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Exploring Growth Trajectories of Informal and Formal Mathematics Skills Among Prekindergarten Children Struggling With Mathematics

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Abstract

Growth in two subscales, Informal and Formal Mathematics Skills, of the *Test of Early Mathematics Abilitity–3* (TEMA-3) was explored in a sample of 281 children. Children were identified as either typically developing (TYP; n = 205) or having mathematics difficulties (MD; n = 76) based on their total TEMA-3 score at the end of prekindergarten. Their average level of informal and formal mathematics skills, growth rate over time, and rate of acceleration of growth were estimated using conventional growth modeling while controlling for the effects of gender. Results indicated that children with MD had significantly lower informal and formal mathematics knowledge than did TYP children at the end of kindergarten. However, for informal mathematics skills, children with MD grew at a significantly faster rate than did TYP children, and the rate of acceleration was also significantly faster for children with MD. In contrast, both the rate of growth and acceleration of growth in formal mathematics skills were significantly faster for TYP children than they were for children with MD. Implications for early MD identification and interventions are discussed.

Keywords

informal mathematics skills, formal mathematics skills, mathematics difficulties

Early mathematical competencies are foundational to developing subsequent competencies in mathematics because mathematics learning occurs in progression, with advanced mathematical concepts, such as multi-digit calculations, and directly building on basic concepts, such as understanding of place value and mastery of arithmetic facts. For example, Duncan et al. (2007) demonstrated that early mathematical competencies were the best predictors of later mathematics achievement even when general cognition, attention, and socioeconomic background were controlled. Similar findings have been reported in other longitudinal studies where early mathematical competencies predicted later mathematics achievement up to fourth grade (e.g., Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Jordan, Kaplan, Locuniak, & Ramineni, 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Koponen, Aunola, Ahonen, & Nurmi, 2007; Krajewski & Schneider, 2009; Passolunghi, Vercelloni, & Schadee, 2007). More importantly, early mathematical competencies predict rate of growth in mathematics achievement (e.g., Jordan et al., 2007, 2009), suggesting that children who have weak early mathematical skills are likely to continue to fall behind their peers and may never catch up.

Given the significant relation between early mathematical competencies and subsequent rate of growth in mathematics achievement, it is not surprising that prior research has consistently found that children with mathematics difficulties (MD) show difficulties in various domains of early mathematical competencies, such as counting, comparing quantities, and recalling arithmetic facts (e.g., Desoete, Ceulemans, De Weerdt, & Pieters, 2012; Geary, Hamson, & Hoard, 2000; Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Mazzocco & Thompson, 2005). Although it is evident that individual differences in early mathematical competencies emerge as early as the preschool years (e.g., Aunio, Hautamäki, & Van Luit, 2005; Howell & Kemp, 2010), and that early mathematical competencies are the best

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predictors of early and later mathematics achievement, little is known about how early mathematical competencies develop among young children.

Early mathematical competencies are broadly categorized as informal and formal skills. Informal skills refer to the "notions and procedures acquired outside of the context of schooling" (Ginsburg & Baroody, 1990, p. 2) that reflect "implicit understanding" (p. 3), and formal skills refer to the "arithmetic skills and concepts the child learns in school" (p. 2) that reflect "explicit understanding" (p. 3). Early informal mathematics knowledge serves as the basis for developing early formal mathematics knowledge, and both are foundational to developing advanced formal mathematics knowledge later in school (e.g., Ginsburg, Klein, & Starkey, 1998; Griffin & Case, 1997; National Mathematics Advisory Panel, 2008). In this study, we focus on how informal and formal knowledge develop in children who are evidencing MD as indicated by scores on a standardized mathematics assessment at or below the 10th percentile. While other studies have examined development of mathematics skills in undifferentiated samples of young children (e.g., Ryoo et al., 2014), a developmental study of young children with MD will shed light on early screening and intervention targets, which in turn may help reduce the prevalence of MD through early identification and may facilitate better mathematics learning trajectories and outcomes for children with MD through instructional supports. Therefore, the primary goal of the present study was to explore the growth trajectories of children with MD in both informal and formal mathematics skills as assessed by the *Test of Early Mathematics Abilitity–3* (TEMA-3; Ginsburg & Baroody, 2003) in comparison with typically developing (TYP) peers.

