Using Teaching Through Problem Solving to Transform In-Service Teachers' Thinking about Instruction

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Many teachers' experiences in problem-based classrooms are limited, therefore, mathematics teacher educators face a significant challenge when trying to inspire in-service teachers (ISTs) to teach through problem solving. To address this challenge, the professor in a mathematics education course designed and implemented an instructional model using the following principles as a guide; modelling problem-based instruction, creating opportunities for reflection, and building a discourse community. Assessment results showed that ISTs were better able to create problem-based lessons after going through the instructional model. Additionally, an analysis of writing samples revealed several changes in ISTs' thinking. Most notably, ISTs discussed a new role for communication in the classroom and emphasised the need for students to develop their own solution strategies. Their writing also provided evidence that participation in problem-based learning and critical reflection during the instructional model were key factors that led ISTs to think differently about instruction.

Keywords problem solving · in-service teacher education · reflective writing

Introduction

Mathematics educators have long promoted reforms for teaching mathematics focused on problem solving (Boaler, 2016; Common Core State Standards Initiative (CCSSI), 2010; National Council of Teachers of Mathematics (NCTM), 2000). Yet, teacher educators face a difficult challenge when trying to inspire in-service teachers (IST) to use a problem-based approach to instruction (Han, Yalvac, Capraro & Capraro, 2015; Lloyd, 2002). Historically, in the United States, the teacher's role in the mathematics classroom largely involves the explanation of mathematical procedures, while the students' role is primarily to listen and apply new procedures to similar problems (Beswick, 2012; Hiebert & Grouws, 2007; Stigler & Hiebert, 1999; Umbeck, 2011). The notion that explanation comes before problem solving is deeply rooted in many of the curriculum materials used in schools and is commonly found in the opinions of school administrators, parents, and students (Handal, 2003; Inglis & Miller, 2011; Pilgrim, 2013). Without a core belief that learning mathematics is a process that best occurs through solving genuine problems, matched with skills in planning and delivering problem-based lessons, ISTs often maintain the status quo by using a traditional approach to instruction.

When using the term teaching through problem solving or problem-based instruction, I am referring to a method of instruction where students learn mathematics while solving genuine tasks. A genuine task is one in which the solution method is not known in advance and one in which students do not perceive there is a single, correct solution path (Hiebert et al. 1997; Jonassen 2000; NCTM 2000). In other words, in a genuine task there is something for students to figure out, to grapple with, to explore. As students struggle to find a solution, and later discuss various methods for solving a task, they build procedural and conceptual understanding about mathematics (Cai, 2003). In addition, students gain a sense of pride and satisfaction from solving a task for which they were not told a solution path (Stylianides & Stylianides, 2014). Through this process, students are "doing mathematics" while simultaneously developing mathematical understanding. Traditional instruction and problem-based instruction are guided by two distinct ways of thinking about learning. In the first, lessons generally begin with explanation and end with pseudo-problem solving. In the second, lessons begin with problem solving and end with explanations, often ones provided by the students themselves (Marshall & Horton, 2011). Making a transition from a traditional approach to envision learning as the result of problem solving, requires, as Hiebert et al. (1997, p. 22) claim, "a fundamental change in our perceptions of teaching." Therefore, at the core of mathematics education reforms should be an emphasis on changing how ISTs view the role of problems in the classroom.

In my teaching prior to this study, the focus of my IST mathematics education courses was on building mathematical understanding through problem solving. I did not, however, make an explicit attempt to challenge ISTs' thinking about selecting tasks and guiding instruction in a problem-based classroom. I noticed a pattern where ISTs would say they were learning from and enjoying the problem solving we engaged in during class, but they were resistant to try this form of teaching in their own classrooms because they did not believe their students could solve a problem without first being shown how to solve it. Their resistance was influenced by years of working in schools that promote the gradual release of responsibility framework (Fisher & Frey, 2013), commonly referred to as I do, we do, they do, and you do. This framework is in direct conflict with the ideals of teaching through problem solving because each lesson begins with the teacher demonstrating to the class a set procedure for solving a given problem. Because there is an inherent conflict between this approach and teaching through problem solving, I realised that just engaging in problem-based instruction was not enough. I needed to inspire a change in thinking about how mathematics should be taught before ISTs would consider implementing teaching through problem solving in their own classrooms.

The current study was designed to motivate ISTs to think about learning in mathematics as a process that occurs through problem solving. As Cai (2010) argues, the problem-based teacher has two main roles, selecting genuine tasks and guiding classroom discourse. Recent research has emphasised the need to focus on problem posing in mathematics teacher education (Ball & Forzani, 2009; Cai, Hwang, Jiang, & Silber, 2015; Crespo, 2003). Many teachers are not used to posing problems and have not developed the necessary skills to create genuine tasks for their students to investigate (Singer & Voica, 2013). Therefore, as a part of the current study, I designed an instructional model to help ISTs create genuine tasks and use them successfully in the classroom. In this model, ISTs reflected deeply on what makes a given task effective, created a rubric for evaluating the effectiveness of a given task, and used this rubric as they constructed tasks to use in the classroom.

The instructional model was implemented with a group of ISTs enrolled in a summer graduate-level course in mathematics education. The effectiveness of the model was examined by analysing pre- and post-assessment results measuring ISTs' ability to construct a genuine task and to develop a plan for teaching with that task. Additionally, ISTs were asked to record their thinking in journals to document the ways their thinking changed during the implementation of the instructional model. This writing was later analysed to determine patterns in how ISTs' thinking changed and to understand why ISTs' believed the instructional model was successful in facilitating changes in their thinking. The work in this study was guided by the following research questions:

1. Do ISTs design problems that are more genuine after working through the instructional model?
2. What new thinking about effective problems emerged during the instructional model?
3. What elements of the instructional model facilitated changes in ISTs' thinking?

Supporting a Change in ISTs' Thinking

Past research seeking to investigate changes in mathematics teachers' thinking, beliefs, and practices have identified three important principles that help produce change: (1) modelling the desired instructional methods, (2) building opportunities for teacher reflection, and (3) creating an effective discourse community (Artzt, Armour-Thomas, & Gurl, 2015; Putnam & Borko, 2000; Richardson & Anders, 1994; Wilson & Cooney, 2002). These three principles were used as a framework for guiding the design of the instructional model used in this study. The paragraphs below describe the research supporting the use of these three principles and briefly explain how the principles were used in this study.

