Flexible understanding consists of complete and incomplete flexibility. SOLO taxonomy level for students who have flexible understanding is relational. Multi-structural level refers to students whose comprehension is incomplete flexible, while uni-structural level refers to students whose understanding is compartmentalized. The findings of this study can be used as a guide to assess the depth of students' mathematical understanding and a foothold in developing learning mathematics based multiple representations.

Keywords: mathematical understanding, multiple representation task, solo taxonomy

INTRODUCTION

The use of various mathematical representation forms is important to be considered in mathematics learning. It could be derived both from aspects of the process and mathematical learning evaluation. Associated with the process of learning mathematics, Bal P (2015), Jao (2013), and Mhlolo et al. (2012) suggest that students work more on various forms of mathematical representation.

On the other hand, in assessing the quality of students' mathematical understanding, it is compulsory to develop task-based multiple representations. The reason is that when students solving multiple representation tasks, they can form a network of connections between mathematical concepts with various representational structures (Barmby et al., 2007). In addition, various forms of representation usage on the task or problem can be identified students' difficulties in understanding mathematical concepts.

Specifically, about the relationship between understanding and multiple representations that the level of understanding is determined by the number and strength of connections between mathematical concepts and external representational structures (Goldin, 1998). Some researchers have examined this connection. For example, Gagatsis et al. (2006) find that there was a compartmentalization phenomenon in understanding the concept of addition and subtraction. The characteristic of this phenomenon is the lack of connection between mathematical content and various forms of representation. Adu-Gyamfi et al. (2017) found that students built different connections between graphical and symbolic representations. Previous research has not yet discussed the quality of students' understanding of building connections between the two representations. Furthermore,

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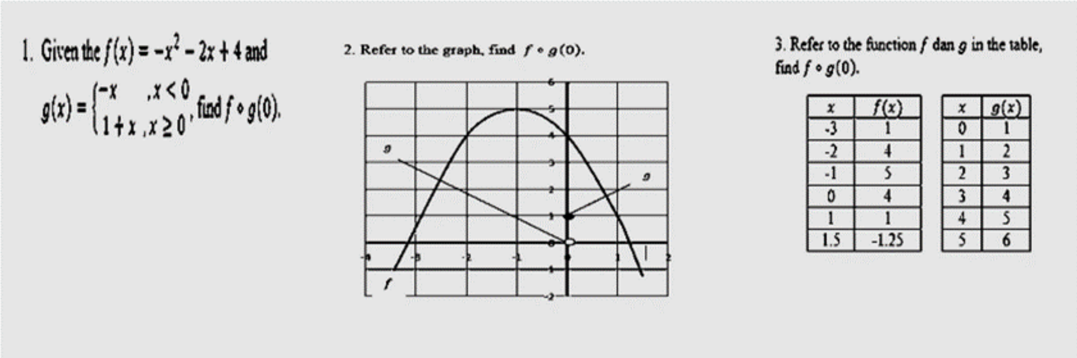

Figure 1. Multiple Representation Tasks

Rahmawati et al. (2017) suggest to research about the characteristics of the subject deeply as the level of ability that might affect the representation translation process and this can be seen when students work with mathematical multiple representation tasks. Therefore, this study focuses on knowing the characteristics of students' understanding in solving multiple representation tasks.

This study is crucial in order to recognize the strong connection between mathematical concepts and various representational structures (Adu-Gyamfi et al., 2017). The aim is to see the real difference among each level of students' mathematical understanding dealing with multiple representation tasks. Assignment of representational tasks was considered in this study because curriculum reform in Indonesia emphasizes more on multiple representation-based learning. However, the effectiveness of the learning needs to be seen from the quality of students' mathematical concepts understanding through creating their characteristics.

The framework used to create the characteristics of students' mathematical understanding in solving multiple representation tasks is the Structure of the Observed Learning Outcome (SOLO) taxonomy (Jimoyiannis, 2011). SOLO taxonomy could be employed to evaluate students' understanding in solving multiple representation tasks and categorize them into five levels, starting from pre-structural, uni-structural, multi-structural, relational and abstract extended (Chick, 1998). Working with various forms of representations brings about varying degrees of performance and complexity. Therefore, this article is interested in making her understanding characteristics based on SOLO taxonomy.

