Using Gestures and Diagrams to Support Students With Learning Disabilities Enrolled in Algebra II

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In this exploratory case study, the researchers conducted a qualitative analysis of six teaching sessions to describe the use of gestures and diagrams by a teacher and her students with learning disabilities when engaging in secondary level algebra (e.g., Algebra I & II content). The teacher and her students used gestures, such as pointing and moving a pen in an arching motion to demonstrate mathematical relationships within equations (e.g., distributive property), and diagrams, such as circling and drawing arrows to facilitate students' progress with logarithms. The teacher used gestures and diagrams to support her students with learning disabilities to organize their thinking processes and understand relationships between problem elements. The findings from this study describe the procedural and conceptual progress made by two students with LD when their teacher utilized gestures and diagrams to teach logarithms, reciprocals, and functions.

Keywords: **Algebra, Diagrams, Gestures, Learning Disabilities, Mathematics**

Introduction

In the United States, more students with learning disabilities (LD) are now being required to pass secondary mathematics courses including a second year of algebra (i.e., Algebra II) to earn a high school diploma (Achieve, 2015). While these students often receive extra support in small-group or one-on-one settings, they are often placed in general education classrooms with students without disabilities due to current educational policies (e.g., Individuals with Disabilities Education Improvement Act, 2004). With these increased expectations — and opportunities for students to engage with more challenging curriculum — as well as continual pressure on students and their teachers due to high-stakes testing, there is a need to understand how students with LD can succeed with complex mathematics in these secondary settings (Confrey, Nguyen, Lee, Panorkou, Corley, Maloney, 2012, Every Student Succeeds Act, 2015; Ysseldyke et al., 2004). More specifically, more information is needed on research-supported interventions (e.g., gesturing and diagramming) that can promote access to secondary school mathematics content for students with LD (Marita & Hord, 2017; Foegen, 2008).

Students with LD and Secondary Mathematics

In the United States, the label of learning disability is used to refer to students who demonstrate difficulties with academics, yet score higher on intelligence tests than students with intellectual disability (Gresham & Vellutino, 2010). These

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students (i.e., about five to eight percent of students in the United States) are often identified after a lack of responsiveness to extra interventions for remedying previous and continuous struggles in mathematics or, less recently, by a discrepancy between the student's scores on intelligence tests and their academic performance (Geary, 2004; Gresham & Vellutino, 2010). While these students have a history of struggling in school, they are also quite capable as learners and can experience high levels of success with strategic interventions even at the secondary level (Marita & Hord, 2017; Kroesbergen & Van Luit, 2003). The struggles of students with LD are often more profound when given multi-step problems due to the tendency many of these students have to struggle with working memory, which is the processing of information, storage of that information, and subsequent integration of multiple pieces of information (Baddeley, 2003; Swanson, & Beebe-Frankenberger, 2004). In multi-step, problem-solving situations (as are common in algebra classes), it can be overwhelming for these students to understand what the problem is asking, break down the problem, and to determine the series of steps to reach the solution (Andersson, 2008; Confrey et al., 2012).

Mathematical Strategies for Students with LD

Under current educational policy in the United States (e.g., Common Core Standards), students are frequently expected to solve lengthy, multi-step problems and provide explanations for their work displaying an understanding of complex and abstract mathematical concepts (Confrey et al., 2012). In response to the implementation of this challenging curriculum and the difficulties students with LD face with multi-step problems, special education researchers have developed interventions utilizing two-dimensional diagrams to help students process, store, and integrate information (see Baddeley, 2003) as they engage in challenging problems (for review, see Marita & Hord, 2017). These diagrams can help students more easily see connections between concepts and relieve the burden on working memory in these situations (Barrouillet, Bernardin, Portrat, Vergauwe, & Camos, 2007; Swanson, & Beebe-Frankenberger, 2004). For instance, if a student is trying to remember information in a multi-step problem, they can simply write the information from that step in a diagram for later use. Then, the students can focus full attention on processing information from subsequent steps and continually write that information down in the diagram as they make progress through the problem. Eventually, the students can then look at the information they have organized visually in a diagram and integrate all parts of the problem to make connections between concepts and solve the problems (e.g., van Garderen, 2007; Xin, Jitendra, & Deatline-Buchman, 2005).