The first principle guiding this work entails that teacher educators model the instructional techniques they are suggesting ISTs use with their own students (Bailey & Taylor, 2015; Putnam & Borko, 2000). Goldsmith & Schifter (1997, p. 25) argue "Teachers seeking to change their practice may not have useful images from their personal experience to guide the creation of a focused and productive classroom culture." Since most ISTs were students in traditional classrooms and teach in schools dominated by the same type of instruction, they may not have an accurate vision of what problem-based learning looks like. In fact, ISTs may not have considered that learning can begin with problem solving or they may believe that assigning word problems after describing a possible solution method is an example of teaching through problem solving (Chapman, 1999). Therefore, it is necessary to provide a model of what problem-based learning is before expecting ISTs to create a vision for teaching and learning that is centred on problem solving.

In a study investigating the success of a professional development program, Borasi, Fonzi, Smith, and Rose (1999) found an essential factor leading to changes in teachers' use of reform lessons was that the participants experienced the innovative lessons before using them as teachers. Likewise, Hart (2002) examined the reasons why the Atlanta Math Project lead to changes in teachers' practices and found that 90% of the teachers said that modelling of the strategies was an important factor in creating change. Taken together, these studies suggest that modelling the desired instructional methods is a necessary component needed to create change. In the current study, a problem-based approach to instruction is used throughout the instructional model, not only when ISTs are learning about mathematics content, but also as they are learning about posing problems and teaching through problem solving.

Simply modelling desired instructional methods is likely not enough to promote change without also creating opportunities for ISTs to critically reflect on these new experiences (Franke, Carpenter, Levi, & Fennema, 2001). After conducting an extensive review of the literature on teachers' beliefs, Wilson and Cooney (2002, p. 142) noted that reflection was a prominent theme throughout the literature. As many research studies have documented, reflection is closely related to learning (Artzt, Armour-Thomas, & Gurl, 2015; Borasi et al., 1999). For example, Artzt (1999) used structured writing assignments to assist preservice teachers in reflecting before and after teaching. She found that the reflective writing provided individuals with the opportunity to question their teaching and to seek improvements. The current study also uses writing to assist in the reflection process by asking ISTs to document how their thinking is changing throughout the course.

When teachers reflect on their experiences in writing their opportunities for learning increase (Bangert-Drowns, Hurley, & Wilkinson, 2004; Cooney & Shealy, 1997; Sanchez & Lewis, 2013). As the National Commission on Writing Report (2003, p. 13) states, "Writing is not simply a way for students to demonstrate what they know. It is a way to help them understand what they know. At its best, writing is learning." Requiring ISTs in this study to use writing promoted learning by providing them with the opportunity to reflect on their experiences, document changes in their thinking, and create a framework for their future approach to instruction. Not only is writing a helpful tool in learning, it also allows the researcher to hear directly from the participants about the changes that are taking place. As Hart (2002) explains, researchers must seek to understand from the teachers' perspective, how they have changed and what factors impacted or deterred that change. The writing completed in this study was used for these same purposes.

Lastly, researchers have documented the important role the discourse community can play in changing teachers' thinking and ultimately their beliefs and practices (Richardson & Anders, 1994; Wineburg & Grossman, 1998). A successful discourse community draws on the diverse expertise of its members to develop rich conversations and new insights about teaching and learning (Putnam & Borko, 2000). Borko (2004, p. 7) emphasises the need to create a "supportive yet challenging" environment where ISTs critically analyse issues in their teaching. To make this possible, an environment must be created where individuals feel free to take the risks required to build new understandings (McLaughlin & Talbert, 1993). Richardson (1992) points out there is an inherent struggle for the leader in such settings, as they desire to change participants' beliefs in particular ways, while also wanting the participants to be meaningfully involved in shaping the change. He argues that at times the teacher educator focuses so much on empowering teachers that no new pedagogical ideas are explored. To address this concern, past studies have suggested creating a balance between these two extremes (Putnam & Borko, 2000). It was with this same lens that the instructional model in this study was developed, paying attention to creating specific changes in ISTs' thinking, while also allowing them to have a meaningful role in creating the change.

The Present Study

The participants in this study were seventeen ISTs enrolled in a graduate level mathematics education course about teaching algebra in K-12 classrooms. The author was the instructor of the course, which encompassed twelve three and a half hour meetings during the summer term. All ISTs in the course were given the opportunity to participate in the study and all accepted. The ISTs ranged from teaching kindergarten to twelfth grade, and the sample included three males and fourteen females. The algebra course is generally the first in a series of five mathematics education courses taken by ISTs seeking a mathematics education teaching certificate or a master's degree in mathematics education (additional courses beyond the five are needed to complete the master's degree). The main goal of these five courses is to prepare ISTs to teach successfully through problem solving.

The Instructional Model

The implementation of the instructional model was based on the following essential questions:

1. How do you know if a given task is an effective one?
2. What is the teachers' role in supporting learning in a classroom using problem-based instruction?

The instructional methods used in the class were consistent with the principles of problem-based learning such that ISTs were not presented with solutions to the essential questions. Instead, these questions were posed as genuine tasks and ISTs used their prior knowledge and experience, as well as each other, to construct solutions. As will be discussed in more detail below, opportunities for reflection and communication were incorporated throughout the model.

The five stages in the instructional model are explained in the following section. Stages one, two, and three were centred around the first essential question, and asked ISTs to engage in problem solving, examine research on problem-based learning, and to develop criteria and a rubric for assessing the effectiveness of a given task. The final two stages addressed the second essential question regarding the role of the teacher when teaching through problem solving.

Stage One: Engage in Problem Solving. The first portion of the instructional model involved solving and discussing genuine tasks. One portion of each class session was devoted to solving the problem or problems of the day. During this time, the course instructor modelled as closely as possible research-based practices that support problem-based learning (Carpenter, Fennema, Frank, Levi, & Empson, 1999; Lampert, 2001; Smith & Stein, 2011; Van de Walle, Karp, & Bay-Williams, 2013). Although some portion of every class session was devoted to problem solving, the problem solving conducted during the first two classes was essential because it provided a model for how to teach through problem solving, allowed ISTs to experience being a student in a problem-based classroom, and created a shared experience that served as a foundation for future class discussions about problem-solving instruction.

The problem of the day during the first class was *the handshake problem* (D'Angelo & West, 1997), and the problem used in the second class was *the border problem* (Boaler & Humphreys, 2005). These problems were strategically selected because they exemplify many of the qualities of effective tasks I wanted to emphasise throughout the course. For example, the border problem is a particularly strong example of a problem that can be solved using a variety of solution strategies. Students' strategies typically range from counting by ones (a method accessible to virtually every student) to applying area formulas.

Even though engaging in problem-solving is important, Wilson and Cooney (2002) argue that modelling desired strategies is not enough. They suggest that opportunities for meaningful reflection must also be incorporated into these problem-solving sessions. Therefore, to encourage reflection, ISTs responded to the following prompt, "What about this task made solving it a worthwhile learning experience?" As was typical throughout the course, ISTs were asked to respond by writing in their journals first, then ideas were shared through a whole class discussion.

