Electromagnetic Induction from a New Perspective

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Abstract
The well-known historical development of Electromagnetism, strongly influenced by the work of Faraday and Maxwell, has led to the introduction of the magnetic field as an important component to explain the diverse phenomena of electromagnetic induction. This historical development is compared in the form of a thought experiment with a possible different course in which the work of Ampere and Weber would have influenced the former development with far-reaching consequences. Two examples are given to show how Weber´s work can be applied to explain the phenomenon of self-induction and the interaction between parallel current carrying conductors. The source for further information to explain different induction phenomena like mutual induction, motional emf and unipolar induction is indicated.

Keywords: Electromagnetic induction, Weber´s law of force, Newton´s 3rd law, mutual induction, interaction between parallel currents.

THE HISTORY OF ELECTROMAGNETIC INDUCTION

If one looks at chapters on „Electromagnetic Induction" in common textbooks, one finds - if anything - a largely uniform representation of the historical development of this branch of physics. This story began in 1819, when Oersted observed, it is told by chance, that a magnetic needle was deflected by an electric current. This made it clear that there is a connection between the area of natural magnetism, well known for many centuries, and the then new phenomenon of the electric current. This finding aroused great interest among experts at that time and prompted Faraday in particular, to investigate the connection. In the following year he published his first findings, (Faraday, 1821) which finally led to a result, known today as "Faraday's Law" (Faraday,1832). In American textbooks one can read that at the same time the same results were found by the American scientist Joseph Henry (1797-1878).

The development was formalized when, in 1864, Maxwell set up his famous "Maxwell's equations", predicting the existence of wave propagation in space; these were confirmed by the experiments of Heinrich Hertz. Added to this were other results such as the Biot-Savart law, Amperes law and the Lorentz force. The corresponding equations are well known and therefore need not be listed here.

A feature that all these laws and equations have in common is that the field concept is accepted as fundamental and consequently the previously prevailing action-at-a-distance theories were discredited. In all these laws the magnetic field or the magnetic flux takes the role as the crucial partner in the interaction with electrical phenomena, such as moving charge carriers, or electric fields.

A Possibly Different Story - As A "Thought Experiment"

If it is true that Oersted made his discovery accidentally, then on this basis it seems permissible to imagine another course of historical development as a thought experiment. As it turns out, this could have had far-reaching consequences.

In this other story it must first be assumed that Oersted did not make his discovery in 1819. Perhaps the magnetic needle was located a little further from the circuit in question and therefore Faraday would not have been encouraged to investigate this new phenomenon.

In addition to Faraday, the work of French physicist Ampère was also strongly influenced by Oersted´s discovery, so these influences also would have been absent. Here, the new, imaginary story is continued, in which it is to be assumed that Ampère, for whatever reason, would have carried out his experiments at that time without Oersted's discoveries. As early as 1820, Ampere discovered that two parallel current-carrying conductors interact with each other, attracting or repelling depending on the direction of the current (Ampère, 1820). Ampère interpreted this interaction and all the other time independent phenomena discovered by Faraday as an interaction between infinitely small current elements.

He succeeded in formulating a law - the original Ampère´s law - on the basis of his experiments and using various newly developed measuring devices to study this interaction. This law allowed him to make quantitative statements about the interaction between two dc current elements at any orientation in space (Ampère, 1822). His law of force has a more complicated form than, for example, the Law of Gravity or the Coulomb Law, since it contains the relation of three angles arbitrarily arranged in space. This law was relatively well received at the time, as evidenced by the following quotation from a statement by Maxwell (Maxwell, 1954).

> *"The experimental investigation by which Ampère established the law of the mechanical action between electric currents is one of the most brilliant achievements in science. The whole, theory and experiment, seems as if it had leaped, full grown and full armed, from the brain of the 'Newton of Electricity'. It is perfect in form, and unassailable in accuracy, and it is summed up in a formula from which all the phenomena may be deduced, and which must always remain the cardinal formula of electrodynamics."*

Two points are important in assessing Ampère's work. On the one hand, Ampère insisted that all forces occurring in nature must be governed by Newton's principle of "action equals reaction" in its strict form; that there can be only attractive and repulsive forces, whose line of action coincides with the line connecting the interacting partners.

On the other hand, Ampère assumed that all phenomena of natural magnetism were due to the interaction between electric currents. To explain permanent magnetism, he postulated the existence of microphysical currents inside the magnetic material, and for the explanation of terrestrial magnetism he hypothesized that there should exist an electric current in the interior of the Earth (Ampère, 1822).