In elementary grades, this approach is supported by research extensively (for review, see Hord & Xin, 2013) and preliminary research in secondary grades has suggested that diagramming may be important in designing interventions for students to engage in secondary level algebra courses (e.g., Ives, 2008). For example, Ives (2008) supported students with LD, when engaging in multi-step algebra equations, with diagrams for organizing problem information into more manageable pieces of information. As with other diagramming studies in special education (e.g., van Garderen, 2007; Xin et al., 2005), the burden of processing, storing and integrating multiple pieces of information was potentially alleviated when students were able to place information in diagrams, stop and think about the visual representations they created at a pace that worked well for them, and problem-solve one step at a time or at a pace that was suitable for their current knowledge and skill level of the particular topic (Barrouillet et al., 2007; Swanson, & Beebe-Frankenberger, 2004).

When paired with two-dimensional representations and strategic verbal language, gestures (i.e., physical movements such as moving of fingers, hands, arms, pens, etc.) have also been supported by special education researchers as being useful for students with LD in algebra settings (Hord et al., 2016). In a pilot study by Hord and colleagues (2016), students with LD benefitted from teachers' use of gestures over equations to help students make connections between the information presented in equations and the problem solving processes that could potentially lead to solutions for the problem. In this study, there was an apparent application of research conducted in the field of psychology regarding the use of gestures to support working memory and student learning in general (see Cook, Duffy, & Fenn, 2013; Goldin-Meadow & Alibali, 2013). Gestures can make multi-step situations or other complex content more accessible by conveying information in ways that are easier for students to process (Alibali et al., 2013). For example, rather than using technical terms to explain concepts — especially as students are becoming familiar with a concept — gestures can make mathematics more accessible when teachers say things like "this right here (while pointing to a part of an equation) connects to this part (while pointing to another part)" (Hord et al., 2016; Goldin-Meadow & Alibali, 2013). When teachers communicate in this way, students do not have to deal with excessively burdening technical terms in situations when they may not be ready to use those terms easily. Of course, students will need to eventually be able to speak using those terms (Confrey et al., 2012), but gestures can provide access as students are struggling with new topics (Alibali et al., 2013; Barrouillet et al., 2007).

Purpose of Study and Research Questions

In general, the use of gestures to support language and gestures in combination with diagrams and language can be useful for teachers and learners for making mathematics more accessible (de Freitas & Sinclair, 2012; Rasmussen, Stephan, & Allen, 2004). Psychology researchers have demonstrated how gestures can help students to more easily process complex information and make connections (Cook et al., 2013; Goldin-Meadow, & Alibali, 2013), and this research has been utilized in pilot studies of students with LD learning more fundamental algebra content such as the distributive property (Hord et al., 2016). However, as students advance to more complex concepts in secondary school, more research is needed to e.g., determine how effective special education teaching principles (gestures and diagrams) can be leveraged with increasingly abstract and challenging concepts such as content required in Algebra II courses (Foegen, 2008). In this study, the researchers will further explore the use of diagrams and gestures — and how these visuals can be combined with verbal language — for the teaching and learning of multi-step algebra problems likely to create working memory-related difficulties for students with LD. The researchers will focus on problems that are representative of secondary level algebra content (including Algebra II content). The specific research question of this study is as follows: How can gesturing and diagramming provide support for students with LD as they solve and discuss multi-step, secondary-level algebra problems?

Method

The researchers conducted an exploratory, qualitative microanalysis of six one-on-one teaching sessions of two students with LD. The researchers used a case study methodology in order to describe the use and the potential impact of gestures and diagrams paired with verbal communication with the students as they engaged in academic content from Algebra I and II. This study was designed to explore and describe this topic to provide a foundation for further research, utilizing more macro-level designs, of this under-studied topic of how these interventions may provide access to secondary level mathematics (Foegen, 2008; Stake, 2010).

