A Co-Requisite Model for Developmental Mathematics: Innovative Pathway Leads to Positive Outcomes for Education Majors  
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Abstract

The traditional mathematics pre-requisite model creates obstacles for pre-service educators by increasing the number of semesters to graduation. Extended time results in additional financial burdens for students in developmental courses along with increased risk of student drop out. Education majors, with little or no room for electives in their programs of study, have been particularly impacted. The pilot of a co-requisite model, involving the collaboration of two academic departments, has proven successful in enhancing students’ academic achievement while addressing social and emotional needs of students.
Introduction

A popular saying often attributed to Albert Einstein speaks of the futility of expecting different outcomes without changing the process or procedure being followed. In other words, if we are dissatisfied with the results we are experiencing, we must be willing to make changes. Such dissatisfaction with student performance in developmental mathematics courses and subsequent credit-bearing mathematics courses led faculty at a public mid-western university (MWU) to examine the university’s approach to developmental education.

Historically, MWU had taken a traditional approach to developmental education in mathematics. Students who failed to meet an established cut-off score on a standardized test were required to successfully complete a series of pre-requisite courses prior to enrollment in a credit-bearing mathematics course. A lack of success in pre-requisite courses or in the subsequent credit-bearing course meant additional semesters at MWU and increased financial burdens for many students.

A number of MWU education majors, especially those pursuing degrees in elementary or special education, scored below the established cut-off score required for credit-bearing mathematics courses and were required to enroll in developmental education courses. Because these education students have few or no electives in their programs of study, the additional developmental education pre-requisite courses impacted their progress towards graduation and teacher certification. Consequently, MWU faculty made the decision to pilot a co-requisite model as an alternative to the traditional pre-requisite model for education majors. Promising results from the pilot led to the recent formalization of the model and the courses that are part of the model.

This article will provide a brief overview of developmental education, a discussion of co-requisite models for developmental education, a description of the process followed in implementing a co-requisite model at MWU, and a summary of the results from the pilot of the model. Finally, a discussion of the lessons learned by MWU faculty is provided.

Developmental Education

History of Developmental Education

When universities were initially established in the United States, over 200 years ago, the only admission requirement of students was the ability of their families to pay tuition fees. It was not unusual for a student from a wealthy family to be accepted for enrollment even though the student lacked the academic preparation necessary for success at the college level (Arendale, 2011). Accepting students who were academically underprepared for the rigor of post-secondary coursework contributed to the financial stability of the institution; however, this practice caused great concern for the success of these students. Eventually, institutions found it necessary to offer college-preparatory courses, to a large population of entering freshmen. As this population continued to grow, institutions established full college-preparatory programs and departments to assist underprepared students in achieving their academic goals. Over time, such assistance became known as “remedial” education, having a focus on building content knowledge that students had not yet mastered due to a lack of prior schooling or to students’ fragmented understanding.

Through the years, America witnessed numerous historical events having a direct impact on education at secondary and post-secondary levels. Following World War I (1914-1918),
education systems began to require formal schooling through the 12th grade, a recommendation made by education reformer John Dewey. This change generated an increase in college enrollment causing colleges “to lower standards to accommodate many of the applicants” (Tucker, 2012, p.4). World War II (1939-1945) generated a demand for mathematicians causing a need for increased numbers of mathematics teachers at the secondary level and encouraging a larger enrollment of college freshmen in calculus courses. Those students unprepared for calculus studies were enrolled in remedial college-level algebra courses. A greater number of military veterans began to enter college following the creation of the G. I. Bill, though many lacked academic preparation due to their military service. The launching of the Russian satellite, Sputnik, in 1957 brought about political concern for the Soviet Union’s lead in the space race along with public blame on American education. Each of these events, along with others, brought about education reform at both the secondary and post-secondary levels.

Reform at the secondary level seemed to occur consequently with societal needs. Lambdin and Walcott (2007) described reforms at the secondary level as six phases: drill and practice, meaningful arithmetic, new math, back to basics, problem solving, and the current standards and accountability phase. Instructional practices in American high schools experienced change through each phase; however, university classrooms remained unchanged causing a growing gap between high school achievement levels and expected college entry levels.