Early Mathematical Competencies: Informal and Formal Mathematics Knowledge

Informal mathematics knowledge refers to the mathematical concepts and skills primarily acquired prior to entering school, and many children show their informal mathematics knowledge before they begin kindergarten (Fuson & Hall, 1983). Informal mathematics knowledge develops through everyday experiences with quantitative concepts in various ways, such as (a) self-initiated or spontaneous interactions with the environment via observations and reflections (e.g., adding an item to a collection results in "more") and (b) informal instruction via informal conversations with adults or siblings, or hearing about using numbers to count, telling time, and comparing sizes (e.g., Ginsburg & Baroody, 2003; Libertus, Feigenson, & Halberda, 2013; Rousselle & Noël, 2007). Thus, informal mathematics knowledge is acquired without the use of conventional symbols, such as Arabic numerals and operation signs, and the development of this informal knowledge does not depend on formal school instruction but is facilitated by individual experiences in the environment.

Krajewski and Schneider (2009) identified three stages of informal knowledge development, with each stage serving as a developmental precursor to the next stage. In Level 1, children discriminate quantities (i.e., more, less, the same; Ginsburg & Baroody, 2003). Children at this stage can remember and reproduce small quantities (approximately one to four items) without counting them, indicate the correct amount when items are either removed or added to an existing set, and recite counting sequences, but they do not use number words to describe quantities (e.g., Jordan, Huttenlocher, & Levine, 1992, 1994). In TYP children, infants as young as 6 months of age can distinguish small quantities (two vs. three objects; Mix, 2010). This discrimination gets refined over time, and children begin to use language, such as *more* and *less*, to compare small and large quantities, and can recite counting sequences by ages 2 and 3 (National Research Council [NRC], 2009). By age 2, children also begin to show basic understanding of addition and subtraction concepts with concrete objects, which becomes explicit by age 3 (Clements & Sarama, 2009, 2014).

This precounting number knowledge provides the basis for the verbal and counting-based knowledge in Level 2. In this stage, number words are linked with quantities that children can count by matching number words (e.g., one, two, three) to discrete quantities. Children also develop basic counting concepts, such as one-to-one correspondence (i.e., only one number word is assigned to each counted object) and cardinality (i.e., the value of the final number word represents the total quantity). By age 3, TYP children can verbally count to 10. Around 3.5 years of age, children begin to coordinate words and objects and develop understanding of one-to-one correspondence and cardinality, which typically reaches proficiency at age 4 (Clements & Sarama, 2009; Mix, 2010). In Level 3, children develop an understanding of the relations among numbers (e.g., five chips are divided into three chips and two chips) and develop concepts of basic operations, such as addition and subtraction. Typically, this occurs at between 4 and 5 years of age, and children begin to add and subtract mentally without concrete objects (Clements & Samara, 2014).

Formal mathematics skills refer to the competencies primarily acquired in school and include the understanding and use of conventional written symbols, such as Arabic numerals, operation signs, and written computations (Ginsburg & Baroody, 2003; Purpura, Baroody, & Lonigan, 2013). Formal mathematics knowledge develops as children learn written symbols, which are mapped onto their counting-based informal mathematics knowledge of number words and quantities, and becomes integrated (Baroody, 1983; Kolkman, Kroesbergen, & Leseman, 2013). TYP children can learn to read many numerals up to 10 by age 4, and can add and subtract written numerals within five. At age 5, children can read and write numerals for the 10 numbers, solve written numerical addition and subtraction problems, and use = and ≠ symbols (NRC, 2009).

This development of formal mathematics knowledge increases the efficiency and fluency of the prior counting-based informal mathematics skills. For example, the use of written numerical symbols and calculation procedures allows children to develop efficiency with operations involving large numbers, and learning base 10 and place value concepts allows children to write multi-digit numbers and understand regrouping (Baroody, 1983). In addition, the development of early formal mathematics knowledge is critical to the subsequent learning of more advanced formal mathematics. For example, calculations with decimals directly draw upon understanding of the base 10 system and place value, mastery of arithmetic facts, and calculation skills.