Participants and Setting

The study was conducted in a one-on-one setting in a suburban secondary school (grades 9 through 12) in the United States. The participants in this study were enrolled in an Algebra II course (that involved some remediation of Algebra I concepts) in an inclusive classroom of students with LD, struggling learners without disabilities, and students who were considered to demonstrate average achievement in mathematics. In this classroom, the instruction and supports were delivered by both a general education and special education teacher. The students also received daily extra instruction in mathematics in a small group setting and frequent after-school one-on-one mathematics tutoring. The study was designed to take place during events that happen typically in the school day to not interfere with the education of the participants. These students received one-on-one instruction as a normal part of their weekly routine in school.

Reviews of students' academic records were completed to gain insight into the background of the participants, including special education history, test scores, and current goals and objectives in the students' Individualized Education Plans (i.e., legal documents outlining the school's plan for educating a student with a disability). The students' teacher, the first author in this study, identified these students for participation in this study due to their labels as students with LD, current enrollment in Algebra II, and their difficulties with succeeding in inclusion, general education settings without extra one-on-one support.

The researchers selected two high school aged students identified as students with LD (Wilma and Megan) for this study. Both participants' special education files indicated a history of difficulties in mathematics and a need for specially designed instruction in the area of mathematics skills, particularly when multiple steps were required, when working with unfamiliar content, or in other situations in which working memory was likely to be taxed (see Barrouillet et al., 2007; Swanson & Beebe-Frankenberger, 2004).

Wilma was a female, African-American student in the eleventh grade that the school-identified, at the end of her tenth grade year, as a student with LD. Wilma's educational records indicated that she profits from a "slower pace of instruction, specially designed instruction or reteaching as necessary, guided instruction, modeled problems, guided practice and corrective feedback." On a recent academic achievement test, Wilma scored at the seventh percentile on mathematics calculation and at the 19th percentile on applied problems. Wilma had IEP goal for mathematics for her to "demonstrate an understanding of the material by choosing the correct formula/tool to solve and follow the appropriate sequential steps to solve the problems".

Megan, a Caucasian-American in the 11th grade, was school-identified as a student with LD when she was in the 3rd grade. Megan's educational records indicated a history of difficulty solving for unknowns with multi-step word problems and struggling with independent mathematics work in general. Her records also indicated that she benefitted from visual representations during instruction as well as having more time than her peers to work through problems and process how to solve an equation requiring more than one step. With mathematics that was difficult for Megan she tended to have more success with mathematics in situations where there was slower pace of instruction, specially designed instruction or re-teaching as necessary, guided practice, corrective feedback and the opportunity to work with instructors individually or in a small group. On a recent standardized achievement test, Megan most recent score in mathematics was at the 26th percentile. She also had IEP goals for mathematics including a specific goal for multi-step problems regarding the use order of operations to solve or choosing other appropriate steps to solve multi-step mathematical tasks.

Intervention

The authors of this study acknowledge the various models of disability that researchers often debate such as the medical model, social model, and neurodiversity (see Lambert, 2018). However, during the design of the intervention, the teaching methods were not planned based on the specific characteristics of any these models. The first author was familiar with these students and we planned our lessons solely based on the characteristics the students displayed in the context of learning algebra and the curriculum demands facing these students in school.

The first author of this study was the teacher of the participants in this study. She worked as a special education teacher at the participants' school. She taught the students one-on-one, while an observer was present to assist with data-collection and other research tasks, for approximately 30 minutes for six sessions during a study period. The students were not pre-tested or post-tested as a part of this study.

During the sessions, the teacher utilized a combination of using explicit instruction followed by conceptual questions to assess student understanding of the mathematical concepts. The teacher often broke the problems into smaller, more manageable pieces of information, explained each component of the topic, and modeled the concept (procedurally and conceptually). The teacher and her students used materials such as pens, pencils, paper, and the students' homework. The students were provided with a graphing calculator throughout the study.

When deciding on academic content for this study, the researchers considered the curriculum the students were expected to learn at this point in their education and the students' current levels of understanding that made these topics difficult, yet within reasonable reach given the necessary opportunities and support. Based on these considerations, the researchers chose to focus on solving equations, graphing equations, solving systems by graphing, solving polynomial equations, exponential functions, logarithmic functions, solving exponential and logarithmic equations. In each case, the teacher connected lessons to prior knowledge from previous sessions such as learning to solve different type of equations with solving for the variable, graphing, finding domain and range. The teacher reviewed the material with the participants between three and five times per week.