**Traditional Pre-requisite Model**

Developmental mathematics education originated as remedial courses designed to address academic deficiencies, due to the lack of preparation of students entering college. College Algebra began as a remedial course serving to prepare students for calculus. Traditionally, remedial courses were pre-requisite courses for an algebra intensive pathway. Eventually, College Algebra became the required gateway course for most major programs of study. Students unprepared for College Algebra were enrolled in pre-requisites of Intermediate and Introductory Algebra. In addition to the number of high school graduates entering college underprepared, the number of non-traditional students continued to grow creating a diverse population of students with academic deficiencies. Educational doors opened to a new population including immigrants with language barriers, military veterans, and those seeking a career change.

Those serving this diverse population realized that these students shared additional characteristics, aside from academic deficiencies. Students enrolled in remedial pre-requisite courses reported a lack of self-esteem, lack of support from family and friends, trouble adjusting to university culture, time-management issues and mathematics anxiety (Bitner, Austin, & Wadlington, 1994; Boylan, 2009; Castator, 1995). The traditional pre-requisite model of remediation was not designed to offer student services in addition to academic courses. In the last 10 years, such traditional models have been accused of creating obstacles for students, preventing students from reaching their academic goals.

Traditional pre-requisite remedial algebra preparation models have been identified as creating financial burdens on students and institutions by extending time to graduation. Those opposing college-preparatory programs argue that such programs “cost taxpayers twice” and only serve to re-teach what should have been learned in high school (Saxon & Boylan, 2001, p. 2). Those supporting such programs argue that the traditional system is flawed and should be reformed to eliminate obstacles and address student needs beyond content understanding. For these reasons, remedial courses and programs have evolved into developmental courses and
programs designed to address the social, emotional and intellectual needs of students while removing unnecessary obstacles.

Another obstacle, cited in current research, is the tradition of requiring all students to complete a college-level algebra course (Cooper, 2014). College Algebra was once the prerequisite for calculus, as calculus was the required gateway course for many majors for most university students. Eventually, College Algebra became the acceptable gateway course for most students. Students unprepared for College Algebra were placed into developmental algebra courses, based on scores earned on a standardized assessment such as the ACT. Higher education has since seen a demand for mathematics courses that more appropriately align with and prepare students for chosen careers and professions. As a result, institutions have begun to offer basic statistics and quantitative reasoning courses as alternatives to College Algebra for students not entering a science, technology, engineering or mathematics (STEM) field. Though these new gateway courses are now available, the prerequisite algebra pathway has seen little change. Until recently, developmental algebra courses have been accepted as appropriate prerequisites for a variety of gateway courses, even though developmental algebra courses were designed to prepare students only for college-level algebra. In addition, the ACT was designed to measure preparedness for college-level algebra and there is no evidence it will predict success in other gateway courses (Clough & Montgomery, 2015).

Co-Requisite Model

One result of current research in developmental education is the effort to enroll students directly into college-level courses rather than expecting students to complete a sequence of one to three developmental courses prior to the gateway course. Rather than following the prerequisite model by re-teaching high school mathematics content in preparation for college-level content, co-requisite models allow students to immediately begin work at the college-level while receiving just-in-time academic assistance along with support and services designed to address social and emotional needs as well.

Co-requisite models can be implemented using a variety of structures. For example, students may be required to enroll in parallel, supplemental courses or to participate in non-course-based options such as self-paced instruction hosted in university computer labs. Mandatory tutoring that accompanies the credit-bearing course is another strategy used as part of co-requisite models. Some institutions offer developmental support immediately after the scheduled credit-bearing course ends while others add additional hours to courses (Core Principles for Transforming Remediation, 2015).