For example, after solving the handshake problem, many ISTs commented on the important role communication played in solving the problem. Many believed they could not have solved the problem alone, but through sharing ideas with their peers they were able to develop a solution method.

Stage Two: Examine Educational Research. After experiencing problem-based instruction, students examined relevant research on the topic. Chapman (2002) argued that to create change, teachers' current beliefs need to be deconstructed, and afterwards, reform-based beliefs must be constructed. For this reason, the first reading was the chapter, *What's Going Wrong in the Classroom,* from Boaler (2008). Although ISTs may already be aware of the shortcomings found in many classrooms in the United States, including their own, this chapter was used to bring these to the forefront and provide an argument for examining a conceptually new approach to instruction. Once the need for change was established, the next set of readings (assigned as homework in the second and third sessions) presented an alternative method of instruction based on teaching through problem solving.

These readings included the first two chapters from *Making Sense: Teaching and Learning Mathematics with Understanding* (Hiebert et al., 1997). The first chapter, titled *Critical Features of the Classroom*, discusses the features of the classroom needed to learn mathematics with understanding and argues that problem solving must be at the centre of the mathematics classroom. The second chapter, titled *The Nature of Classroom Tasks*, describes genuine tasks and explains the characteristics of effective tasks for teaching mathematics. ISTs also read, *Why is Teaching with Problem Solving Important to Student Understanding* (NCTM, 2010) and *Designing and Implementing Worthwhile Tasks* (Breyfogle & Williams, 2008). In sum, these articles and book chapters provided the ISTs with evidence about the types of tasks that lead to the development of mathematical understanding.

As part of each homework assignment, ISTs were encouraged to reflect on the reading by choosing two ideas from the reading to write about. During the ensuing class discussion, ISTs were selected or volunteered to share an idea with the class. To help keep these discussions focused on the ideas and concepts in the readings and not solely on personal experience, , each IST alerted the class to a specific page number and passage from the reading and shared their thinking about that particular passage. Other students in the class then added on to or questioned what was shared. In addition to this, I posed discussion and/or writing prompts for ISTs to contemplate. My goal with these prompts was to encourage ISTs to reflect on their experiences solving problems during class and to encourage them to make connections between these experiences and the viewpoints expressed in the readings. For example, after reading *What's Going Wrong in the Classroom* (Boaler, 2008), I gave the class the following prompt to write about, "How are the problem-solving experiences we have engaged in during class similar to or different from the description in the text?" Reading, reflecting on, and discussing this research helped prepare ISTs for the work in stage three.

Stage Three: Create Criteria for an Effective Problem. At the beginning of stage three, the following question was posed to the class, how do you know if a given problem is an effective one? My first goal in this stage was for ISTs to use their prior problem-solving experiences and their knowledge from reading about problem-based learning to create a list of criteria that could be used to determine the effectiveness of a task. I emphasised that they must be able to support their selection of the criteria using mathematics education research. Consequently, although a structure was provided for answering this question, ISTs were in control of the outcome.

The ISTs in the course worked individually at first, then in small groups, and finally as a class to develop a list of criteria. The decision to require a single list helped support our discourse community by creating a shared experience in which individuals' diverse expertise and experiences allowed others to build new insights. Through discussion, ISTs were able to refine and redevelop their ideas about effective problems. For example, a criterion suggested often was the problem must have a real-world context. While there are benefits to problems presented with a real-world context, through discussion and by providing examples of effective problems that do not involve a real-world context, the class eventually agreed this should not be one of the criteria used on the class rubric.

During each stage, ISTs were asked to write about how their thinking was changing. For example, after working in small groups to create a shared list among the group members, students were asked to reflect in writing on how their thinking about effective tasks changed during the small group discussion. This type of in-class reflective writing took place in ISTs' journals which were used regularly for writing during class. Other reflective writing took place on assignments that were completed at home and submitted to the instructor for feedback.

To develop ISTs' thinking about genuine tasks more deeply, the class created a grading rubric. The rubric required ISTs to describe what each criterion would look like across varying levels of mastery. This task proved to be very difficult as ISTs struggled to understand each criterion more deeply. The final rubric is shown in Table 1 and represents one group's search for understanding about what makes a task effective. It is important to note that the goal of the exercise was not to create a *perfect* rubric, but was to create a discussion about effective tasks, and for everyone in the class to feel ownership of the rubric and to understand each element that was included.

Stage Four: Identify the teachers' role in teaching through problem solving. The goal in stage four was centred on the following question, "What is the teachers' role in developing learning in a problem-based classroom?" To help ISTs discover the actions and thinking necessary in a problem-based classroom an activity was conducted where three different classroom scenarios using problems to guide instruction were presented. Each scenario described a fourth-grade classroom where students had not previously been exposed to adding fractions with unlike denominators. The scenarios ranged from describing a more traditional classroom to a more reform-centred classroom. The activity began by asking ISTs to explain in writing how they would teach a lesson about adding fractions with unlike denominators using the following problem: "On Saturday, Tom ate one and one-half candy bars. On Sunday, Tom ate three-fourths of a candy bar. How much candy bar did Tom eat on the weekend?" This writing provided a record of their current thinking about what instruction would look like in their classrooms when using a problem.

Next, ISTs were asked to read the three teaching scenarios and identify similarities and differences between the teachers' role in each. After discussing their thinking as a class, ISTs were asked to reflect on how they originally thought they would use the problem in class and to detail how their approach would change after completing the activity. It was only after this exploration into the teacher's role in the classroom that ISTs were asked to read Smith and Stein's (2011) text, *Five Practices for Orchestrating Productive Mathematics Discussions.* This text was the foundation for thinking about how to implement problem-based lessons. As Smith and Stein describe, the teachers' role involves anticipating likely responses, monitoring students' progress, selecting and sequencing students for presenting solution strategies, and making connections visible between the strategies presented and mathematical concepts. The progression from the activity to reading in stage four mirrored the tenets of problem-based learning with exploration coming before explanation (Marshall & Horton, 2011).

Stage Five: Teach through Problem Solving. Finally, each IST was ready to put their new learning into action by creating a genuine task and using it to teach a problem-based lesson. These lessons were taught to their peers in the graduate level course where this study took place, which provided everyone with the opportunity to experience and reflect on, both what it was like to teach through problem solving and what it was like to be a student who is learning through problem solving. ISTs were encouraged to search for genuine tasks in curricular materials and on the internet or create their own. In addition, each lesson was videotaped, and ISTs watched the video of their teaching and reflected on the experience in writing. They were also asked to discuss whether the task successfully met the criteria in the class-generated rubric and whether they successfully implemented the five practices.