Independent and Dependent Variables

Our independent variable was the use of gesturing and diagramming to support the conversations between the teacher and her students about mathematics. We considered the use of hands, fingers, pencils, or other physical objects supported by the teacher or the students' hands to draw attention to mathematical notation or to represent mathematical concepts to be gestures. We considered any drawing on paper or physical representation on a graphing calculator used to organize information visually in ways that supported students' thinking process to be diagrams. The dependent variable was the demonstration of progress by the student as a learner of mathematics regarding procedural or conceptual understanding such as solving a problem and being able to speak effectively about the concepts involved in the problem.

Procedures

The researchers audio recorded the teaching sessions. Data was also collected via field notes taken during audio recordings of one-on-one teaching sessions and analysis of student work samples from these teaching sessions. The researchers took pictures throughout the teaching sessions with a document camera to record when the teacher and the students used gestures and diagrams when conversing about Algebra I and II problems (e.g., reciprocals, logarithms, function operations, and inverse operations).

Data Analysis

The researchers transcribed the teaching sessions. Then, the coding process began with an overview of the entire set of data to gain a holistic view in order to establish codes based on trends the researchers observed. The transcriptions were then coded and the codes were eventually combined into themes that emerged during data analysis. The researchers used the field notes, record reviews, and work samples, along with the coded transcriptions, to provide support for or against the developing themes.

Interpretive Validity

During each teaching session, an independent observer was present. After each session, the first author discussed her interpretation of what occurred in each session with the independent observer. The first author utilized the perspective of the independent observer to support her own interpretation of the session and her initial work to code instances within the sessions. Also, after the researchers had agreed upon coding, themes, and an overall interpretation of data, they consulted with a local special education teacher, as an external auditor, to monitor the interpretive validity of the study including a discussion of the researchers' inferences about the data (Brantlinger, Jimenez, Klinger, Pugach, & Richardson, 2005; Maxwell, 1992). The researchers consulted with the external auditor until a consensus was reached on all instances of teaching and learning that were included in the study regarding classification for coding, support for themes that emerged, and for the overall findings of the study. The interpretations of the findings, upon which a consensus was reached, are presented in the following section.

Results

Descriptive data is reported from six teaching sessions of eleventh grade students with LD enrolled in Algebra II. The included sessions focused on the mathematics topics of reciprocals, logarithms, function operations, and inverse operations. The findings indicated that the teacher's use of gesturing and diagrams often occurred in situations when the student was struggling to interpret information within the problem or how to begin the problem. The teacher would often pair gesturing and diagramming with spoken language to communicate information in accessible ways to alleviate demands on students' working memory (Alibali et al., 2013; Cook et al., 2013). The students also used gestures and diagrams to demonstrate their understanding of the material. Each key example depicting the findings will be presented with pertinent background information on the context of the mathematical situation. The following excerpts were chosen based on the use of teacher and student gestures and diagrams and the significance of both the struggle and success of the students with the content.

Megan's Work on Logarithms and Reciprocals

Megan and her teacher, Jan, a special education teacher in Megan's school, were working on logarithmic equations. When working on changing an equation from logarithmic form to exponential form, Megan did not remember where to begin. She had notes and the answers from some of her previous work, but could not remember how she obtained the answer. Therefore, Jan began the session with a review. The directions read, "Put each of the following in exponential form," and the first problem was $log_4 64 = 3$.

> Jan: When the problem is asking us to put it in exponential form, what do we know our answer will have? [POINTING to the word, exponential]
>
> Megan: An exponent…
>
> Jan: Good, what number do we see first? [POINTING to the 4]
>
> Megan: 4
>
> Jan: [circles the 4] Good… So, write your 4 first. Then, we are going to do a swirl. [draws an arrow going around the problem] (see Table 1, row 1)
>
> Jan: If we draw an arrow with a loop around the problem, what number do I come to next?
>
> Megan: 1

Jan: So, what will our 1 be in our problem?