Other former physicists expressed similarly positive views of Ampère's work like those of Maxwell. It is therefore permissible to suppose that the basic ideas of Ampère would have determined the course of further development, namely the phenomena of induction as an interaction between electric currents, and for which Ampère introduced the name "electrodynamics".

What Really Happened

The development took a different course. Oersted made his discovery first, Faraday followed with the law of induction and so the idea crystallized into what was characterized as "electromagnetism": The magnetic field became an important partner of all electromagnetic interactions, often without mentioning that usually moving charge carriers are the cause of a magnetic field. And for the forces that occur, the principle "action equals reaction" applies only in its soft form. In terms of amount, strength and counter force they are the same, but they are no longer rectified.

In 1846 Wilhelm Weber presented his force law (Weber, 1846). His starting point was Faraday's law and Ampère's law. These appeared unconnected, but Weber suspected that they had to be based on a common fundamental law of electrodynamics.

He developed an impressive measuring device - a precision mechanical masterpiece - with the help of which he was able to determine with great precision the interaction of two suspended circuits, rotatable about the same axis. And since Weber was not only a great experimenter, but also an equally great theoretician (he was an assistant to Gauss), he succeeded in deriving the presumed fundamental law from his measurements.

This law is an extension of Coulomb´s Law, and that means first of all, that as in electrostatics the Newtonian action / reaction principle applies in its strict form: the forces between interacting partners are not only of equal size but act exclusively in the direction of the interacting partners.

To Coulomb's law are added two new terms: the first contains the factor $-v^2/c^2$, the second the factor $+a/c^2$. Weber's Fundamental Law describes the mutual force $F_{1\text{->}2}$ and $F_{2\text{->}1}$ between two charge carriers q_1 and q_2 at their mutual distance r_{12} and reads as follows:

$$\vec{F}_{1>2} = \frac{q_1 q_2 r^0_{12}}{4\pi\varepsilon_0 r^2_{12}}\left(1 - \frac{\vec{v}^2_{12}}{2c^2} + \frac{\vec{r}_{12}\cdot\vec{a}_{12}}{c^2}\right) = -\vec{F}_{2>1}$$

$F_{1\text{->}2}$ means the force from particle 1 acting on particle 2 and accordingly for $F_{2\text{->}1}$

The terms v_{12} (dr/dt) and a_{12} (d^2r/dt^2) denote the relative velocity and the relative acceleration between the interacting partners. The term r^0_{12} denotes the unit vector for the distance between the interacting partners. The constant c, first introduced by Weber, was later experimentally determined by him together with Kohlrausch as being identical in physical dimension and size with the speed of light (Weber, Kohlrausch, 1857). As the constant c is so large, all changes to the Coulomb force caused by these two new terms in Weber´s equation are very small. This is in line with the fact that magnetic forces are much smaller than Coulomb forces.

A frequent objection to Weber´s law, which was raised very early on, concerns the question of a distinction between an action-at-a distance theory and a field theory. After the publication of Maxwell's equations and the experiments of Hertz it was known that waves are possible in space and a theory of proximity was used to describe the change of electrical quantities as continuously propagating in space and time. Weber´s force law is in this sense an action-at-a-distance-law. It makes no statements about how a change propagates in space, how, for instance, the equality of $F_{1>2}$ and $F_{2>1}$ is achieved.

However, when using this law, one does not have to presuppose that in the thoughts of Weber this equality is reached in zero time and at infinite speed. On the contrary, Weber and Kirchhoff considered independently of each other a possible change in voltage/current along a conductor and on the basis of Weber's law of force they derived the equation known today as the telegraph equation (Kirchhoff, 1857). Weber predicted that a voltage/current change would propagate along a zero-resistance conductor at the speed of light.

From a transmitted voltage change along a conductor with R=0, it is not far to wave propagation in space. However, at the time of Weber the idea of an aether was universally presupposed and there were no certainties about the properties of such an aether. In his first large publication of 1846, Weber showed how, using his law, it was possible to derive Faraday's law and Ampère's law. This was confirmed by Maxwell in the last chapter of his book (Maxwell. 1954).

In the following two examples, we show how mutual induction and the interaction between parallel current carrying conductors can be derived from Weber’s force law.

WEBERS LAW AND THE PHENOMENON OF MUTUAL INDUCTION

We consider two sections of two separate closed circuits, a primary circuit P and a secondary circuit S, with the same number of positive lattice units and free electrons per unit-length (fig.1). For the sake of simplicity, it is assumed that by applying an external voltage to the circuit P, all free electrons of this circuit P are uniformly accelerated from rest.

