A Forgotten Controversy in Basic Classical Mechanics: The Difference Between the Energy Equation and The Center - of - Mass Equation

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Abstract
In this paper, we indicate the difference between energy equation and the center of mass equation by an especially example. According to our opinion, in teaching mechanics, we should more clearly an integral of Newton's second law and the energy equation. Maybe this leads to greater clarity in the notions of system, work and energy.
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Introduction

One of the forgotten concepts in basic classical mechanics is the difference between the energy equation and the center of mass equation. This subject leads to informal concept called to "Pseudowork" (Sherwood, 1984; Sherwood, 1977). This story is absent in mechanics' textbooks. The differences between the center of mass equation and the work-energy equation can be appreciated by writing both equations for a variety of situations, usually such differences can arise for deformable systems and also for rotating rigid systems.

Some people in their reports, have pointed out that we frequently do not properly distinguish between the work-energy equation of mechanics and a particular integral of Newton`s second law (Erlichson, 1963; Peachina, 1978). In fact, in teaching mechanics, we should more clearly distinguish between them and this leads to greater clarity in the notions of system, work, and energy (Mungan, 2017).

Take the second law for a system of particles,

$$\sum F_{i\,external} = Ma_{cm}$$

And integrate through a displacement of the center of mass point (interchanging summation and integration):

$$\int (\textstyle\sum F_{i\,external}).dr_{cm} = \int m\frac{dv_{cm}}{dt}.dr_{cm}$$

$$\textstyle\sum(\int F_{i\,external}.dr_{cm}) = \Delta\left(\frac{1}{2}mv_{cm}{}^{2}\right) \qquad (1)$$

The term on the left-hand side is the total "pseudowork" (Peachina, 1978). It is not equal to the total real work done on the system, because the forces have been multiplied by the center of mass displacement rather than by their individual displacements. The right-hand side of the equation is not in general equal to the kinetic energy change of the system, since it involves only the center of mass speed. The displacement dr_i of the point of application of the i^{th} force is not necessarily equal to the displacement dr_{cm} of the center of mass point, so that for the ith force,

$$\int F_{i\,external} \cdot dr_{cm} \neq \int F_{i\,external} \cdot dr_i \qquad (2)$$

In section IV, we investigate the distinction between dynamical equilibrium point and maximum height point against a conservative force.

Illustrative Example

Suppose a rod with mass m and length L that locating in perpendicular situation according to following figure. The rod released from this state and our goal is calculation of speed of center of mass of rod while collide to earth.

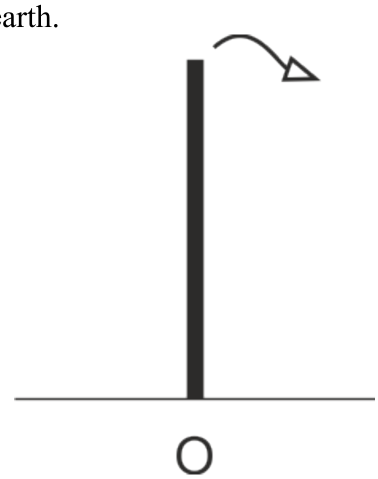

***Figure 1**. A rod with mass m and length L that released from rest.*

According to energy conservation can be writing

$$\Delta U = -\Delta K \qquad (3)$$

Where, ΔU is the change of potential energy and ΔK is the change of kinetic energy of the system and so

$$\frac{mgl}{2} = \frac{1}{2} I_o \omega^2 \qquad (4)$$

Where, I_0 is the rotational inertia around a point of O. Here $I_o = \frac{1}{3} ml^2$ and ω is the angular speed of the rod. Then speed of the center of mass is equal to

$$v_{cm} = \frac{l}{2}\omega = \frac{1}{2}\sqrt{3gl} \qquad (5)$$

If we concentrate on the concept of the system them we can consider whole of the system as a system so that it has pure rotational motion around point of O. Now we write the equation of the center of mass based on the equation (1),

$$\frac{mgl}{2} = \frac{1}{2} I_{Cm}\omega^2 + \frac{1}{2} mv_{Cm}^2 \qquad (6)$$

Here, rotational inertia is equal to,$I_{cm} = \frac{1}{12}\, ml^2$ so $v_{cm} = \frac{1}{2}\sqrt{3gl}$ which consistent to equation (5). For interpretation of equation (6), one can say that the point of center of mass is system, clearly this point has rotation and transmission too, which the right-hand side of equation (6) verify it.

To more clarity, we obtain speed of another point on the rod. By the same conditions, our aim is yielding speed of point A.

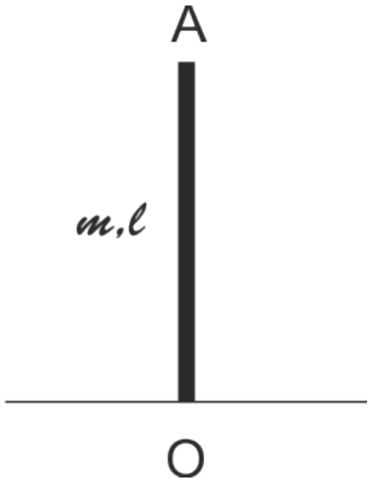

Figure 2. *A rod with mass m and length l which released at rest and we are going to obtain speed of A while the rod reach on the surface of earth.*

We use relation (3) again and yielding

$$v_A = l\, w = \sqrt{3gl} \qquad (7)$$

For the point of A as a system, equation (1) can be

$$\int (\sum F_{i\, external}).\ dr_A = \Delta\left(\frac{1}{2}\, m\, v_A^2\right) + \Delta\left(\frac{1}{2}\, I_o\, \omega^2\right) \qquad (8)$$

Not that in equation (8). Rotational inertial is around point of o rather than A. The left-hand side of above equation equals to

$$\int (\sum F_{i\, external}).\ dr_A = mgl + mgl \qquad (9)$$

In right-hand side of equation (9), the first part is related to rotation motion and second is transmission motion.

Therefore, final equation is equal to

$$mg(2l) = \frac{1}{2}\, I_o\, \omega^2 + \frac{1}{2}\, m\, v_A^2 \qquad (10)$$

So,

$$v_A = \sqrt{3gl}$$

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