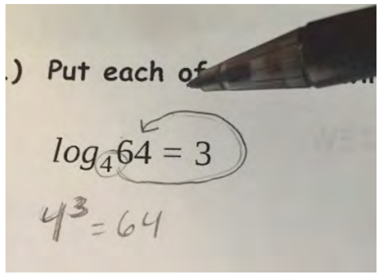
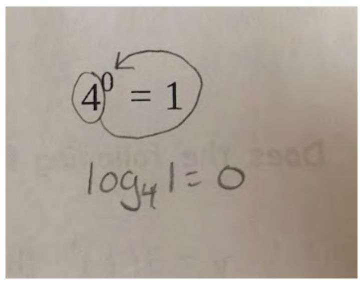
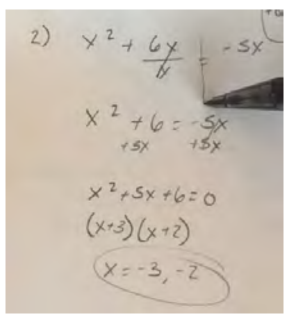
Megan: The next number… The exponent…

Jan: Where does the arrow point to? What is the answer?

Megan: 0… It equals 0.

Jan: Make sure you plug it in your calculator to make sure 4 to the 0 power equals 1.

Table 1. ***Jan and Megan's Use of Gestures and Diagrams***

Task: Put equation into exponential form Teacher diagram: Drawing a loop with an arrow	
Task: Put equation into logarithmic form Student diagram: Drawing a loop with an arrow	
Equation: $x^2 + 6 = -5$ Teacher diagram: Drawing a line to organize the problem	

Megan did a series of four more problems, utilizing the diagramming technique she learned from Jan, making a swirl around each one to write the equation in exponential form.

Megan then reached the next section of her homework that asked her to change an exponential equation into logarithmic form. Megan asked, "How do I do this one?" and, sensing some worry in Megan's question, Jan replied, "You can do this! Circle the first number you see and make your swirl." Then, Megan re-engaged and completed the rest of her homework; she continued with making swirls around each problem and plugging the equations in her calculator to ensure they were correct (see Table 1, row 2). Listening to Jan's explanations, observing Jan's gestures, and utilizing the diagram of writing the swirl seemed to help Megan complete the problems and remember the proper steps to take. At first, the logarithm seemed overwhelming for Megan, but organizing the information on paper with a diagram seemed to make the multi-step problem easier to process, store, and integrate (see Baddeley, 2003) and facilitated her progress forward in the session.

During another teaching session, when working on reciprocals, Megan was struggling to set the equation equal to zero by getting the problem elements within the equation to one side of the equal sign. Jan simply drew a vertical line down through the equal sign and asked Megan to move everything to one side of the line. The two-dimensional visual of writing down a line on the paper helped Megan better organize the information on paper so she could concentrate on where and how to move the problem elements she needed to move (see Table 1, row 3). This seemingly simple diagramming technique, similar to other studies where spatial organization and partitioning of equations supported students' thinking processes (e.g., Ives, 2008), was essential for helping Megan continue to make progress in the session.

Wilma's Work on Functions

During one of the sessions, when reviewing problems on inverse operations, Wilma could not remember what the word, inverse, meant. Jan utilized a simple diagramming technique (i.e., drawn arrows) with a function and the inverse function with this problem: "Find the inversion of each side" for the equation, $y = x^2 + 2$.

Jan: These two are inverse of each other. What do they look like? [POINTING to both functions]

Wilma: Opposite…

Jan: Yes, inverse is the opposite! Looking at the equation, how would we get the opposite?