Changing the Model, Changing the Outcomes

The Department of Academic Enrichment at MWU has been serving students with developmental needs for more than 50 years. Traditionally, students have been placed into developmental algebra courses when the reported ACT math score was below the placement score required for all gateway mathematics. Depending on the reported ACT score, students were expected to complete from one to three developmental algebra courses before enrolling in the required college-level course, offered in the Department of Mathematics and Computer Science. As noted earlier, a number of education majors were placed in these developmental courses.
Each year the MWU Department of Academic Enrichment holds a department retreat where members of the entire department come together to share program reports, research, legislative updates, and university goals. At the spring 2014 retreat, members reviewed and discussed the implications of an education bill recently approved by state legislators. This bill requires higher education to identify and reduce the use of methods shown to be ineffective in supporting students or that delay student enrollment in credit-bearing courses. Best practices in developmental education were discussed at the spring retreat and ideas for improvement were shared among faculty and staff. In an attempt to meet the expectations of the state bill and the MWU vision to improve graduation rates, members of the department agreed that steps should be taken to reduce time to graduation by decreasing time spent in developmental mathematics courses.

Academic advisors identified education majors as having the least time for elective credits in the current program of study. In addition, new state level teacher certification rules require all prospective teachers to pass a series of rigorous assessments, the first of which is an assessment of general education knowledge and skills. Initial pass rates for the mathematics portion of this assessment were very low. These two factors, along with the knowledge that preservice teachers maintain the highest levels of math anxiety compared to students of other undergraduate majors (Gresham, 2007), led to the suggestion that a co-requisite model be implemented for education majors to provide these students with a successful pathway through their required gateway mathematics courses, while reducing time to graduation and alleviating financial burdens.

The next step was for the Academic Enrichment mathematics program coordinator to consult the mathematics education coordinator in the Department of Mathematics and Computer Science. Though the two academic departments had occasionally worked together to ensure curricular alignment from pre-requisite courses to gateway courses, the departments had not worked collaboratively on such a huge project. Changing the developmental model would have an impact on both departments so it was imperative that both departments contribute to the foundation of this project.

The Department of Mathematics and Computer Science offers several mathematics courses that fulfill the general education mathematics requirement at MWU. It was agreed that a co-requisite developmental lab would be paired with a Contemporary Mathematics course. Contemporary Mathematics is a mathematics survey course focused on quantitative reasoning. Topics in the course include set theory, statistics, probability, measurement, and geometry, and students are assessed through course assignments, projects, and tests. Contemporary Mathematics (CM) is required of all MWU elementary, middle school, and special education majors because the topics studied are more relevant to future educators. In addition, the course is a popular general education option for students in other teacher education programs. Historically, Contemporary Mathematics had been one of the general education mathematics courses for which Introduction to Algebra was required as a pre-requisite.

The co-requisite model for the CM course was piloted over four semesters, beginning the fall 2014 semester. Students who did not meet the required ACT mathematics placement score, or did not have a score on file, were enrolled in a CM course, carrying three college credit hours, meeting on Monday, Wednesday, and Friday each week. These students were also enrolled in a developmental mathematics lab (DML), carrying two elective credit hours, meeting on Tuesday and Thursday of each week. Students enrolled in the DML received traditional instruction,
alongside students who met the recommended placement score, in the program specific CM course.

Just-in-time content support was provided through student-centered activities and tasks in the co-requisite DML course. For this to be successful, instructors of the gateway CM courses shared course objectives, syllabi, and instructional schedules with the DML instructor. This allowed the DML instructor time to plan appropriate activities that would align with the content covered in the CM course at the appropriate time. The DML instructor was able to identify student prior knowledge and implement pre-teaching tasks to prepare students for instruction received in the CM course. If a CM instructor needed to alter the schedule, he or she simply sent a short email notifying the DML instructor so that adjustments could be made prior to lab meetings.

Co-requisite Lab Description and Example Activities

In addition to academic deficiencies, students with developmental needs deal with mathematics anxiety, low self-esteem, time-management troubles, and other non-cognitive issues. The DML course was designed to address the social, emotional and intellectual needs of students while supporting the development of both conceptual and procedural content knowledge. In addition to helping students develop understanding of the mathematics curriculum, the DML served to provide students with materials, tools, and resources for success in the gateway course and, in a broader sense, in all university courses and experiences. Because teachers who suffer from mathematics anxiety unintentionally transmit their fears to students and rely on traditional methods of instruction (focusing on algorithms, expecting seatwork, relying on lectures), the DML also served to expose education students to a variety of instructional strategies appropriate for use in future classrooms (Aslan, 2013; Bekdemir, 2010).