Informal and Formal Mathematics Skills Among Children With MD

Converging evidence from prior research suggests that children with MD have difficulties with various domains of informal and formal mathematics skills, including reading and writing numbers, mastering counting principles, and mastering arithmetic facts. Furthermore, these difficulties are present as early as preschool and kindergarten (e.g., Butterworth, 2005; Desoete et al., 2012; Geary et al., 2000; Geary et al., 2004; Mazzocco & Thompson, 2005). These difficulties in informal and formal mathematics learning create initial disadvantages for children because early mathematical competencies are strongly related to the rate of growth of mathematics skills and later mathematics achievement. Consequently, children with MD may experience "Matthew Effects," in which the achievement gap between children with MD and their TYP peers widens as "the rich get richer, and the poor get poorer" (Stanovich, 1986).

Prior studies examining growth trajectories of children with MD compared with their TYP peers document a consistent achievement gap at kindergarten that extends to upper elementary grades (e.g., Jordan, Kaplan, Nabors Oláh, & Locuniak, 2006; Jordan et al., 2009; Morgan, Farkas, & Wu, 2009, 2011; Murphy, Mazzocco, Hanich, & Early, 2007). For example, Murphy et al. (2007) identified children with persistent MD (severe MD: below 10th percentile, less-severe MD: between 10th and 25th percentile) based on their performance on the TEMA-2 and examined the differences in their growth trajectories on TEMA-2, compared with TYP peers from kindergarten to third grade. The initial performance differences on TEMA-2 continued into third grade for both children with severe and less-severe MD. In addition, the growth rate for children with severe MD slowed down, widening the achievement gap between the TYP children and children with severe MD even further. Although prior studies provide evidence that children with MD show not only poor achievement in early mathematical skills as early as kindergarten but also slower growth compared with TYP peers (e.g., Jordan et al., 2006; Jordan et al., 2009; Morgan et al., 2009, 2011; Murphy et al., 2007), these studies only examined the overall growth trajectories of broad early mathematical skills. Thus, it remains unclear whether children with MD have difficulties in informal mathematics skills, or formal mathematics skills, or both, and how their growth trajectories change over time.

A few available studies that have investigated informal and formal mathematics knowledge separately or reported separate data for each domain provide some insight into the magnitude of difficulties that children with MD experience, but findings are mixed. These studies with school-aged children found that children with MD and TYP children had comparable informal mathematics skills, but that children with MD lacked proficiency in formal mathematics skills (e.g., Rousselle & Noël, 2007; Russell & Ginsburg, 1984). For example, working with second- and fourth-grade children, respectively, Rousselle and Noël (2007) and Russell and Ginsburg (1984) found that children with MD performed comparably to their TYP peers on informal mathematics tasks, such as quantity comparison with pictures of sticks. However, children with MD performed significantly lower on formal mathematics tasks (e.g., numeral comparisons, recalling addition arithmetic facts) than did TYP children. The authors suggested that children with MD had specific difficulties in accessing semantic information for numerical symbols, but their informal mathematics knowledge was intact. These findings support the claim that children with MD have specific difficulties in formal, but not informal, mathematics skills.

Other studies, also with school-age children, report that children with MD lack proficiency in both informal and formal skills (e.g., Landerl, Bevan, & Butterworth, 2004; Mazzocco & Thompson, 2005). Landerl et al. (2004) found that third and fourth graders with MD had general difficulties in both informal and formal mathematics knowledge, such that children with MD were slower at counting dots, reciting number sequences, and reading and writing numbers than were TYP children. Children with MD were also significantly slower and less accurate at recalling arithmetic facts than were TYP children. Yet, one longitudinal study of children from kindergarten to second grade reported that children with MD had difficulties in both informal and formal mathematics knowledge in kindergarten, but that they outgrew difficulties in informal mathematics knowledge by second grade (Desoete et al., 2012). Although prior studies are consistent in suggesting that children with MD have difficulties in formal mathematics skills (which is expected given current methods of identifying MD, for example, below 25th percentile on mathematics achievement), mixed findings exist for informal mathematic skills. Such conflicting findings exist without a clear pattern to explain the results, as the studies used various age groups and various measures of informal and formal mathematics knowledge.