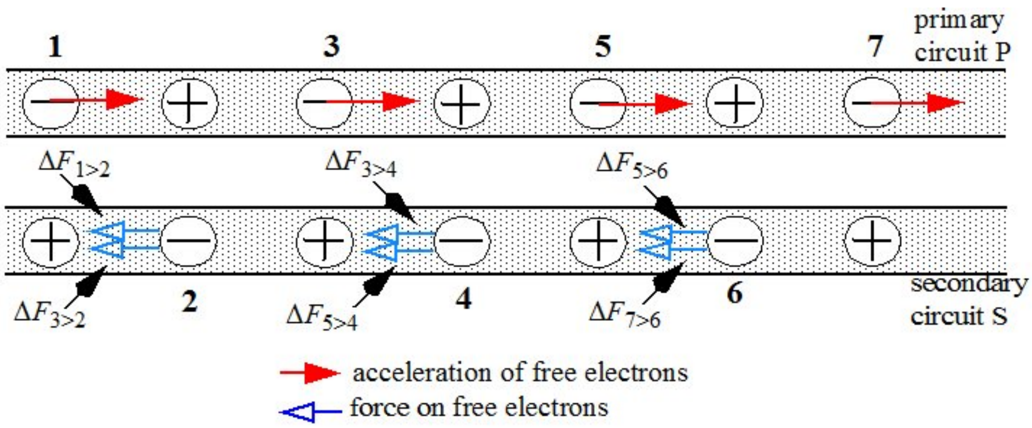

Figure 1. *Section of 2 separate conductors with accelerated electrons in P (see text)*

We consider now the interaction between the accelerated electron 1 of circuit P and the initially stationary electron 2 of circuit S. The acceleration, as seen from electron 2 in S, is negative (the distance is reduced). If Weber´s equation is applied to this situation with v_{rel}= 0, only the acceleration term has to be considered. Therefore, Weber's equation results in a reduction of the repulsive interaction between electrons 1 and 2.

Since the interaction with all neighboring conductor elements is not changed, this reduction means an accelerating force = $\Delta F_{1\text{-->}2}$ against the direction of the accelerating electrons in the primary circuit.

If we now consider the interaction between the accelerated electron 3 in the primary circuit and the initially stationary electron 2 in the secondary circuit, then this acceleration is positive as seen from 2 (the distance increases) and, according to Weber, this results in an increase of the repulsive force between 3 and 2 = $\Delta F_{3\text{-->}2}$. The same considerations can be applied to the interactions as indicated in figure 1 between the accelerated electrons 3, 5 and 7, and the initially stationary electrons 4 and 6 respectively, and so on to all electrons in the secondary circuit S. For all those electrons, these two changed interactions add up to an accelerating force against the direction of the developing flow in the primary circuit, which means an induced current in the secondary circuit.

For proof of a quantitative agreement with the experimental fact in accord with these first qualitative considerations, reference should be made to the literature (Assis, 1994).

Weber's Law and the Interaction between Parallel DC-Currents

In traditional courses the interaction between parallel currents is explained on the basis of magnetic field lines and the magnetic force (~ vxB). The result can be stated as: Parallel current-carrying conductors attract each other if the currents flow in the same direction and repel each other if the currents are anti-parallel.

When starting with Weber´s force law, the question to be answered is: Which relative velocities and relative accelerations occur between two constant DC currents (1) and (2), flowing in parallel or anti-parallel directions?

In the laboratory system, the answer is: there are only constant drift velocities and there are no accelerations. When looking for relative terms the answer is different, and this answer can be found using graphical means.

An acceleration is found graphically by first taking as vectors the velocities at two closely spaced points (1) and (2) of a trajectory with corresponding times t_1 and t_2. These vectors are then shifted to a common starting point in the middle between the two points (1) and (2). Their difference Δv corresponds to the mean acceleration with respect to the time interval between t_1 and t_2. Figure 1 shows this procedure, which is often used in class with the example of a circular motion.