METHOD

The explanatory method in qualitative research was used to obtain the students' understanding characteristics in solving multiple representation tasks. Twenty-five students mathematics major in the Universitas Islam Negeri Malang participated in solving multiple representation tasks. The mathematical translation task can be seen in **Figure 1**.

This task was classified as a non-routine task because mathematics books and mathematics learning rarely ask students to find the composition function value from three different forms of representation. Shortly after the students solving the task, an interview was conducted to investigate and confirm the mathematical understanding which determined their task solving. The questions posed around the understanding of register representation, the mathematical concepts represented by the register and the connection between the mathematical concepts with different representation registers.

Data on written work and interviews data characteristics were reduced, coded and described based on the SOLO taxonomy level. The SOLO taxonomy response indicator was developed from (Chick, 1998) described in **Table 1**. **Table 1** contains indicators of student understanding in solving multiple representation tasks at each level of thinking. The indicator is used to assess the student's response, so as to know the position of the thinking level of each category of acquired understanding.

Table 1. Indicators of Student's Understanding in Solving Multiple Representation Tasks based on SOLO Taxonomy

Level response	Indicators
Pre-structural	• Lack of knowledge in representation register terms. • Having a little information about representation register, nevertheless cannot explain mathematical concept represented by the register itself. • Trying to get the tasks done, but failed to find out the right answer.
Uni-structural	• Only understand specific representation register (symbolic,table,or graphic). • Doing composition function concept interpretation only from the registered particular type of representation. • Understanding compositional function concept from one kind of representation only.
Multi-structural	• Understanding symbolic, table, and graphic representation register. • Doing interpretation of symbolic, table, and graphic representation register to come across composition function concepts. • Understanding composition functions concept from various types of representation (graphics, symbolic and tables). • Connecting the concept of composition function to two forms of representations only.
Relational	• Understanding symbolic, table, and graphic representation register. • Doing interpretation towards symbolic, table, and graphic representation register to discover composition function concept. • Understanding composition function concept from many kinds of representations (symbolic, table, and graphics). • Connecting composition function concept to three representation forms.
Extended Abstract	• Understanding the register of all mathematic representations which reflects real function. • Doing interpretation towards register of all mathematic representations to abstraction composition function concept. • Understanding composition function concept of all mathematic representation varieties. • Creating flexible connection among mathematic representation and being able to generalize all mathematic concepts.

FINDING AND DISCUSSION

Based on data reduction derived from 25 students, it was revealed that there were two types of students' mathematical understanding in solving multiple representations tasks, which were flexible understanding and compartmentalized understanding. The flexible understanding character was flexible in making the relation or the composition function concept network through various forms of representation. The characteristic of compartmentalized understanding was the students' ability to make the relation between the composition function concepts withal one form of representation but failed to find a connection to other forms of representation.

Students who were able to determine $f \circ g(0)$ graphic, symbolic, and table representations were classified as having a complete flexible understanding (CFU), meanwhile, those who were only able to determine $f \circ g(0)$ from two of three representations were categorized as having an incomplete flexible understanding (IFU). Students who could determine $f \circ g(0)$ from one particular form of representation, for instance, graphic, symbol, or table regarded having a compartmentalized understanding (CU). Students who could not determine $f \circ g(0)$ absolutely classified as failed.

Quantitative data showed that from 25 students solving multiple representation tasks about composition function, there were 7 (28%) students who had flexible understanding, 12 (48%) students who had compartmentalized understanding, and 6 (24%) students who were failed to solve the tasks. Students who have a flexible understanding are divided into two, namely complete flexible (12%) and flexibly incomplete (16%). Students who have a compartmentalization comprehension are divided into three, namely compartmentalized into symbolic (36%) and compartmentalized on the graph (0%) and compartmentalized on the table (12%).

Students categorized as incomplete flexible understanding could only find $f \circ g(0)$ from symbolic-table, table-graphs. In the compartmentalized understanding category, students' understanding of composition functions is more compartmentalized on symbolic representation, less in table representation, and not in the graphical representation.