Data Sources and Analysis

To examine whether the problems designed by ISTs were more effective after the implementation of the instructional model, I administered identical assessments at the beginning and end of the course. Each assessment asked ISTs to create a mathematics problem and to answer the following questions designed to provide insight into how the problem would be used in the classroom:

1. What goals do you want to accomplish with this problem?
2. What prior knowledge would you expect students to use to engage with the problem?
3. What solution strategies would you expect students to develop?
4. What connections involving the strategies listed in your response to Question 3 would you want to emphasise while using this problem to teach a class?

To assess the quality of these mathematics problem descriptions, a rubric was developed and utilised (see the Appendix for the rubric and an explanation of how it was created). The rubric included five criteria, and each criterion could be awarded a score ranging from 0 to 3. Each researcher independently scored each assessment collected as part of the current study. This included two assessments for each of the 17 ISTs who participated in the study with five items to score on each assessment for a total of 170 items scored. When comparing these scores, the researchers found that the interrater agreement was 81.18% (Tinsley & Weiss, 2000). The researchers discussed each difference until reaching an agreement. The majority of the disagreements, 68.75% occurred on the pre-assessment. We hypothesised the reason for this was that many answers on the pre-assessment lacked detail, which in turn made it more difficult to accurately assess them. When both assessments were considered together, the proportion of disagreements found on each item were as follows: item 1 (9.38%), item 2 (28.13%), item 3 (25%), item 4 (3.44%), and item 5 (31.25%). Differences in scores were then analysed by conducting an ANOVA to compare the ISTs' pre- and post-assessment scores to determine whether their understanding about teaching through problem solving improved during the study (Welkowitz, Cohen, & Lea, (2012). To look for improvements in learning by criterion, a repeated measures ANOVA was conducted with a within subjects factor of time (pre, post) and performance on each criterion as within subjects measures.

Table 1
Criteria and Grading Rubric: Is this an Effective Problem?

Criterion	High (2)	Medium (1)	Low (0)
Is the problem problematic/ challenging?	The problem is intriguing and rigorous.	The problem is either intriguing or rigorous, but not both.	The problem is neither intriguing nor rigorous.
Does the problem allow students to make connections between prior knowledge and the current problem?	The problem clearly embeds the use of prior knowledge to make the connections needed to solve the task.	There is little evidence about how the student will use their prior knowledge to solve the task.	The problem does not seem like it will allow students to make connections with prior knowledge.
Can the problem be solved using various methods?	There are multiple ways to solve the problem that are accessible to students in the same class.	There are multiple ways to solve the problem, but it is unlikely students in the same class will develop them.	There is only a single solution strategy that will likely be used by students.
Will the problem promote communication and reflection?	The problem is challenging, can be solved with multiple methods, and therefore, should promote communication and reflection.	The problem may promote communication and reflection for some of the students.	The problem does not promote communication or reflection. Likely it is either too difficult or too easy for students.
Does the problem leave mathematical residue?	The problem promotes the development of problem solving skills and promotes a connection between a process and the problem.	The problem promotes the development of problem solving skills or promotes a connection between a process and the problem.	The problem does not promote the development of problem solving skills or promote a connection between a process and the problem.

To answer research questions two and three, ISTs' writing was examined through a thematic analysis of students' written journals (Creswell, 2003). Throughout the first portion of the course, ISTs were asked each day to reflect on how their thinking about problems was changing after completing the discussions, readings, and problem-solving that was taking place in the class. To provide a guide for this writing, Ritchhart, Church, and Morrison's (2011) thinking routine, *I used to think ..., now I think ...* was used. This thinking routine is specifically designed to help individuals reflect on changes in their thinking. Later in the course, ISTs were asked to use these informal writing assignments to respond to the following prompt, "Explain how your thinking about effective problems has changed or evolved from the beginning of class (use your baseline response) until now". This writing was used to identify patterns in how ISTs' thinking about problems changed while participating in the instructional model, therefore answering Research Question 2.

For the last research question, two writing samples were used in the analysis. The one described above, and a reflective response given after the activity in stage four of the instructional model. The writing prompt asked, "Look back at how you originally thought you would use the problem in your class and write how your approach would be different now that we've done this activity. What has changed in your thinking about what a problem-based teacher needs to do to be successful?"

To begin the analysis, the writing samples were read in their entirety to get a sense of the data (Tesch, 1990). Next, the samples were read more carefully to derive codes related to specific statements made in the writing (Miles & Huberman, 1994). Descriptive coding was used to assign a basic topic to the statements ISTs' made that related to how their thinking about effective problems changed throughout the course (Saldana, 2009). All the codes used in this study emerged during the coding process. An example of the coding used is described here. Consider the following excerpt, "My point of view has now changed . . . . A good problem needs to stimulate the students' curiosity (1) and make it their goal to find the answer . . . The problem needs to provide students various ways to resolve it (2), giving them the opportunity to share their ideas with others (3)." In the excerpt, three statements were coded. Statement (1) was assigned the code *curiosity*, statement (2) was assigned the code *multiple strategies*, and statement (3) was assigned the code *sharing ideas*. Throughout the process, codes were refined, collapsed, and revised. For example, the code *sharing ideas,* was ultimately combined with other codes and given the description, *sharing solution methods*.

The next step in the analysis involved searching for themes. In this process, several codes describing a similar aspect within the data are combined to create a theme. For example, the codes, sharing solution strategies, listening to others, and discussing accuracy were combined to create the theme *communication*. For research question two, three themes were identified. The researcher examined each in relation to the whole dataset to ensure they reflected the data set as a whole (Braun & Clarke, 2006). Finally, each theme was defined, and exemplars were identified (Hsieh & Shannon, 2005). These themes represent the ways that ISTs' thinking about problems changed during the instructional model.

