What Is Shadow?

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Abstract

This paper has two parts. In the first part, intended for physics students as well as for engineering students whose primary interest is not physics, a new lesson related to shadow properties is given as a part of lectures on light. The effects resulting from a shadow ’motion’ observed on a screen were looked into. The relationship between the parameters that define the screen placement and the parameters related to the conditions of shadow formation is given. The possibility of information transfer in the optimal geometrical conditions is analyzed as well. The second part of the paper contains homework assignments. Assignments designed for physics students differ from those proposed for engineering students. The assignments are intended to encourage students to reason and to take a creative approach.

Keywords: Shadow, velocity, light.

INTRODUCTION

In lectures about light, the speed limit provokes students’ interest at all educational levels. Generality of the limit raises even stronger curiosity. For creative students, it is very important to have an opportunity to work on a problem on their own. This is particularly useful for students with non-physics major, for example electrical engineering. For those majoring in informatics, there are numerous lectures related to the speed of data transfer. Even more, technology advance in informatics is closely related to data transfer speed.

In general, existence of a boundary or of unavoidable limitations that emerge from Mother Nature’s constraints is sometimes more important than information about a phenomenon or an effect itself. We will see that, if presented properly, a shadow as well as light may help students in understanding such limitations. In addition, students may learn to be skeptical as well as to recognize and explain illusions.

In addition, engineering students have significant programming knowledge and are familiar with computer animations tools. To make a course more interesting it is very convenient to offer lectures that are not included in the basic curriculum but are connected to it and represent its supplement. Such is the case with light and shadow as its direct consequence. In such a case, after basic information about a phenomenon, assignments for semester papers, diploma or master thesis could be specified.

In this paper, we will first present suggested supplement to the lectures on light and then give guidelines for related assignments.

Supplement about Shadow

Introduction

As an introduction to a lesson on shadow, it is beneficial to initiate a discussion among students about their understanding of a shadow. A convenient example to start a discussion is an eclipse of the Sun, eclipse of the Moon, as well as movement of a shadow projection along a meadow or a mountain cliff, the examples that can help students perceive different shadow projection velocities. An interest provoked in this manner should be supported with a brief theoretical introduction. There are numerous reviews that address a shadow (Gibbs, 1997; Penrose, 2004; Stachel, 2002). Of course, all papers and research related to light are indirectly tied to this problem as well (Cooke, 1968; Aoki, 2008; James, 1999; Liberati, 2002).

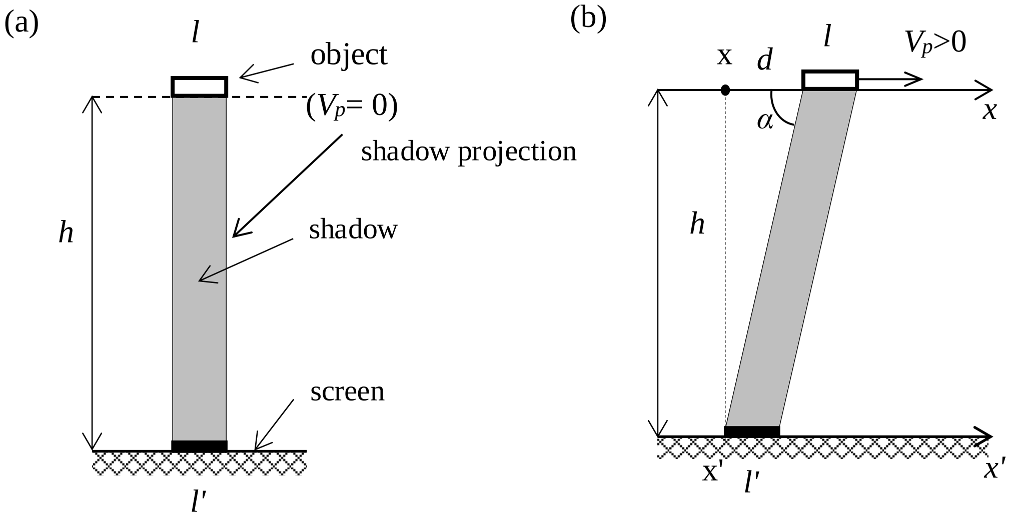
The most often used definition, i.e., description of a shadow is: A shadow is a dark area where light from a light source is blocked by an opaque object. It occupies all of the three-dimensional volume behind an object with light in front of it. The cross section of a shadow is a two-dimensional silhouette. Or a reverse projection of the object blocking the light (Gibbs, 1997).