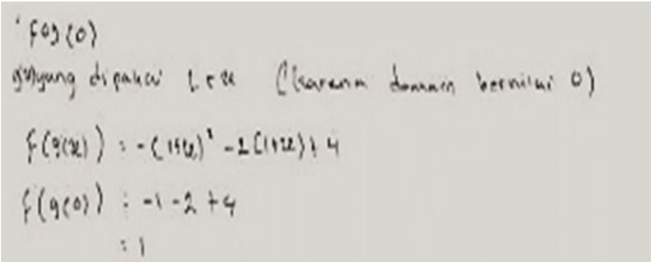

Figure 2. The Example of CFU Students' Work in Task 1

Flexible Understanding

Students who had a complete flexible understanding (CFU) can solve multiple representation tasks. The CFU students were able to establish the connection of the concept of composition function with various forms of representation such as symbolic, graphics and table. Meaning that CFU students have an understanding of the composition of deep composition function (Barmby et al., 2007) and flexible (Ainsworth, 1999; Eisenberg & Dreyfus, 1991; Warner et al., 2009).

The CFU students did interpretation against the register representation to accessing the concept of composition function. As in graphical representation, it concludes that the horizontal axis represents the function domain, the vertical axis represents the co-domain function, the line graph represents the linear function of one variable and the parabolic graph represents the squared function. In the table representation, CFU students have made a correspondence between the values in the left column and the right column, for instance, it is obtained $g(x) = 1$.

In symbolic representations, CFU students could see the correlation between domains and co-domain. They understood that the domain value from the function $g(x)$ has function value at $g(x) = 1 + x$. This indicates that CFU students understand the register of symbolic representations, tables and graphics (Adu-Gyamfi, 2016) and are able to interpret the registers (Boote, 2012). The procedure done by CFU student in determining $f \circ g(0)$ on the representation graphic was started by spelling out $f \circ g(0)$ into $f(g(0))$. Furthermore, CFU student explained that $g(0)$ would be done earlier through looking at the domain (horizontal axis) at zero, then pulled up and met at co-domain (vertical axis) at one, then it was obtained $g(0) = 1$. On the next step, CFU students enacted that $g(0) = 1$ as a domain for $f(x)$. Its domain produced a co-domain which was the same as one. In another word, it could be written $f(g(0)) = f(1) = 1$.

The same process also did by CFU students on finding. $f \circ g(0)$ from the table.The students picked $x = 0$ on the left column of function table g and got $g(x) = 1$, then move onto the table f and picked $x = 1$ on the left column so that $f(x) = 1$ was acquired. In **Figure 2** it seemed that the process of finding $f \circ g(0)$ on symbolic representation tended to be procedural specifically by substituting $g(x) = 1 + x$ onto $f(x) = -x^2 - 2x + 4$ so that we got $f(g(x)) = -(1+x)^2 - 2(1+x) + 4$.

Furthermore, $x = 0$ was substituted to $f(g(x))$ so that $f(g(0))$ was obtained. Referring to the process which was done, it could be assumed that CFU students have understood that the concept of composition function $f(x)$ and $g(x)$ had a rule that x worked on $g(x)$ and $g(x)$ worked on $f(x)$ which eventually formed $f \circ g(x)$. It can be said that this concept could actually be applied to the three representations. On the interview section, CFU student made the conclusion that all three representations had the same content. The CFU students are able to maintain the semantic structure of the three representations (Goldin, 1998). Based on the description of the work and interviews, CFU students could be categorized into the relational level of SOLO taxonomy.

Unlike CFU students, IFU students were only able to find $f \circ g(0)$ from only two representations are symbolic-tables and graphs. IFU students were right in finding $f \circ g(0)$ either symbolic representation and table or graphic and table. They were capable to recognize the registers of both representations. They were capable of interpretation composition function concept from the second register of the representation and being able to connect the composition function concept with both representations.

After an investigation through interviews with IFU students working on symbolic and tables, information was derived that they had no idea at all on how to find $f \circ g(0)$ from the graphic. This condition is due to when working with composition functions, it focuses more on symbolic representation (Knuth, 2000). They realized that symbolic and graphical representations were equivalent. Hence they assumed that $f \circ g(0)$ from the