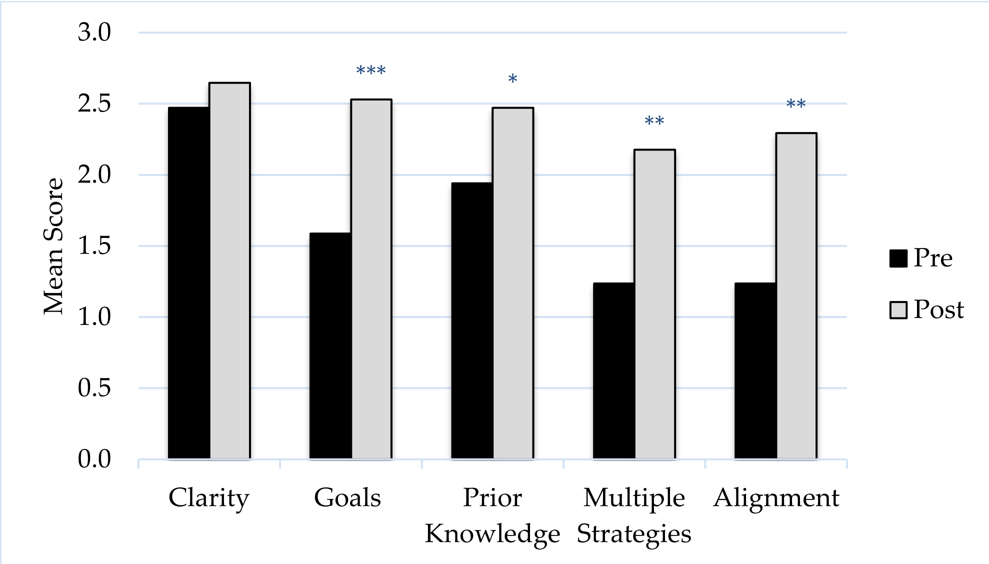
The analysis process used to answer research question three was analogous to the one used to answer research question two. In this analysis, two themes emerged describing the aspects of the instructional model that ISTs' said facilitated changes in their thinking.

Results

Pre- and Post-Assessment Results

Results of the ANOVA comparing pre- and post-assessment total scores revealed that the scores for all 17 ISTs increased from the pre- to post-assessment, with an average increase of 3.76 on the 15-point scale. Overall, there was a statistically significant increase in the average assessment scores from the pre-assessment ($M = 8.47$, $SD = 3.89$) to the post-assessment ($M = 12.24$, $SD = 3.32$), $F(1, 16) = 47.75$, $p<0.001$.

The results of the repeated measures ANOVA on each criterion show that there was a small, not statistically significant gain from the pre- to post-assessment on the first criterion that measured the clarity of the task (Figure 1). For the other four criteria on the rubric there were statistically significant gains from the pre-assessment to the post-assessment. The largest gains were found on the criterion that measured how closely the goals were aligned to the task and whether the goals included a focus on conceptual understanding. On the pre-assessment, ISTs often limited their goals to procedural ones, while on the post-assessment their goals commonly included both procedural and conceptual outcomes. There were also statistically significant increases on the last three criteria measuring whether the problem likely builds from students' prior knowledge, whether a variety of solution strategies are provided that will help build understanding, and whether the goals are aligned with the strategies provided.



Note. *$p < 0.05$; **$p < 0.01$; ***$p < 0.001$.

Figure 1. Mean scores by rubric criterion and assessment.

Changes in ISTs' Thinking

The results just described provide evidence that ISTs' ability to create genuine problems improved from the beginning to the end of the course. In the second research question, the focus shifted to consider what changes in ISTs' thinking about problems happened during the course. Three themes emerged from the analysis: 1) the need to have a genuine task, 2) the need for a problem to promote communication, and 3) the need for the problem to promote the use of multiple solution strategies.

Genuine Tasks. The first pattern that emerged from the analysis of ISTs' writing dealt with their understanding about the need to select problems that are genuine in nature. The use of the word genuine is based on Hiebert et al.'s (1997) discussion of the nature of classroom tasks (see pp. 7-

8 & 17-27). Hiebert et al. (1997) describe three aspects of genuine tasks, 1) the task is problematic, meaning students see it as interesting and there is something for them to make sense of, 2) the task connects with students' current level of knowledge, and 3) the task must require that students think about important mathematics. The analysis demonstrated that fifteen out of the seventeen ISTs described their thinking about the nature of a problem changed to include ideas from Hiebert et al.'s discussion of genuine tasks.

Many ISTs discussed their new understanding that a problem should be problematic in nature. As one IST explained,

> One thing that I have learned after reflecting on the criteria was my understanding of the word problematic. As discussed specifically in the book *Making Sense*, problematic means that students will see the task as interesting. They will have enough prior knowledge to begin to solve the problem, but they will need to work at the problem to find something new out.

Another IST commented,

> The problem being problematic was one area that I was aware of, but, I feel that I didn't see the problematic-ness in its totality; in other words, if the problem needed a solution, I considered it problematic. Yet, after our discussion, I now see problematic as part of a problem that challenges the student in a higher level. It brings out a sense of unsure feelings that inspires the student to strive for the answer.

As these quotations signify, one way in which ISTs started to envision effective problems was an understanding that the problem must interest students and inspire them to want to find a solution.

Another IST contrasted her previous idea about what a challenging problem would look like with her newly emerging ideas by writing the following,

> I used to think problems should be challenging, but challenging would take the form of wordiness, ambiguity, tricky numbers, and complex situations. Now I think that good problems should ignite and intrigue students into wanting to know more. They should be cognitively demanding and have low entrance points and multiple exit points.

This IST makes a distinction between problems that are challenging because they might "trick" students and problems that are challenging because they leave students "wanting to know more." This distinction is important because the main objective of problem solving should be to build mathematical understanding, therefore, the challenges students encounter while problem solving should lead to developing their understanding.

Communication. The second pattern that emerged from ISTs' writing was an emphasis on the important role communication plays in learning. Thirteen out of seventeen ISTs described a change in their thinking related to the use of communication in the classroom. These comments often involved a discussion about what type of communication they desired to have in the classroom. For example, "I've realised how important it is to have discourse that is grounded in the situation or problem at hand, rather than superficially talking about procedures." As another example, "I believe that a good problem should engage students in conversations and discussions by expanding, clarifying, and supporting their problem-solving methods." As these quotations demonstrate, ISTs began to see the role for communication in the classroom as centred on the problem-solving experiences of the class.

This is a shift from what some ISTs thought at the start of the course. As one IST described,

> I used to think that students' work should be addressed individually and not shared. To my understanding now, the problem needs to provide students with various ways to resolve it, giving students the opportunity to explain how they construct their own method as well as compare and contrast their experiences and ideas with others.

This IST pinpoints a change in her thinking from a belief that work should not be shared to a new vision for the classroom where sharing and comparing various solution strategies is an essential part of learning mathematics. Overall, the writing by ISTs demonstrated a change in thinking where communication became a more central component in the classroom.

Multiple Solution Strategies. The final pattern that emerged from the writing centred on the strategies used to solve problems. Eleven out of seventeen ISTs indicated a change in their thinking from privileging a single solution strategy to valuing multiple solution strategies. For example, one IST wrote,

> I used to think good problems should encourage a definite course of action in resolving it, as this will eliminate confusion. Now I think good problems should be approached by students in multiple ways using different solution strategies.

Another IST commented, "I believed that math was right or wrong, I never once considered the grey area that allowed me to understand that problems can be solved in different ways." As these examples demonstrate, ISTs' thinking began to shift to envision a classroom where multiple solution strategies were accepted and encouraged. Smith and Stein (2011) discuss the importance of sharing various strategies by explaining this provides students with the opportunity to make connections among strategies that highlight important mathematical concepts.