Though there were many areas of importance, instructional activities in the DML course focused heavily on mathematical language development. Stahl, Simpson, and Hayes (1992) emphasized the importance of student ability to understand and use the language of mathematics including mathematical symbols. Student ability to draw from multiple definitions and examples including synonyms and antonyms is vital for effective communication in current courses and for success in subsequent courses.

For this reason, each new unit was introduced in the DML by focusing on vocabulary fluency through student-centered activities. For example, a unit on geometry was introduced first by establishing definitions of specific terms that would be used throughout the unit. Students were given a list of important terms along with the textbook definition of each term. In small groups, students worked together to develop their own appropriate definitions of these terms so that they could personalize the definitions and make connections to their own prior knowledge and experiences. Finally, students were asked to take photos of objects representing each term and to share with the class their reasons and justification for selecting the objects photographed. This activity allowed students to personalize their learning of the vocabulary while building an understanding of mathematical language through communication with classmates.

Allowing students to work collaboratively promotes communication in a non-threatening situation where students can receive feedback and accept responsibility for their learning (Osterholt & Barratt, 2010). For example, while completing a lesson on probability, students were randomly placed into small groups and each student was randomly assigned a specific responsibility (driver, recorder, starter, or judge) within the group. The task of each group was to
participate in a remote control car race, in order to collect data that would later be used to investigate the probability and odds of different cars winning a race. While in the small groups, students encouraged their team drivers by providing driving tips, cheers, and pats on the back. They also worked together to ensure data was recorded correctly and that decisions were made fairly. The activity provided an opportunity for students to become comfortable with their teammates and to build relationships that would be supportive in the future as well. After the activity, teammates worked together to solve mathematical problems. During this time, communication was comfortable for students because the teams had shared in the fun activity first. Students felt comfortable in asking mathematical questions and offering suggestions to their teammates. Discussions moved through phases that started with the activity itself, turned to the mathematics content and language, and ended with plans to collaborate for other shared tasks and courses. Students not only developed conceptual knowledge but learned the value of cooperation as well. At the same time, these education majors experienced instructional strategies they can eventually implement in their own classrooms once they become in-service educators.

Formative assessment was used daily in the developmental lab, often through the use of technology including iPad apps such as Kahoot and Nearpod. Students often reported, through anonymous course evaluations, enjoying the use of technology and the participation in simple, friendly, no-pressure competitions. Regular formative assessment provided information to the lab instructor that was then used to develop new tasks to address content areas where students continued to struggle. Grade evaluation in the developmental lab was based only on class work, lab attendance and summative assessments completed in the gateway course. No summative assessments were administered in the developmental lab. Rather than spending valuable class time on additional summative assessments, the time was spent on building content knowledge and understanding. Student performance on unit exams was assessed in the gateway course and scores were then shared with the lab instructor.

Pilot Results

In order to determine the effectiveness of the co-requisite model, faculty at MWU designed a study to investigate the relationship between student participation in a developmental lab and student achievement in the associated Contemporary Mathematics course. The study also sought to examine the relationship between possible placement variables and achievement of education students enrolled in Contemporary Mathematics and participating in the co-requisite model of instruction.

A total of 54 education majors participated in the co-requisite pilot; however, for the purpose of this study, students without both an ACT mathematics score and a high school grade point average on file with the university were eliminated from the study. The subjects of this study were 43 education majors having both an ACT mathematics score and a high school grade point average on file. Two of the 43 participants were males. All subjects of this study had a reported ACT mathematics score less than 22, placing these students into the developmental mathematics program. Education majors with areas of emphasis in elementary, early childhood, middle school, music, art, special education, and career and technology education participated in the study.

Quantitative data for this study came from a total of four semesters of a pilot co-requisite mathematics program. The data include student high school grade point average, ACT mathematics sub-scores, daily attendance rates in the developmental mathematics lab, and final
gateway mathematics course scores. All quantitative statistical analyses were completed using the SPSS Statistics 23 software package.