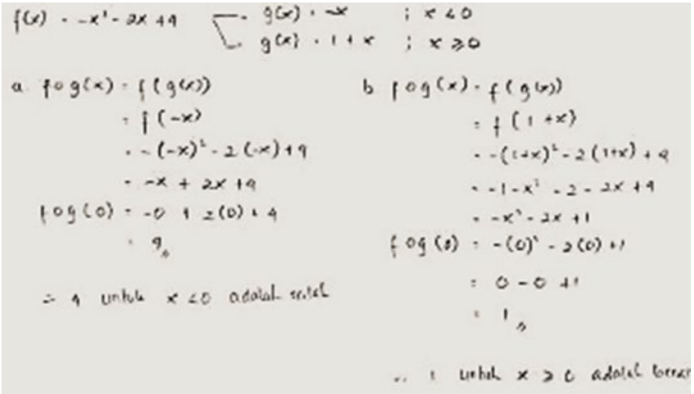

Figure 3. Example of SCU Students' Work Result

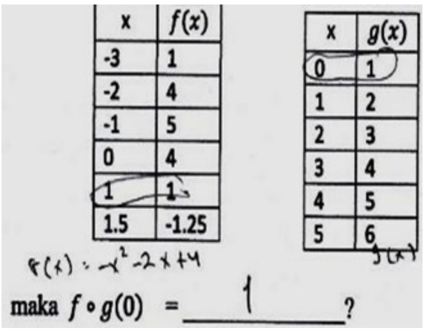

Figure 4. Example of TCU Students' Work Result

graphic was the same as one as well, but could not prove from the graphic. It also happened to IFU students who were only able to find $f \circ g(0)$ from the graphic and table.

A condition when the IFU students could not find $f \circ g(0)$ from the symbolic one was astonishing. This was in contrast to the quantitative data, that the students were frequently correct in finding $f \circ g(0)$ which was from symbolic. This finding is in contrast to the findings of of Greer (2009), that students more effectively work with the symbolic rather than tables and graphics. These findings need to be explored further in the future research. Referring to the characteristics of work and interviews, IFU students could be categorized into multi-structural levels in the SOLO taxonomy.

Compartementalized Understanding

The quantitative information showed that in part students have compartmentalization understanding (CU) and most frequencies are compartmentalized on symbolic representation (SCU). These data imply that the SCU students' learning experience about composition functions mostly use symbolic representation rather than tables and graphs (Greer, 2009). The CU students' work results in symbolic representation can be seen in **Figure 3**.

From **Figure 3**, the students seem to understand dependence relationship among x with $g(x)$ and $g(x)$ with $f(x)$. Implicitly, it shows that SCU students understand about symbolic representation registers (Adu-Gyamfi & Bossé, 2014; Boote, 2014) and are able to interpretation composition functions concept.

There were three students who had compartmentalized understanding on the table (TCU). The three students did not write too much of their answers on paper. They worked directly on the table by encircling a pair of values in the table $g(x)$ and $f(x)$ table. Example of student working of TCU could be seen in **Figure 4**.

There were three students who had compartmentalized understanding on the table (TCU). The three students did not write too much of their answers on paper. They worked directly on the table by encircling a pair of values in the table $g(x)$ and $f(x)$ table. Example of student working of TCU could be seen in **Figure 4**. Confirmation through interviews obtained explanation that student of TCU firstly worked on the table and assumed that for $x = 0$ it would be $g(x) = 1$. Hereinafter, $g(0) = 1$ became the domain of $f(x)$ table through choosing $x = 0$ on the $f(x)$, it was obtained that $f(x) = 1$. Eventually, the students assumed that $f \circ g(0) = 1$.

Based on the working result and interview, it could be assumed that TCU students understood table register (Adu-Gyamfi & Bossé, 2014) and were able to interpret the of composition function concept (Boote, 2012).

The quantitative data also showed that the students have no graphic compartmentalization understanding (GCU). The students' failure in a finding $f \circ g(0)$ of the graphic was because they admitted difficulty and had no idea at all. There were still some GCU students trying to translate the graphics into its symbolic but failed. Result of previous research mentions that the translation process is considered difficult by students (Greer, 2009). Confirmation through interviews obtained information that GCU students claimed that $f \circ g(0)$ could not be found from a graphical representation and could only be found in a symbolic representation. Based on the description of the work result and interviews, the characteristics of students' understanding of CU both SCU and TCU could be classified into the uni-structural level of the SOLO Taxonomy.

CONCLUSIONS

This research found some characteristics of students' mathematical understanding in solving multiple representation tasks. Characteristics of students' mathematical understanding namely complete flexible understanding, incomplete flexible understanding, and compartmentalized understanding. SOLO taxonomy level for students who have a complete flexible understanding of relational and students who have an incomplete flexible understanding is in multi-structural level. Students who have a compartmentalization understanding go to the uni-structural level. The multiple representation tasks in this study have limitations to assess all mathematical understanding indicators of students at the level of abstract extended. Therefore, there is an opportunity for further research to examine the characteristics of students' mathematical understanding at the level of extended abstract. The findings of this study can be used as a guide to assess the characteristics of students' understanding of learning based on multiple representations.