In-service teachers in this sample also expressed why encouraging students to develop their own strategies for solving problems is important. One IST described a revelation that she had during the course declaring that, "Students are capable of becoming responsible for their own knowledge acquisition." While another wrote, "Through my experiences reading, discussing, and completing activities I realised that good problems empower students to be self-thinkers and give them autonomy over how they want to solve these problems." These quotations signify a thought process about teaching that focuses on the student and the students' thinking and moves away from the teacher as the centre of the classroom.

What Facilitated Change?

The final research question was designed to understand from ISTs' points of view, the aspects of the instructional model that facilitated changes in thinking. Two themes emerged from the analysis. The first, identified the important role reflection played in ISTs' development, and the second, identified the importance of engaging in a problem-based approach to instruction.

The use of reflective writing was identified as one aspect of the course that led ISTs' thinking to change and grow. As an example, in stage four, ISTs were asked to describe how they would teach a problem-based lesson, and then describe how the lesson would be different after reading the scenarios presented in the instructional model. One IST explained the important role of reflection in the following,

> The funny thing is that I thought I planned an excellent lesson. Why? Because I included many elements that are effective in the classroom such as cooperative learning and provision of math tools. Reflecting after the discussion made me critically analyse myself in a productive way because no one (like the professor) pointed out what I was missing. I came to my own conclusions.

This IST emphasises how important it was for her to draw her own conclusions without being told by someone else how to think. She also describes the important role reflection played in this process. Being given the time to reflect allowed her to evaluate and modify her original thinking based on the new information presented in the discussion (Bean, 2011; Ritchhart, Church, & Morrison; 2011). Other comments also explained how reflection played a role in the learning that took place during the instructional model. As an example, one IST wrote,

> It wasn't until I started to write about how my old thinking compares to my new thinking, that I realized how much I was learning. Taking the time to step back from my experiences allowed me to pinpoint how my thinking had changed.

Again, this IST explains how the opportunity to reflect in writing was instrumental to the learning that occurred during the class.

The second theme that emerged from the analysis was the role that modelling teaching through problem solving played in changing ISTs' thinking. As one IST wrote,

> I think that the research for problem-based learning points out that students are capable of becoming responsible for their own knowledge acquisition, and thus, they must own and create the process themselves. In hindsight, I think that was the goal of this project, we developed our own understanding of assessing and creating problematic problems with the ultimate goal that we move from the intuitive level towards embedding problem solving in our classrooms successfully.

As is pointed in the previous quotation, it was the use of teaching through problem solving in the instructional model that helped this IST build understanding and form new ways of thinking about instruction.

Another IST described how her experiences participating in problem-based learning were essential in helping her understand the powerful role discussion can play in mathematical learning.

> I had always heard that children should be allowed to share their answers in order to promote communication and help them understand math better. Now, I understand the importance because I experienced it as a student and it was refreshing to see how the idea that I thought was so correct, sounded so wrong when explained to my classmates.

This quotation demonstrates the importance of pairing an examination of research with personal experience. This IST was aware of the research suggesting the value of communication in the classroom, but it was her experience, not the research, that enabled her to understand why communication is a valuable tool in learning mathematics. Without experiencing a problem-based classroom, she may never have developed an understanding of the research.

Conclusion

This study investigated an instructional model designed to change the way ISTs' think about using problems in the mathematics classroom.

Writing was a central component of the work in this study. It was used to encourage ISTs to reflect on the ways in which their thinking changed throughout the course. As Fulwiler and Young (1982) describe, writing allows individuals to "distance themselves from experience and helps them to interpret, clarify, and place value on that experience" (p. *x*). By using reflective writing, ISTs were provided a pathway for building understanding out of their experiences. Additionally, the writing provided a record of ISTs' thinking (Bean, 2011) that was then used by the researcher/instructor to identify patterns in ISTs' thinking and ultimately determine if the instructional model accomplished its goals. The patterns that emerged from ISTs thinking help demonstrate the successful aspects of the instructional model, while the information missing from their writing helps pinpoint what improvements need to be made to the instructional model in the future.

Examining what was missing from ISTs writing helped me identify possible directions for future research. As an example, in this study, ISTs rarely discussed an effective problem as needing to leave mathematical residue, despite the fact that this was a common theme in the course readings (Hiebert et al., 1997). Future research should investigate ISTs understanding about this important aspect of problem-based learning. It is my feeling that ISTs may assume all problems leave mathematical residue. While this may be true on some level, we can be certain that not all problems leave the same amount or type of residue. The mathematical residue obtained through problem solving should not be left to chance, but instead, should be shaped by the teacher. Therefore, in future iterations of the instructional model, I plan on incorporating readings from and discussions about the five strands of mathematical proficiency (National Research Council, 2001). The five strands, conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition are detailed in *Adding it Up: Helping Children Learn Mathematics* (National Research Council, 2001). After including the five strands and deeper discussions about mathematical residue, future research can investigate whether ISTs understand the diversity of learning that is possible while problem solving and examine whether ISTs understand the ways a teacher can shape the mathematical residue that comes out of problem solving.

Creating problems is often a time-consuming process, and some ISTs do not feel confident in their ability to create effective problems from scratch (Murphy, 2016). Furthermore, this study focused on the creation or examination of a single problem. However, future iterations of the instructional model should include opportunities for ISTs to examine existing curriculum materials to identify, revise, and sequence a series of problems that would lead students to build mathematical understanding. Efforts to assist ISTs in gaining the skills needed to quickly assess, revise, and sequence already constructed problems will likely increase their desire to use teaching through problem solving in the classroom. Future research should move beyond examining single problems and investigate how to best prepare ISTs to use a series of problems designed to develop students' conceptual and procedural understanding of mathematics.

In closing, while the results show that ISTs in this study wrote more effective problems and were able to document how their thinking about problems changed, it is important to realise that these changes may not reflect changes in practice. This intervention is likely a first step along the pathway to creating a problem-based classroom. With the new ideas developed during the instructional model about how students learn, ISTs will likely be more open and willing to experiment with teaching through problem solving. If this experimenting is well-supported (for example, through additional coursework), ISTs should continue along the pathway, and ultimately create an instructional environment in their own classrooms focused on problem solving.

References

Artzt, A. F. (1999). A structure to enable preservice teachers of mathematics to reflect on their teaching. *Journal of Mathematics Teacher Education, 2*(2), 143-166. doi.org/10.1023/A:1009999004407

Artzt, A. F., Armour-Thomas, E., & Gurl, T. (2015). *Becoming a Reflective Mathematics Teacher: A Guide for Observations and Self-assessment*. New York, NY: Routledge.