Present Study

Taken together, although there is strong evidence that children with MD show deficits in early mathematical competencies and show slower growth rates than do their TYP peers, further investigation is warranted for several reasons. First, most longitudinal studies of children with MD have focused on their growth in broad early mathematics skills, and only a few studies have investigated informal and formal mathematical skills separately. Even so, conflicting findings exist regarding whether children with MD have deficits in either or both informal and formal mathematics knowledge. Finally, most studies have focused on school-age children with and without MD while many early informal and formal mathematics skills are acquired prior to entering kindergarten.

Therefore, in the present study, we explored the longitudinal growth trajectories of informal and formal mathematics skills among children with MD and TYP children from prekindergarten to first grade. This represents a critical period for development of early mathematics skills as informal mathematics knowledge is solidified and formal mathematics knowledge develops in the preschool and early elementary years. Exploring growth trajectories of informal and formal mathematics skills at this critical developmental period should provide valuable information on early screening and intervention targets for MD. For example, if children with MD do not show difficulties in informal mathematics knowledge, early intervention should focus on building formal mathematics skills by leveraging children's strong informal mathematics skills, and early MD screening efforts should also target formal mathematics skills. Such information can provide more accurate and detailed guidelines on intervention and curricula focus for both practitioners and researchers. Providing focused and refined early intervention on a specific domain may be more effective and efficient in helping children to overcome initial disadvantages, which, in turn, facilitate better mathematics learning trajectories and lead to more positive outcomes for young children struggling with mathematics.

Method

Participants

Data in the present study were collected as part of a larger study exploring the roles of teacher training and classroom activities on children's mathematics skills from prekindergarten to first grade. Teachers in state funded and Head Start preschool programs volunteered to participate in the larger study. Children included in this study were children who were identified at prekindergarten entry as typically developing (without an identified disability), and spoke English as their primary language. At the time of recruitment, the sample included 389 children (182 males and 207 females) with an average age of 54.46 months (range = 47–59 months, $SD = 3.47$) in 46 prekindergarten classrooms across six counties in two Midwest states. Children were followed for 2 years, from prekindergarten to the end of first grade.

By the end of the prekindergarten year (Time 1), 29 children dropped out of the study; two children did not meet eligibility for kindergarten enrollment at Time 2, and 25 children either moved to nonstudy classrooms or moved out of the region. Further attrition at the end of kindergarten (Time 2, $n = 54$) and at the end of first grade (Time 3, $n = 12$) was due to children moving out of the region or unsuccessful attempts to contact them. Of the 294 children, 13 students had missing data at more than one time point, leaving 281 children as the final sample for the present study. To examine any potential nonrandom missing data, we compared the means between the retained group and the missing group on TEMA-3 at each time point. At each time point, the mean difference was not significant (all $p > .05$); we, therefore, omitted these cases.

Measure

The TEMA-3 (Ginsburg & Baroody, 2003) is a standardized instrument designed to assess informal and formal mathematics skills for children aged 3 years 0 months to 8 years 11 months. The test consists of 72 items, divided into two broad categories: 40 items that assess informal mathematics skills (e.g., counting on fingers, subitizing, and comparing sets of dots) and 32 items that assess formal mathematics skills (e.g., reading and writing numbers, and single-digit addition, subtraction, and multiplication facts). The administration of the TEMA-3 requires approximately 45 min. The administration and scoring of the TEMA-3 followed standardized protocols. Items were administered using age to determine item entry level. Items before the age-defined starting point were only administered when the child did not establish the basal (five consecutive items answered correctly). Testing stopped when the child reached the ceiling (five consecutive items answered incorrectly). Each child's responses were scored on a record form and raw scores were transformed to Math Ability scores ($M = 100$, $SD = 15$). According to the examiner's manual, the 2-week test–retest reliability of the TEMA-3 is .82. Cronbach's alpha for this sample was .92 at Time 1, .94 at Time 2, and .95 at Time 3.