Disclosure statement

No potential conflict of interest was reported by the authors.

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REFERENCES

Adu-Gyamfi, K., & Bossé, M. J. (2014). Processes and Reasoning in Representations of Linear Functions. *International Journal of Science and Mathematics Education, 12*(1), 167–192. https://doi.org/10.1007/s10763-013-9416-x

Adu-Gyamfi, K., Bossé, M. J., & Chandler, K. (2017). Student Connections between Algebraic and Graphical Polynomial Representations in the Context of a Polynomial Relation. *International Journal of Science and Mathematics Education, 15*(5), 915–938. https://doi.org/10.1007/s10763-016-9730-1

Ainsworth, S. (1999). The Functions of Multiple Representations. *Computers and Education, 33*(2), 131–152. https://doi.org/10.1016/S0360-1315(99)00029-9

Bal, P. A. (2015). Skills of Using and Transform Multiple Representations of the Prospective Teachers. *Procedia - Social and Behavioral Sciences, 197*, 582–588. https://doi.org/10.1016/j.sbspro.2015.07.197

Barmby, P., Harries, T., Higgins, S., & Suggate, J. (2007). How Can We Assess Mathematical Understanding? *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education, 2*, 41–48.

Boote, S. K. (2014). Assessing and Understanding Line Graph Interpretations Using a Scoring Rubric of Organized Cited Factors. *Journal of Science Teacher Education, 25*(3), 333–354. https://doi.org/10.1007/s10972-012-9318-8

Chick, H. (1998). Cognition in the Formal Modes: Research Mathematics and the SOLO Taxonomy. *Mathematics Education Research Journal, 10*(2), 4–26. https://doi.org/10.1007/BF03217340

Eisenberg, T., & Dreyfus, T. (1991). On the Reluctance to Visualize in Mathematics. In W. Zimmermann & S. Cunningham (Eds.), *Visualization in teaching and learning mathematics*. Washington, DC: MAA. 127–138.

Gagatsis, A., Elia, I., & Mousoulides, N. (2006). Are Registers of Representations and Problem Solving Processes on Functions Compartmentalized in Students' Thinking? *Relime Número Espec, 9*, 197–224.

Goldin, G. A. (1998). Representational Systems, Learning, and Problem Solving in Mathematics. *The Journal of Mathematical Behaviors, 17*(2), 137–165. https://doi.org/10.1016/S0364-0213(99)80056-1

Greer, B. (2009). Representational Flexibility and Mathematical Expertise. *ZDM—The International Journal of Mathematics Education, 41*(5), 697–702. https://doi.org/10.1007/s11858-009-0211-7

Jao, L. (2013). From Sailing Ship to Subtraction Symbols : Multiple Representations to Support Abstraction. *International Journal for Mathematics Teaching and Learning, 33*, 49–64.

Jimoyiannis, A. (2011). Using SOLO Taxonomy to Explore Students' Mental Models of the Programming Variable and the Assignment Statement. *Themes in Science and Technology Education, 4*(2), 54-74.

Knuth, E. J. (2000). Student Understanding of the Cartesian Connection: An Exploratory Study. *Journal for Research in Mathematics Education, 31*(4), 500-5008. https://doi.org/10.2307/749655

Mhlolo, M. K., Venkat, H., & Schäfer, M. (2012). The Nature and Quality of the Mathematical Connections Teachers Make. *Journal of the Association for Mathematics Education of South Africa, 33*(1), 49–64. https://doi.org/doi:10.4102/pythagoras.v33i1.22

Rahmawati, D., Purwanto, Subanji, Hidayanto, E., & Anwar, R. B. (2017). Process of Mathematical Representation Translation from Verbal into Graphic. *International Electronic Journal of Mathematics Education*, *12*(3), 367-381.

Warner, L. B., Schorr, R. Y., & Davis, G. E. (2009). Flexible Use of symbolic Tools for problem Solving, Generalization, and Explanation. *ZDM—The International Journal of Mathematics Education*, *41*(5), 663–679. https://doi.org/10.1007/s11858-009-0190-8

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