Bailey, J., & Taylor, M. (2015). Experiencing a mathematical problem-solving teaching approach: Opportunities to identify ambitious teaching practices. *Mathematics Teacher Education and Development*, *17*(2), 111-124.

Ball, D., & Forzani, F. (2009). The work of teaching and the challenge for teacher education. *Journal of Teacher Education, 60(5)*, 497-511. doi.org/10.1177/0022487109348479

Bangert-Drowns, R. L., Hurley, M. M., & Wilkinson, B. (2004). The effects of school-based writing-to-learn interventions on academic achievement: A meta-analysis. *Review of Educational Research*, *74*(1), 29-58. doi.org/10.3102/00346543074001029

Bean, J. C. (2011). *Engaging ideas: The professor's guide to integrating writing, critical thinking, and active learning in the classroom*. San Francisco, CA: John Wiley & Sons.

Beswick, K. (2012). Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice. *Educational Studies in Mathematics, 79*(1), 127-147. doi.org/10.1007/s10649-011-9333-2

Boaler, J. (2008). *What's math got to do with it: Helping children learn to love their least favorite subject--and why it's important for America*. New York, NY: Viking.

Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages, and innovative teaching*. San Francisco, CA: Jossey-Bass.

Boaler, J., & Humphreys, C. (2005). *Connecting mathematical ideas: Middle school video cases to support teaching and learning (Vol. 1)*. Portsmouth, NH: Heinemann.

Borasi, R., Fonzi, J., Smith, C. F., & Rose, B. J. (1999). Beginning the process of rethinking mathematics instruction: A professional development program. *Journal of Mathematics Teacher Education*, *2*(1), 49-78. doi.org/10.1023/A:1009986606120

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher, 33(8)*, 3-15.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3(2)*, 77-101. doi.org/10.1191/1478088706qp063oa

Breyfogle, M. L. & Williams, L. E. (2008). Designing and implementing worthwhile tasks. *Teaching Children Mathematics, 15*(5), 276-280. doi.org/10.3102/0013189X08328877

Cai, J. (2003). What research tells us about teaching mathematics through problem solving. In F. K. Lester (Ed.), *Research and issues in teaching mathematics through problem solving* (pp. 241-254). Reston, VA: National Council of Teachers of Mathematics.

Cai, J. (2010). Helping elementary school students become successful mathematical problem solvers. In D. Lambdin (Ed.), *Teaching and learning mathematics: Translating research to the classroom* (pp. 9-14). Reston, VA: National Council of Teachers of Mathematics.

Cai, J., Hwang, S., Jiang, C., & Silber, S. (2015). Problem-posing research in mathematics education: Some answered and unanswered questions. In F. M. Singer, N. F. Ellerton, & J. Cai (Eds.), *Mathematical problem solving: From research to practice* (pp. 3-34). New York, NY: Springer. doi.org/10.1007/978-1-4614-6258-3_1

Carpenter, T. P., Fennema, E., Frank, M. L., Levi, L. & Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Chapman, O. (1999). Inservice teacher development in mathematical problem solving. *Journal of Mathematics Teacher Education*, *2*(2), 121-142. doi.org/10.1023/A:1009948231298

Chapman, O. (2002). Belief structure and inservice high school mathematics teacher growth. In G. C. Leder, E. Pehkonen, & G. Torner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 177-193). Dordrecht, The Netherlands: Springer.

Common Core State Standards Initiative. (2010*). Common core state standards for mathematics.* Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

Cooney, T., & Shealy, B. (1997). On understanding the structure of teachers' beliefs and their relationship to change. In E. Fennema and B. S. Nelson (Eds.), *Mathematics teachers in transition* (pp. 87-110). Mahwah, NJ: Lawrence Erlbaum Associates.

Crespo, S. (2003). Learning to pose mathematical problems: Exploring changes in preservice teachers' practices. *Educational Studies in Mathematics, 52*, 243-270. doi.org/10.1023/A:1024364304664

Creswell, J. W. (2003). *Research design: Qualitative, quantitative, and mixed methods approaches (2nd ed.).* Thousand Oaks, CA: Sage.

D'Angelo, J. P., & West, D. B. (1997). *Mathematical thinking: Problem solving and proofs* (2nd Ed.). New York, NY: Pearson.

Fisher, D., & Frey, N. (2013). *Better learning through structured teaching: A framework for the gradual release of responsibility*. Alexandria, VA: Association for Supervision and Curriculum Development.

Franke, M. L., Carpenter, T. P., Levi, L., & Fennema, E. (2001). Capturing teachers' generative change: A follow-up study of professional development in mathematics. *American Educational Research Journal*, *38*(3), 653-689. doi.org/10.3102/00028312038003653

Fulwiler, T., & Young, A. (1982). *Language connections.* Urbana, IL: National Council of Teachers of English.

Goldsmith, L.T., & Schifter, D. (1997). Understanding teachers in transition: Characteristics of a model for the development of mathematics teaching. In E. Fennema and B. S. Nelson (Eds.), *Mathematics teachers in transition* (pp. 19-54). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Han, S., Yalvac, B., Capraro, M. M., & Capraro, R. M. (2015). In-service teachers' implementation and understanding of STEM project based learning. *EURASIA Journal of Mathematics, Science & Technology Education*, 11(1), 63-76. doi.org/10.12973/Eurasia.2015.1306a

Handal, B. (2003). Teachers' mathematical beliefs: A review. *The Mathematics Educator, 13(2),* 47-57.

Hart, L. C. (2002). A four year follow-up study of teachers' beliefs after participating in a teacher enhancement project. In G. C. Leder, E. Pehkonen, & G. Torner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 161-176). Dordrecht, The Netherlands: Springer.

Hiebert, J., & Grouws. D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 371-404). Charlotte, NC: Information Age Publishing.

Hiebert, J., Thomas, P., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., … Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding*. Portsmouth, NH: Heinemann.

Hsieh, H. F., & Shannon, S. E. (2005). Three approaches to qualitative content analysis. *Qualitative Health Research*, *15*(9), 1277-1288. doi.org/10.1177/1049732305276687

Inglis, L., & Miller, N. (2011). Problem based instruction: Getting at the big ideas and developing learners. *The Canadian Journal of Action Research*, *12*(3), 6-12.

Jonassen, D. H. (2000). Toward a design theory of problem solving. *Educational Technology Research and Development, 48*(4), 63-85. doi.org/10.1007/BF02300500

Lampert, M. (2001). *Teaching problems and the problems of teaching*. London: Yale University Press.

Lloyd, G. M. (2002). Mathematics teachers' beliefs and experiences with innovative curriculum materials. In G. C. Leder, E. Pehkonen, & G. Torner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 149-159). Dordrecht, The Netherlands: Springer.