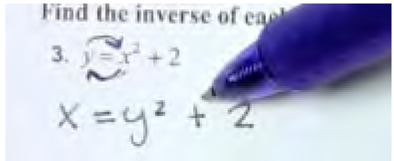
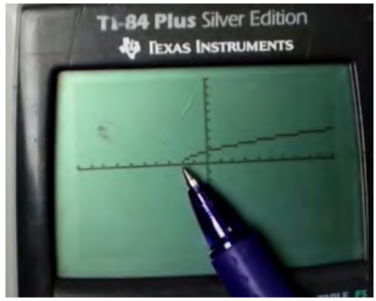
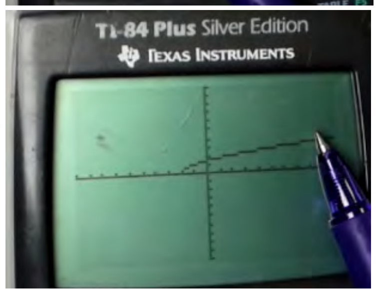
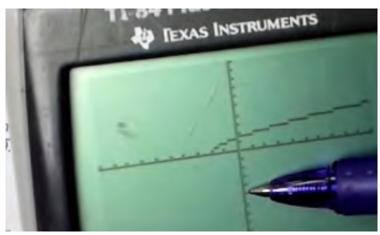
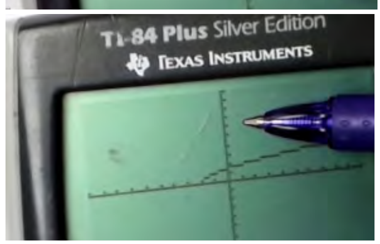
Wilma: I don't remember.

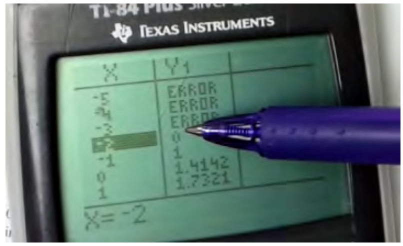
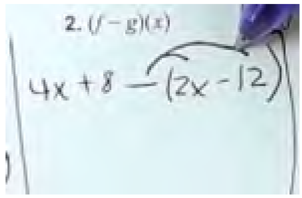
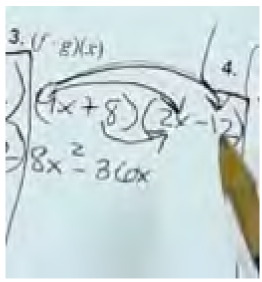
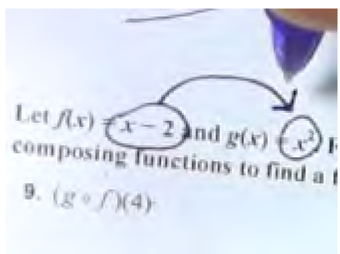
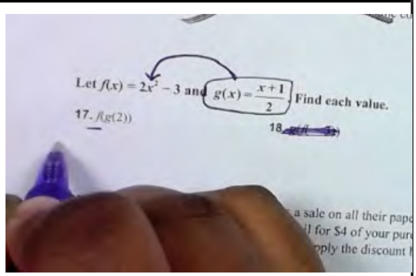
Jan: [POINTS to the x and draws an arrow to the y, then draws an arrow from the y to the x] (See Table 2, row 1).

Wilma: Switch them.

Jan: Exactly, switch and solve.

Table 2. Jan and Wilma's Use of Gestures and Diagrams

Task: Find the inverse of $y = x^2 + 2$ Teacher Diagram: Drawing the arrows to demonstrate flipping the problem elements	
Task: Find the domain of the equation Teacher gesture: Moving the pen along the x-axis	 
Task: Find the range of the equation Teacher gesture: Moving the pen along the y-axis	 

Task: Find the range of the equation Teacher gesture: Pointing to key information	
Task: (f - g)(x) Teacher diagram: Drawing arches to show distribution	
Task: (f - g)(x) Student using gesturing and diagrams to remember distribution and not miss any steps.	
Task: (g ∘ f)(4) Teacher gesture and diagram: Movement along the drawn arrow to demonstrate that f(x) will be placed in g(x).	
Task: f(g(2)) Student gesture: Student using pen movement to gesture to remember the steps to complete the problem	

Wilma switched y and x and continued to need some help through gestures to solve for y. Wilma moved the 2 to the other side of the problem and then required help (through pointing to the squared x) to provide a hint to Wilma regarding the next step in the problem. Wilma went through the correct series of steps after the gestures to solve for y.

Then, Wilma moved on to the next series of problems asking for the domain and range of the function operation. Wilma remembered that domain involved the x-axis and range involved the y-axis, but was struggling to identify what the domain and range was for the problem. Wilma graphed the equation on her calculator.

Jan: Let's look at domain [POINTS to the x-axis]. Does this keep going left, negative, forever?