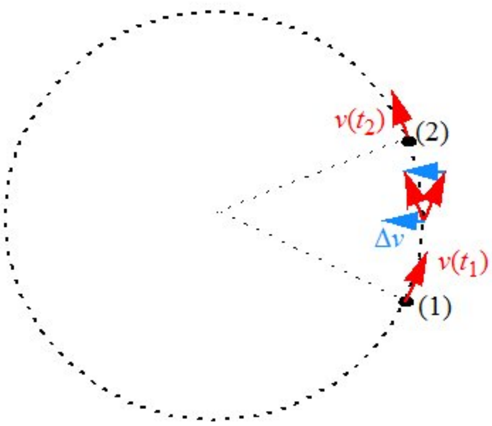

Figure 2. *Graphical method for determining an acceleration (using the example of a circular motion)*

To apply this method to two parallel DC currents (1) and (2) we can for the sake of simplicity assume that the drift velocities of the electrons in both conductors are the same and that the length of the conductors is infinite.

Seen from a chosen element A of conductor 1 (fig.3) all drifting electrons of conductor (2) are at rest, while all the positive charge carriers (lattice components) in this conductor are moving in the opposite direction to the current. Thus, in this case, only the interaction between the negative part of conductor (1) and the positive part of conductor (2) has to be considered.

Figure 3 shows the constant drift velocities of the positive elements of conductor 2 (relative to the laboratory) as seen from A and corresponding relative velocities (relative to A).

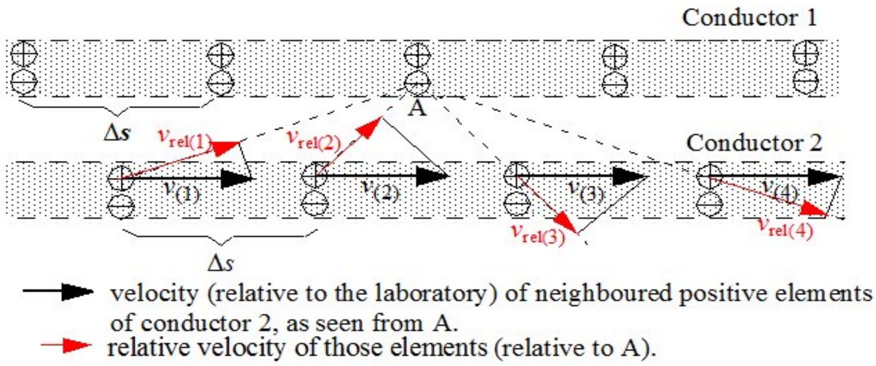

***Figure 3**. Drift velocities and relative velocities of two parallel current carrying conductors (see text).*

Figure 4 shows the result of applying the graphical method, described above, to determine the change of the relative velocity of the selected positive elements, while moving along the distance Δs during a constant time interval Δt.

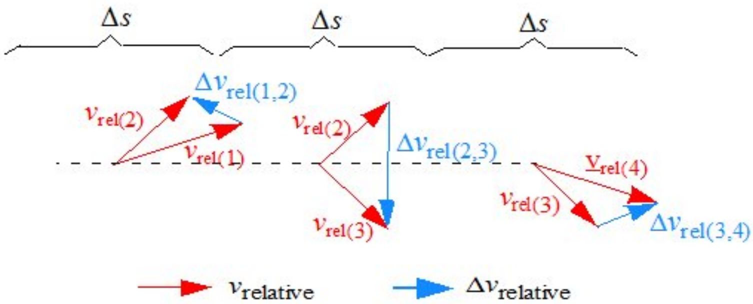

***Figure 4**. Graphical determination of the Δv_{rel} between point A of conductor(1) and four selected elements of conductor (2) while moving along the distance Δs.*

To determine the relative acceleration from these graphically obtained Δv, one must take note of the sign of the velocities. In Weber´s equation the direction from particle 1 to particle 2 is defined as positive. It follows that if particle (2) is moving towards particle (1), the distance is decreasing, and its relative velocity is negative. If it moves in the opposite direction with increasing distance, its relative velocity is positive. Another point not familiar in a laboratory system should be noted. Since in Weber´s equation the relative velocities are defined as dr/dt, only a change in distance is relevant.

A change of the direction of a velocity without changing the distance is irrelevant. Applying this result to the different Δv of figure 4 it follows that for all cases Δv is positive (see fig.5).