One can say that the main topic of our research emerged as a desire to explore in more details the idea that a shadow can travel faster than light. According to the relativity (Einstein, 2006), no physical substance can exceed the speed of light because it would take infinite energy to accelerate anything to such a velocity. Yet the laws of physics pertain only to that which is. That which isn’t is not bound by relativity’s restraint. From the point of view of relativity, a shadow (having no mass) is a non-thing, an existential void.

It is this last fact that speaks in favor of including a shadow as a topic when lecturing about light. If we take that a shadow projection is a result of the geometrical optics, and if we nevertheless assign properties of a real object to it, we can obtain some interesting results. We will see that these results do not contradict the laws of physics, instead they represent excellent exercise in modeling of a phenomenon that has its origin in nature, it can be observed in nature, and can have consequences in understanding nature.

Analysis and Model

We will first look into the case when the screen is parallel to the trajectory of an object of size *l*, as depicted in Fig. 1. The trivial case when the object is at rest ($v_p=0$) and is producing a shadow projection at the screen placed at the distance *h* from the object, is illustrated in Fig. 1a. It is clear that the size of the shadow projection is equal to the size of the object, $l'=l$. The shadow occupies the space between the object and the shadow projection on the screen, as depicted with the shaded rectangle



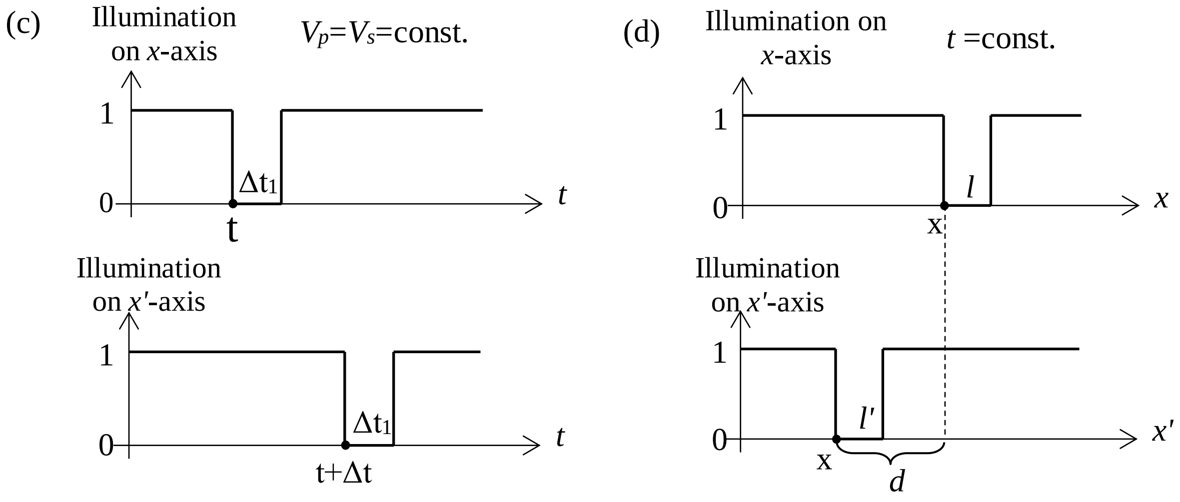


Figure 1. *Shadow formation. The illustrations of an object, its shadow, and shadow projection on the screen for object velocity equal to zero, $v_p = 0$, and larger than zero, $v_p \neq 0$ are given in parts (a) and (b), respectively. For the case when $v_p \neq 0$, the illumination intensity on the screen and along the object's straight line trajectory is shown in graphs (c) and (d). The illumination dependence on time at a given point is depicted in (c), whereas the spatial distribution of illumination at a given fixed moment in time is shown in (d).*