Marshall, J. C., & Horton, R. M. (2011). The relationship of teacher-facilitated, inquiry-based instruction to student higher-order thinking. *School Science and Mathematics, 111*(3), 93-101. doi.org/10.1111/j.1949-8594.2010.00066.x

McLaughlin, M., & Talbert, J. E. (1993). *Contexts that matter for teaching and learning: Strategic opportunities for meeting the nation's educational goals.* Stanford, CA: Center for Research on the Context of Secondary School Teaching, Stanford University.
Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook.* Thousand Oaks, CA: Sage.
Murphy, C. (2016). Changing the way to teach maths: Preservice primary teachers' reflections on using exploratory talk in teaching mathematics. *Mathematics Teacher Education and Development*, *18*(2), 29-47.
National Commission on Writing in America's Schools and Colleges. (2003). *The neglected "R": The need for a writing revolution.* New York, NY: The College Board.
National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
National Council of Teachers of Mathematics. (2010). *Why is teaching with problem solving important to student learning?* Reston, VA: Author.
National Research Council. (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.
Pilgrim, M. E. (2013). Engaging future teachers in problem-based learning with the park city mathematics institute problems. *PRIMUS*, *24*(3), 215-231. doi.org/10.1080/10511970.2013.859190
Putnam, R. T., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning? *Educational Researcher, 29*(1), 4–15. doi.org/10.3102/0013189X029001004
Richardson, V. (1992). The agenda-setting dilemma in a constructivist staff development process. *Teaching & Teacher Education*, 8, 287-300. doi.org/10.1016/0742-051X(92)90027-Z
Richardson, V., & Anders, P. (1994). The study of teacher change. In V. Richardson (Ed.), *A theory of teacher change and the practice of staff development: A case in reading instruction* (pp. 159-180). New York, NY: Teachers College Press.
Ritchhart, R., Church, M., & Morrison, K. (2011). *Making thinking visible: How to promote engagement, understanding, and independence for all learners*. San Francisco, CA: Jossey-Bass.
Saldana, J. (2009). *The coding manual for qualitative researchers.* Thousand Oaks, CA: Sage.
Sanchez, B., & Lewis, K. (2013). Writing to learn: A study of pre-service teachers demonstrated increased content knowledge through the use of structured writing assignments. *Delta Journal of Education, 3*(1), 42-51.
Singer, F., & Voica, C. (2013). A problem-solving conceptual framework and its implications for designing problem-posing tasks. *Educational Studies in Mathematics, 83*(1), 9-26. doi.org/10.1007/s10649-012-9422-x
Smith, M. S. & Stein, M. K. (2011). *Five practices for orchestrating productive mathematics discussions*. Reston, VA: National Council of Teachers of Mathematics.
Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York, NY: The Free Press.
Stylianides, A. J., & Stylianides, G. J. (2014). Impacting positively on students' mathematical problem solving beliefs: An instructional intervention of short duration. *The Journal of Mathematical Behavior, 33*, 8-29. doi.org/10.1016/j.jmathb.2013.08.005
Tesch, R. (1990) *Qualitative research: Analysis types and software tools.* Bristol, PA: Falmer.
Tinsley, H. E. A., & Weiss, D. J. (2000). Interrater reliability and agreement. In H. E. A. Tinsley & S. D. Brown (Eds.), *Handbook of applied multivariate statistics and mathematical modeling* (pp. 95-124). San Diego, CA: Academic Press.
Umbeck, L. M. (2011). Navigating classroom change. *Mathematics Teaching in the Middle School, 17*(2), 88-95.
Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2013). *Elementary and middle school mathematics: Teaching developmentally.* Boston, MA: Pearson Education.

Welkowitz, J., Cohen, B., & Lea, R. B. (2012). *Introductory Statistics for the Behavioral Sciences, Seventh Edition.* Hoboken, NJ: John Wiley & Sons.
Wilson, M. & Cooney, T. (2002). Mathematics teacher change and developments. In G. C. Leder, E. Pehkonen, & G. Torner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 127-147). Dordrecht, The Netherlands: Springer.
Wineburg, S., & Grossman, P. (1998). Creating a community of learners among high school teachers. *Phi Delta Kappan, 79,* 35.

Appendices

Table 1A
Grading Rubric for Mathematical Tasks

Item	(3)	(2)	(1)	(0)
Clarity Does the task clearly set the context and ask a question?	What the students are being asked to do is clear and the context supports the question.	It is generally clear what is being asked, but the true intent of the problem may be debated.	The intent of the problem is not clear or the context is unnecessarily confusing.	It is not clear what the task is asking the student to do.

Goals Can the task lead to the stated goals? Do the goals emphasise mathematical understanding?	The goals focus on building conceptual knowledge. The outcomes would likely result from solving and discussing the task.	The goals focus on mathematical understanding, but it is unclear how the task will lead to this type of understanding.	The goals focus on procedural knowledge without connections, but are related to the strategies.	The goals are not aligned with the task.
Prior Knowledge Does the task build from the prior knowledge students will have?	There is an obvious connection between the prior knowledge assumed by the teacher and the problem.	The prior knowledge stated will be used, however some necessary information is missing.	It is not clear that the prior knowledge assumed by the teacher will be used in solving the problem.	The prior knowledge assumed by the teacher is not connected to the task.
Multiple Strategies Are a variety of solution strategies described that will aid in the development of mathematical understanding?	Multiple, age-appropriate strategies are described using various tools and/or representations.	Multiple strategies are described, but at least one key strategy is missing.	Only a single solution method is provided or several strategies that are likely to appear in class are missing.	No strategies are provided for solving the problem.
Alignment Are the goals aligned with the strategies provided?	The goals are closely aligned with the strategies provided for solving the talk.	The goals are only partially aligned with the strategies provided for solving the task.	Alignment between the strategies and goals is weak.	The goals are not aligned with the task.

To create the scoring rubric and score the pre- and post-assessment, a team of two researchers met biweekly for one month. To begin this work, we used assessment responses that were collected in a previous course (this data was not used in this study) to create the scoring rubric. Our goal was to create a scoring rubric that would assess how well ISTs were able to create an effective problem and demonstrate that it is effective. As the researchers read these responses, they discussed what elements they wanted to include in the rubric. The process underwent many cycles of creating items and scoring criteria, using these to evaluate a sample of assessments, and then suggesting changes to the rubric for future use. This work involved changing both the list of items to be evaluated and the criteria for assessing each item. For example, we started with an item about communication similar to what was found in the class-generated rubric. However, we quickly realised that there was not enough information provided to understand why the IST believed this problem would promote communication. Therefore, we created rubric Item 4, which focused on the various solution methods provided and whether these have the potential to lead to mathematical understanding.

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