A descriptive analysis found the mean ACT mathematics score to be 17.33 with a standard deviation of 1.44. The mean high school grade point average was 3.13 with a standard deviation of 0.48. The mean daily lab attendance rate was 0.79 with a standard deviation of 0.17. The mean gateway course score was 2.95, on a 4-point scale, with a standard deviation of 1.17. Descriptive data is displayed in Table 1.1. The Pearson Product Moment Correlation was computed to describe the strength and direction of the linear relationship between each pair of variables. An analysis of the data revealed a moderate and positive correlation between attendance in the developmental lab and gateway course scores \((r = 0.644, p < 0.001)**\). The analysis revealed a moderate and positive correlation between high school grade point average and gateway course scores \((r = 0.451, p = 0.002*)\). The analysis also revealed a moderate and positive correlation between ACT mathematics scores and gateway course scores \((r = 0.403, p = 0.007*)\). A correlation matrix is provided in Table 1.2.

Table 1.1 Descriptive Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACT mathematics score</td>
<td>17.33</td>
<td>1.44</td>
</tr>
<tr>
<td>High School GPA</td>
<td>3.13</td>
<td>0.48</td>
</tr>
<tr>
<td>Lab Attendance Rate</td>
<td>0.79</td>
<td>0.17</td>
</tr>
<tr>
<td>CM Course Score</td>
<td>2.95</td>
<td>1.17</td>
</tr>
</tbody>
</table>

Table 1.2 Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>ACT</th>
<th>High School GPA</th>
<th>Lab Attendance</th>
<th>CM Course Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACT mathematics score</td>
<td>Pearson Correlation Sig. (1-tailed)</td>
<td>1</td>
<td>0.176</td>
<td>0.053</td>
</tr>
<tr>
<td></td>
<td>n = 43</td>
<td></td>
<td>0.259</td>
<td>0.734</td>
</tr>
<tr>
<td>High School GPA</td>
<td>Pearson Correlation Sig. (1-tailed)</td>
<td>0.176</td>
<td>1</td>
<td>0.226</td>
</tr>
<tr>
<td></td>
<td>n = 43</td>
<td></td>
<td>0.259</td>
<td>0.145</td>
</tr>
<tr>
<td>Lab Attendance</td>
<td>Pearson Correlation Sig. (1-tailed)</td>
<td>0.053</td>
<td>0.226</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>n = 43</td>
<td></td>
<td>0.734</td>
<td>0.145</td>
</tr>
</tbody>
</table>

* p<.01, ** p<.001

To determine the value of using multiple measures as course placement tools, regression analyses were conducting using ACT mathematics scores, high school grade point averages, and DML attendance as the independent variables, and gateway course scores as the dependent variable. Using the Enter Method regression model, a significant equation was found \((F(3, 39)=20.877, p < 0.001)\) with an \(R^2\) of 0.616. The model summary is provided in Table 1.3 and the results for independent variables in Table 1.4.
The quantitative data analysis indicates that there is a correlation between attendance in the developmental lab and performance in the gateway course. The analysis also indicates that using multiple measures, rather than a single measure alone, will contribute to CM course scores. All independent variables were significant predictors of CM course scores: Lab Attendance ($t = 5.568, p < 0.001$), ACT Mathematics Scores ($t = 3.231, p = 0.003$), High School GPA ($t = 2.570, p = 0.014$). Although all students in this analysis had complete data, unfortunately, not all students are able to report those scores. Students who did not complete high school in the United States may not have a traditional high school grade point average to report. Non-traditional students who completed high school many years ago may have a high school grade point average to report, but those high school GPAs may not parallel those of more recent high school graduates as graduation requirements and course offerings have changed over the years. Along with these issues is the concern over the possibility of grade inflation that could impact the reported high school grade point average.
In addition to the quantitative analysis, another indicator of program success may be the overall course completion rate for the subjects of this study. MWU monitors the DFW rate of each course offered. Courses with fewer than 30% of students earning a letter grade of D or F or having withdrawn from the course are considered successful courses. Of the students participating in this study, approximately 86% successfully passed the gateway course with a letter grade of C or higher leaving only 14% in the DFW category. Of these students, 67% earned a letter grade of D meaning less than 5% of the students in this study either failed or dropped the co-requisite courses. The reported DFW rate for students enrolled in the same sections of the CM course but participating in the traditional pre-requisite instead of the co-requisite pathway was 25%.