The schematic shown in Fig. 1b corresponds to the case when the object is moving along the x-axis with the velocity $v_p \neq 0$. The remaining parameters are the same as those given in Fig. 1a, namely the screen is positioned at the x'-axis that is parallel to the x-axis and the distance between these two axes is h. The graphs in Fig. 1c give the dependence of the illumination at a given point in time. The point X' in the shadow projection, which originates from the point X in the object is delayed by $\Delta t = h/c = d/v_p$, where c is the speed of light (Fig. 1c). Therefore, at the same moment (without getting into the precise definition of time equality) the difference between the positions of the object and the shadow projection is $d = hv_p/c$, and the shadow is now depicted by the parallelogram whose slant angle is α ($\mathrm{tg}\alpha = h/d = c/v_p$).

The upper level of the graphs in Fig 1c denotes illumination, whereas the lower level indicates the absence of illumination, i.e., it corresponds to the shadow just below the object on its trajectory or the shadow projection on the screen. The time for which the shadow projection covers the position X' is equal to the time the object needs to pass by the point X and is equal to $\Delta t_1 = l/v_p$. The size of the shadow projection is equal to the size of the object. The dependence of the illumination on the x- and x'-axis at a fixed moment in time, t=const., is given in Fig. 1d. The shadow projection travels with the velocity v_s that is equal to the velocity of the object, v_p.

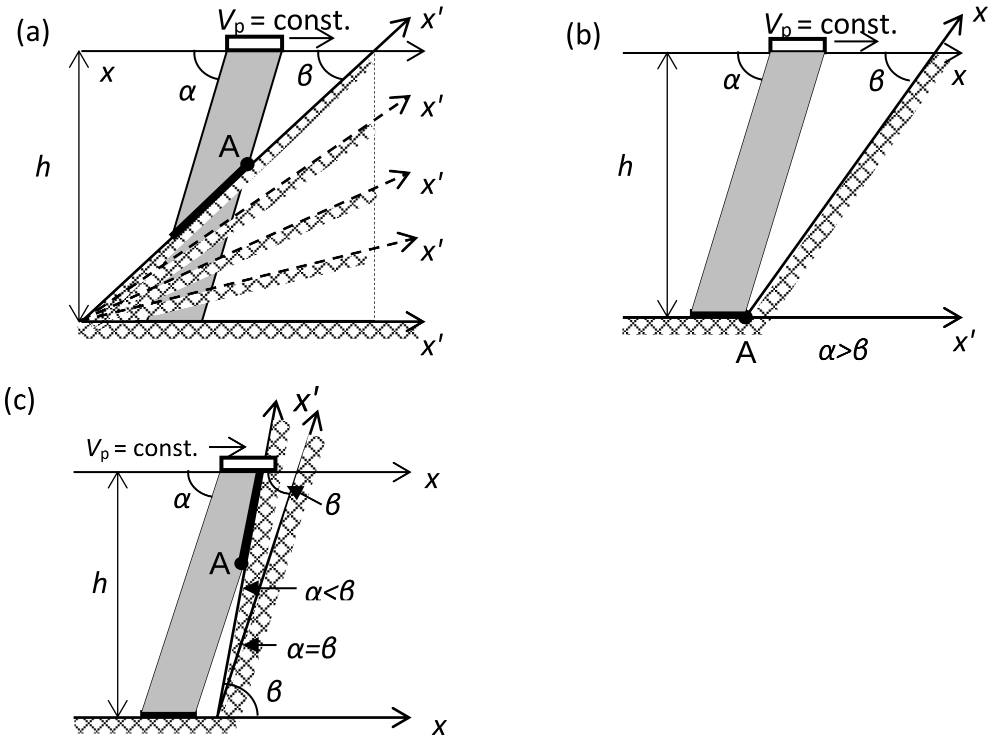


Figure 2. *Shadow on slanted screen. The schematic of a shadow formation on the slanted (a), and partially slanted screen (b) for α>β, c) α=β and α<β.*