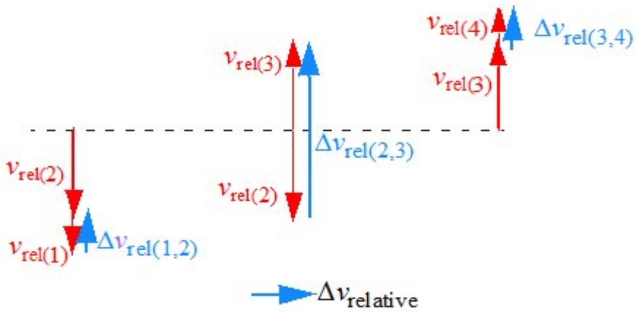

***Figure 5**. Determination of Δv_{rel} respecting the sign of v_{rel}*

These consideration can be applied to all negative elements of conductor (1). A positive Δv for a certain Δt means a positive acceleration. From Weber´s equation it follows that a positive acceleration term gives rise to an increase of the interaction between the negative and the positive elements of the two parallel conductors.

In addition to the acceleration term, the velocity term in Weber´s equation with $-v^2/c^2$ must also be considered. To do so, mathematical tools must be used to integrate over all velocity terms and accelerations terms, assuming conductors of infinite length. This calculation has been done with the result that the positive acceleration term dominates the negative velocity term by a factor 3[1]. Due to this dominant positive acceleration term, and since the other interactions remain unchanged, it follows, as expected, from Weber's equation that an attracting force exists between two conductors with currents drifting in the same parallel direction.

In case of anti-parallel currents, the positive and the negative parts of the conductor (2) move with different relative velocities, as seen from the element A of conductor (1). Therefore, the considerations as displayed in figures 3, 4 and 5 must be doubled: firstly, for the interaction between the free electrons of conductor (1) and the positive lattice elements of conductor (2) and secondly for the interaction between the free electrons of conductor (1) and the free electrons of conductor (2). The former will lead to an increased attraction, the latter will cause an increased repulsion. Due to the higher relative velocity between the electrons in both conductors, the latter will dominate, resulting, as expected, in a repulsive force between two conductors with currents drifting in opposite directions.

The same qualitative consideration as shown above can be successfully applied on the basis of Weber´s equation to all well-known phenomena, such as self-induction, motional emf, Faraday´s paradox and unipolar induction. (*How this is done in detail is available at: http://www.astrophysik.uni-kiel.de/~hhaertel/PUB/induction-alternative.pdf*)

DISCUSSION

Is Faraday's law a universal law or are there constellations in which it does not apply, but in which only the Lorentz force applies? There is currently no scientific consensus on this question. There is agreement, however, that there is no alternative to Faraday's law and the Lorentz force. As can be seen from the treatment of this subject in all available textbooks, these seem to be established facts or laws of nature which need not be further questioned.

But now there is an alternative that was introduced over 150 years ago by Wilhelm Weber but has fallen into complete oblivion.

Does this fact have consequences for teaching? Hardly; the curricula and examination regulations leave little room here. But what could change is the "mind" in which these topics are treated.

Perhaps Faraday's law is not an unquestionable law of nature, but is only a rule that provides more or less amazingly correct results, but nobody knows why?

Perhaps the Lorentz force does not describe a process that occurs in nature exactly the same way, but is also just a rule that, because it works so perfectly, you can only marvel at, but nobody knows why it applies?

[1] Privat communication by Ernesto Martin

Such a new "mind" could prevent students from seeking a deeper understanding of the phenomenon of "induction", and could, in the event of a likely failure, prevent them from losing interest in further learning.

REFERENCES

Ampère, A.M. (1820). *Annales de Chimie et de Physique*, 15, pp. 59-76.

Ampère, A.M. 1822. Expériences relatives à de nouveaux phénomènes èlectro-dynamiques. *Annales de Chimie et de Physique* (20), pp. 60–74.

Assis, A.K.T. (1994). *Weber's Electrodynamics*, Kluwer Academic Publishers, Dordrecht.

Faraday, M. (1821). On some new electromagnetic motors, and on the theory of magnetism. *The Quarterly Journal of Science, Literature, and the Arts,* 12, pp. 74-96.

Faraday, M. (1832). Experimental researches in electricity. *Philosophical Transactions of the Royal Society of London*, 122, pp. 125-162.

Maxwell, J.C.A. (1954). *Treatise on Electricity and Magnetism*. Dover, New York, 2, ar. 528, pp. 175.

Weber, W. (1846). Elektrodynamische Maassbestimmungen, Über ein allgemeines Grundgesetz der elektrischen Wirkung. Abhandlungen bei Begründung der Königl. Sächs. Gesellschaft der Wissenschaften am Tage der zweihundertjährigen Geburtstagfeier Leibnizen's herausgegeben von der Fürstl. Jablonowskischen Gesellschaft (Leipzig), pp. 211–378.

Weber, W. & Kohlrausch, R. (1856). *Annalen der Physik*, 99, pp. 10-25.