Lessons Learned from Implementing the Pilot

As the results above indicate, MWU’s co-requisite model has been effective in terms of promoting student success and completion of a gateway mathematics course while reducing time to graduation and financial burden. The pilot of the model has also provided faculty with a great deal of insight into the successful implementation of a co-requisite model. First, if multiple sections of the co-requisite courses are offered, the model works best when each section of the gateway course is paired with a specific DML section. When this is not possible, then all sections of the gateway course associated with a specific DML course should have the same instructor. When students in a DML course have different instructors for the gateway course, different topics are often being studied. Just-in-time content support really only works if all students are studying the same content.

Frequent communication and ongoing collaboration between the instructors of the gateway and DML courses is essential. Face-to-face meetings prior to the beginning of the semester have allowed faculty to plan together and to establish means of communication. A weekly email from the gateway course instructor provided the DML instructor with an update on the topics to be covered during the week, and this allowed the DML instructor to plan accordingly. Adding the DML instructor to the course management site of the gateway course was another strategy that helped to facilitate communication. Having access to the site allowed the DML instructor to access student grades, and thus, monitor student progress and performance in the gateway course.

In addition, assessments must be carefully planned and coordinated between the gateway course and DML instructors. In the MWU model, summative assessment occurs exclusively in the gateway course and formative assessment is a major component in the DML course. Interestingly, students in the DML course have asked for graded quizzes that help them practice for the summative tests given in the gateway course. This assessment plan has worked to provide students with the information they need to monitor their progress without overwhelming them with multiple summative assessments.

Monitoring attendance in both the gateway course and the DML course has proven to be an important aspect of the model. Not surprisingly, students who were successful in the gateway course attended both the gateway and DML course regularly. In addition, instructors of the courses must communicate with each other on a regular basis about student attendance.

The logistics involved in ensuring students are enrolled in the appropriate gateway and DML courses proved to be more challenging than initially anticipated. Several different processes were investigated and tried before a workable procedure was identified. While course
management and enrollment platforms vary from university to university, spending time upfront to address enrollment issues is essential.

Finally, many models for co-requisite programs are in use at universities. In some of these models, the same instructor teaches the gateway and DML course. For the model described here, a deliberate decision was made to involve two different faculty members representing two departments of the university. Doing this has provided students the opportunity to experience the content from two different perspectives and through two different teaching styles. This practice supports a variety of student learning styles and needs.

**Next Steps - Extending the Model**

Based on the success of the pilot of a co-requisite model for the Contemporary Mathematics course for education majors, the co-requisite option has been opened to students with majors other than education. Enrollment in the co-requisite sections of the gateway course and the DML course has grown tremendously and several sections of each course must now be offered. In addition, the success of the pilot has led MWU to begin offering a co-requisite option for a second course, Introduction to Mathematical Modeling. The same structure of offering the gateway course through the Department of Mathematics and Computer Science and the DML course through the Department of Academic Enrichment is being utilized with this new course. The DML course is structured in a similar manner as the course used with Contemporary Mathematics. This fall, the Department of Mathematics and Computer Science will use a co-requisite model for Basic Statistics. For this course, both the gateway course and the DML will be offered in the Department of Mathematics and Computer Science. Data will be collected and analyzed for each of these new models.

**Conclusion**

The co-requisite model implemented by the MWU has resulted in positive outcomes for students. Education majors have been supported in achieving success in gateway mathematics courses and in gaining the knowledge required for successful completion of required teacher certification assessments. In addition, the credit hours previously devoted to pre-requisite developmental math courses can now be spent on other required or elective courses. The success of the pilot of the model has resulted in the model being extended to include majors other than education and to additional mathematics courses. A deliberate, yet informed, move away from traditional practices and the willingness to undertake the work involved in trying something new has led to very different, and very positive, results for students.
References