An analysis becomes much more interesting if a shadow projection travels along a screen inclined with respect to the object trajectory by an angle β, as illustrated in Fig. 2a, or on a screen that is partially inclined for better effect illustration (Fig. 2b). In order to examine properties of a shadow projection on such a screen we must take into account the size of the shadow projection, l', that is being formed on the screen, $l'=l/\cos\beta$, as well as the angle between the sideline of the shadow and the screen. To put it simple, the closest to the complete description is the analysis of the motion of the point A along the screen given in Fig. 2, where the point A is the cross-section between the shadow sideline and the screen. There are several ways to mathematically relate the velocity of the point A along the screen, v_A, and the angles α and β (i.e., v_p, c, and β). For example, taking into account previously established relationships between the parameters in question:

$$v_A = \frac{1}{\frac{\cos\beta}{v_p} - \frac{\sin\beta}{c}} = \frac{v_p}{\left(\frac{1}{\tan\beta} - \frac{1}{\tan\alpha}\right)\sin\beta} \qquad (1)$$

Note that the length of the screen or other length related geometrical factors do not affect the velocity v_A. It is clear that the angle α ($\mathrm{tg}\alpha = c/v_p$) takes the value in the range between $\pi/2$ ($v_p=0$) and $\pi/4$ ($v_p=c$). However, the screen can have various inclinations as well; namely, the angle β can have any value between 0 and $\pi/2$. The point A's velocity dependence on the angle β is given in Fig. 3.