Wilma: Yes

Jan: Does it keep going right forever? [MOVING THE PEN ALONG the x-axis]

Wilma: Yes

Jan: What does this mean about the range? If it goes both ways on the x-axis forever… [MOVING THE PEN BACK AND FORTH on the x-axis] (See Table 2, row 2).

Wilma: It's the all real one [drawing the symbol].

Jan: Yes, all real numbers… Now, range… [MOVING THE PEN ALONG the y-axis] Does it keep going up forever? [POINTING to line] (See Table 2, row 3)

Wilma: Yes.

Jan: Does it go down forever?

Wilma: No, it starts there. [POINTING to where it stops]

Jan: What point is that?

Wilma: -2

Jan: That is the x value. What is y? [POINTING to the y value]

Wilma: 1.5

Jan: Let's look at the table. When x is -2, y is… (see Table 2, row 4).

Wilma: 0

Jan: Good, okay, back to the graph… Where does the graph start on the y-axis?

Wilma: 0

Jan: Is it getting bigger or smaller? [MOVING THE PEN UP the y-axis]

Wilma: Bigger, so, y is greater than 0

In this case, Jan paired gestures with the two-dimensional representation on the calculator (e.g., pointing to key elements of the problem with her finger and moving her pen sideways along each axis) and strategic use of verbal language (often asking questions to stimulate Wilma's thinking processes and facilitate her progress).

Wilma continued to persist with the problems and seemed to benefit from frequent and seemingly subtle gestures to help her remember her next step or realize how something was connected within the problem.

When working through function operations for this problem, f(x) = 4x + 8 and g(x) = 2x-12 for (f – g)(x), Wilma set up the equation of 4x + 8 - (2x - 12). Wilma was having difficulty finding a way to further simplify the problem. Jan noticed her struggling and offered some guidance.

> Jan: Can't we distribute this negative sign? [POINTING to the negative sign before the parenthesis of (2x-12)]
>
> Wilma: I don't know what that is.
>
> Jan: [drew distribution lines] (see Table 2, row 5)
>
> Wilma: Oh, oh… I know that.

Then, gesturing and diagramming on her own, Wilma completed the problem with ease and did the next problem, which involved distributing twice (see Table 2, row 6). Wilma understood how to distribute and the proper steps to take; she just did not understand the vocabulary word, distribute. Jan utilized arrows and gestured over the arrows to represent the concept of distribution and, then, Wilma immediately could finish the problem. Further down the worksheet, Wilma began to compose functions.

> Jan: Do you remember anything about this problem? g of f of 4
>
> Wilma: Not really… It's confusing. I don't get it at all.
>
> Jan: We're going to look at it just like how you read a book [POINTING to the problem from left to right]. So, g is our base. This is your base [POINTS to g(x)] and this [POINTS to f(x)] is what we will plug into our base [POINTS to g(x)]. We will take this [circling f(x)] and plug it in [drawing an arrow for x in g(x)]. Does that make sense? (see Table 2, row 7).
>
> Wilma: Yes.
>
> Jan: So, go ahead and write that. So, this is your base [POINTING to g(x)] and we are plugging in this [POINTING to f(x)], where x is, and then adding the squared.
>
> Wilma: (writes the equation)
>
> Jan: Perfect. So, do you understand that the first function is our structure; and the second is what we are plugging in; and then the number is last for x? [POINTING to g(x) and f(x)].
>
> Wilma: Yes.

Wilma continued with the problems and continued circling the function and drawing an arrow to the x where she would plug in the function (See Table 2, row 8). Wilma stated that drawing the lines helped her realize what she was moving and where she was plugging it in. In this situation, Wilma benefitted from her teacher's verbal language paired with gestures to demonstrate what she had to do and explain the meaning behind the action, reducing the burden on her working memory and facilitating her progress with the problem (see Alibali et al., 2013; Hord et al., 2016). Wilma often adopted the gesturing and diagramming strategies that Jan used when working independently. Jan's gestures and diagrams, paired with verbal communication, facilitated Wilma's progress in situations where she was struggling, but also important were the new strategies that Wilma acquired for her own use to support her thinking processes as she continued to engage in Algebra II content.