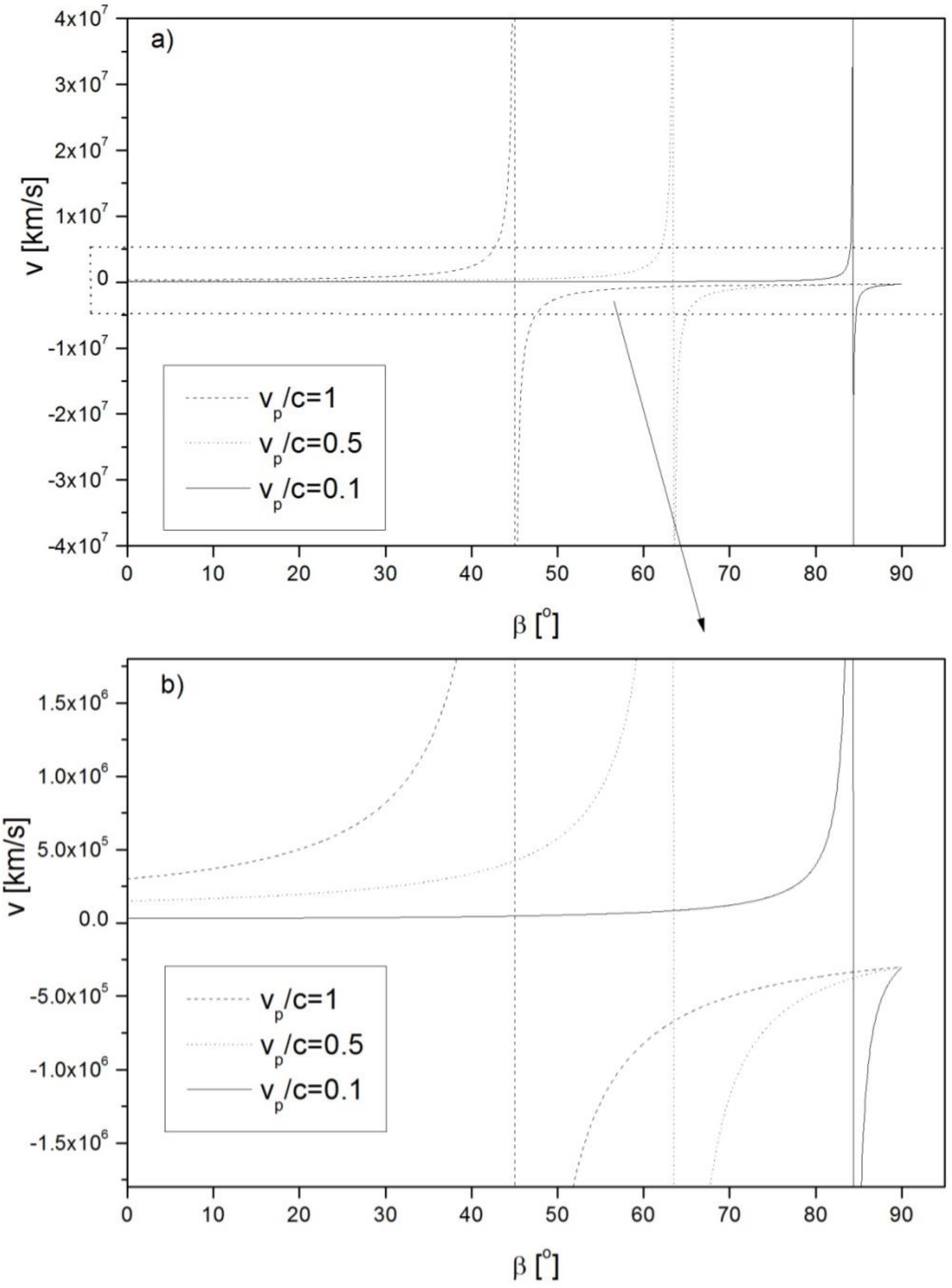


Figure 3. *Shadow velocity. In part (a), the dependence of the point A's velocity on the inclination angle of the screen is given for different ratios between the object's velocity and the speed of light. For better visibility, the characteristic area in part (a) is enlarged and shown in part (b). The vertical lines depict the singularities and are given in colors that correspond to those of the dependences they are related to.*

If $\beta = 0$, then $v_A = v_p$ and that is the case we have already analyzed. If the screen becomes slanted, i.e., when β increases, v_A increases and reaches a singularity, i.e., it becomes infinite for $\beta=\alpha$ (Fig. 2c.). To ease the visualization, in Fig. 3. the singularity is denoted with a vertical line, which has the same color as the change it is related to. On the right-hand side of the singularity $\beta > \alpha$ (Fig. 2c.), the point A begins to move with an infinite velocity but backwards, from the top to the bottom of the inclined screen. Due to the finite velocity of the shadow while it expands from the object to the screen, one of the possible scenarios is that the object had passed the screen before its shadow reached the screen, i.e., there is a delay. Further increase of the angle β causes decrease in the intensity of the shadow projection velocity and for $\beta = \pi/2$ it decreases enough to becomes equal to the speed of light.

This result is strange at first because it contradicts the basic principles of the maximal velocity. However, one should always keep in mind that the calculated velocity does not represent the velocity of a real object. The observed point A, as a part of the shadow projection, does not represent something real, instead it is mass-less, as a shadow itself; therefore, the basic laws of motion cannot be applied to it.

To backup this fact let's take a look at the illumination of a point on the screen, in other words consider the time interval the shadow projection spends at a given point on the screen. It is easy to conclude that it is independent on the angle β, instead it depends only on v_p and l, and it is equal to the time the object needs to pass over a point on the x-axis (as is also the case for $\beta = 0$). This is true for the singularity $\beta = \alpha$, as well.

What is different is the size of the area on the screen that has the information about the presence of the shadow projection l' at a given fixed moment in time. Let's say that using the object's shadow we need to convey the information about the object's motion along the x-axis to every point on the screen. The geometrical parameters of the screen, the length of the object's trajectory, and the object's velocity are now relevant. The point of the exercise is that if $\beta=\alpha$ all the points on the screen at the same moment in time have the information about the motion of the object along the x-axis. The screen slant angle serves as a parameter that can adjust the time delay of the information along the screen, so that in the limiting case, $\beta = \alpha$, all the points on the screen have the information at the same time. Therefore, again, it is not the case of infinitely fast information transfer along the screen, since that is not the direction of information transfer, instead the information is transferred from the object to every point of the screen by the shadow with the finite velocity (in this case equal to c). The similar discussion is applicable for a series of objects traveling along the x-axis.

Problems for Take-home Assignments

The objective of these assignments is for students to reach by themselves the conclusions offered previously in class. It is also very important for engineering students to accept their future career role as those who use laws of physics in solving various problems, as well as those who correctly present a known natural phenomenon. For engineering students, whose courses in physics are of somewhat lower level than those designed for physics students, it is of most importance to relate various knowledge these students acquire. In such a way, physics courses fit better into the primary goal of engineering education.

As a continuation of these concepts the following assignments may be given to engineering students:

1. Modeling of the presented cases with adequate animations. According to my experience, there are two options for this assignment. For engineering students with just basic programming skills the modeling task includes simple reproduction of the lecture material. Namely, a student can illustrate illumination dependence on time for the cases discussed in class. However, if students' programming skills are advanced the problem becomes a nice example how a real physical phenomenon can be implemented into an animation. In such a

case, students can be required to create user interface that provides complete analysis for a chosen input parameters. These assignments are more a programming than a physics exercise; however, in their future carrier most of these students will face problems that require transfer of certain formalism into a software package.

2. Extend shadow property analysis to the case of train of objects of size l_1 separated by l_2, traveling with velocity v_p along an axis inclined with respect to the screen. This can help students in understanding use of light in data transfer. It is of most importance that at the end of the assignment student on his or her own comes to conclude that there is a realistic limit in data transfer, as well as that the limit is the speed of light, not the velocity of an illusion of shadow transfer. This assignment is intended for those engineering students that show high level in understanding physics as well as significant knowledge in programming and graphic design. It is more appropriate for midterm or final papers than for homework assignments.

For physics students, it is sufficient to offer very brief shadow review at the end of the lesson on light, which gives the basic information on speed of light and its travel. It is best to give the dependence of shadow velocity on the screen inclination angle, and provide details only to interested students through mentoring. Of course, the assignments intended primarily for engineering students may be given to physics students; however, physics students are then required to have a significant knowledge in programming and graphic design, usually not offered in their curriculum. If physics student show significant interest for this problem, it is better to combine this assignment with additional theoretical considerations and afterwards only with few selected students extend the activity to animation. Several such assignments for physics students may be:

1. *Considering a variable velocity of the object. This task requires from a student to comprehend direct relationship between the object's velocity and the shape of its shadow.*
2. *Given the shape of the screen, determine the object's velocity that provides the "optimal" shadow. This is an inverse assignment, which asks the students to choose the velocity of object according to the screen shape so that the shadow completely covers the screen. As a continuation of this assignment the complete assignment 1 for engineering students could be given.*
3. *Discuss in class just the non-inclined screen case and the complete analysis of inclined screen case leave for the take-home assignment.*
4. *Leave the mathematically obtained velocity values larger than the speed of light for the students to think about and resolve before it is discussed in class.*

CONCLUSION

Studies in engineering require from students to apply theoretical knowledge in general, therefore, knowledge in physics as well. Mentoring that follows a particular lesson is may be the most important for forming a good engineer. This work illustrates how an inadequate choice of parameters (in this case v_A) may lead to the wrong conclusions in an analysis of the nature of light and its accompanying effects such as the shadow. Drawing the conclusions about the nature of a shadow based solely on the perception may lead to serious misconceptions. A shadow is a stationary effect and it does not move. The illusion of its motion is made of successive independent images that occur one after another. However, a comprehension of a misconception that uses the clear example of a shadow is of great use in observing other effects in diverse fields from astronomy, through informatics, to an everyday life.

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