Discussion

The findings suggest that the participants benefitted from the teacher's gestures and diagrams when struggling to make decisions about how to proceed through mathematics problems or make connections between concepts within the problems. The gestures and diagrams seemed to be most useful for students when working memory was being taxed due to multi-step tasks, when several pieces of information needed to be utilized, and when a concept was unfamiliar and challenging for the student. Most importantly, these students with LD were able to access and experience success with Algebra II content which is often required in many states within the United States for graduating with the high school diploma (Achieve, 2015). These findings provide further support for the use of gestures and diagrams to support the learning of students with LD of challenging content in higher level, secondary mathematics courses.

Gesturing and Diagramming by Teachers and Students

Both students often made rapid progress after observing Jan's diagrams and gestures and subsequently utilized those same gestures and diagrams in their work. As in previous studies (Hord et al., 2016; Ives, 2008), these students with LD seemed to benefit from the working memory support that gestures and diagrams provided in the context of secondary level algebra. For example, Jan used gestures to remind Megan of the steps for changing from logarithmic form to exponential form. Megan continued to use these gestures to remember the steps and supplement her thinking process. Diagramming seemed to have a similar impact. When working on function operations, Wilma did not understand the verbal language explained by Jan; however, when Jan circled one function and drew an arrow to where Wilma would plug it into the second function, Wilma seemed to understand the next steps and was able to complete the problem. Diagramming strategies, such as using an arrow to connect problem elements and lines to separate parts of problems, seemed to convey the content accessibly to the students in order to foster deeper comprehension and decision-making about problem solving processes.

Gestures and diagrams seemed to clarify and support the verbal language used by the teacher. For example, Wilma was initially confused when the teacher discussed distributing. However, upon gesturing the movement of distribution with the pen and utilizing the arched arrows drawn by Jan, Wilma immediately remembered how to proceed and continued to draw her distribution lines to solve the rest of the problems. As in previous studies (e.g., Hord et al., 2016) and for the participants in this study, vocabulary used in isolation from visual representations seemed to create difficulties for the students until the teacher started supporting the vocabulary with meaningful visual representations. After these teaching adjustments, the students began making rapid progress with algebra, and most notably, advancing through Algebra II level content.

Limitations and Future Research

In the study, the findings provide some preliminary insight into how gestures and diagrams can be used by teachers to support students with LD enrolled in Algebra II. However, as are inherent in the exploratory, descriptive design of this study, there are limitations that need to be addressed in future research. The interventions in this study were implemented in one-on-one settings. Future studies should address the potential impact of these interventions with larger groups of students. This study was conducted over a short period of time including only six sessions and future research should focus on understanding how these interventions can impact students with LD over weeks or months as they progress through Algebra II. The sample size of this study was suitable for the exploratory focus of the study, but future studies should include more participants to produce more generalizable results. However, the findings of this study offer potential principles for developing interventions in future studies regarding how gestures and diagrams may provide the support that students with LD need to access and succeed with secondary level mathematics courses, including higher level mathematics courses such as Algebra II (Achieve, 2015; Ysseldyke et al., 2004).

Implications for Practice

Based on the findings of this study and similar studies of students with LD regarding the use the gestures and diagrams, we encourage teachers to carefully watch their students to notice when they may be struggling with multi-step problems in secondary algebra settings. In some cases when students seem overwhelmed, teachers may be able to teach students to use diagrams to segment parts of problems into sections which may help students engage with multi-step problems (Ives, 2008). Yet, as with this study, students with LD may also benefit from diagramming that utilizes arrows and lines connecting problem elements to show conceptual connections and procedural steps. In other cases, it may be that students with LD may benefit from teachers gesturing (with one of their fingers or a pencil or pen) over the top of equations to help students notice important parts of problems or make key connections between parts of equations (Hord et al., 2016). The nuances of how these strategies should be used are still open to interpretation due understudied nature of secondary algebra and students with LD (Foegen, 2008). We also recommend that teachers are open to making adjustments as they teach due to the complex needs and strengths of students with LD. While these students can experience great challenges, they can also demonstrate rapid growth and success when teachers unlock their potential by giving these students the right combination of support and opportunity (Geary, 2004; Foegen, 2008; Lambert, 2018).

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