Table 1. Student Demographics and Raw Scores by MD Status.

	TYP (n = 205)		MD (n = 76)	
Variable	*n*	%	*n*	%
Females	121	59.0	28	36.8
Race				
Caucasian	171	85.9	53	72.6
African American	12	6.0	12	16.4
Hispanic	9	4.5	3	4.1
Other	7	3.5	5	6.8
	M	*SD*	*M*	*SD*
Age (months)				
Time 1	60.26	3.78	60.93	3.42
Time 2	69.99	3.70	70.73	3.28
Time 3	84.29	3.75	85.13	3.35
Informal Knowledge				
Time 1	14.55	4.86	6.22	2.92
Time 2	24.08	5.91	15.19	6.05
Time 3	33.29	3.02	27.55	5.72
Formal knowledge				
Time 1	2.16	1.41	0.21	0.50
Time 2	5.04	2.08	2.63	1.64
Time 3	11.40	5.24	6.21	2.31

Note. Race on nine students (six TYP, three MD) were missing because teachers did not provide information. MD = mathematics difficulties; TYP = typically developing.

Procedures

The University's institutional review board approved this study. Trained researchers and graduate research assistants individually assessed children's informal and formal mathematics skills using the TEMA-3 in spring (March to May) of each year from prekindergarten to first grade. Each child was tested in a quiet area of the school and received stickers and an age-appropriate toy or game upon completion.

Data Analysis

MD status. We used children's mathematics scores at the end of prekindergarten to identify children with MD. The final sample of 281 children were classified as either TYP ($n = 205$) or MD ($n = 76$) based on their mathematics ability total scores on TEMA-3. We used a conservative criterion of the 10th percentile cutoff to designate MD status because using the strict cutoff of 10th percentile is likely to yield students with true MD, who exhibit distinctive academic and cognitive profiles compared with those identified with a lenient criterion (e.g., Morgan et al., 2009; Murphy et al., 2007). Also, using the 10th percentile is likely to yield the percentage of MD identification that is consistent with prevalence estimates (5%~10%; for example, Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). Therefore, children with the total mathematics ability scores at or below the 10th percentile on the TEMA-3 were classified as having MD. The descriptive statistics for gender, race, age, and TEMA-3 scores at each time point by MD status are shown in Table 1.

We examined the relation between MD status and gender, race, age, and informal and formal mathematics knowledge. As expected, TYP children had significantly higher informal and formal mathematics knowledge than did the children with MD at all assessment points (all *p*s < .001). There was no significant relation between MD status and age at the end of prekindergarten, $F(1, 179) = 1.86$, $p = .17$. However, male children were significantly more likely to be identified as MD, $\chi^2(1, N = 281) = 10.95$, $p = .001$. Although the overall chi-square test was statistically significant, indicating that TYP children and children with MD had a significantly different distribution of race, $\chi^2(3, N = 272) = 9.07$, $p = .03$, post hoc tests indicated that there were no significant differences after adjusting the alpha level to $p = .0063$ (.05/8) to control for multiple comparisons.

Time. Because the three time points (i.e., end of prekindergarten, end of kindergarten, end of first grade) in the present study were not equally spaced (i.e., 10-month difference between Time 1 and Time 2, 14-month difference between Time 2 and Time 3), we used children's age at each time point as a proxy for time to accurately represent the intervals between each assessment point. Because we identified children with MD based on their performance at Time 1, child age was centered at Time 2 to minimize the correlation between the intercept and slope in growth models.

Unconditional and conditional growth modeling. First, given that children were nested in classrooms, intraclass correlations (ICCs) were estimated at each time point for both informal and formal mathematics skills to account for instructional effects. Because no significant ICCs were found (all *p*s > .05), we used conventional growth modeling. Second, unconditional growth curve models, which provide estimates of the average level of informal and formal knowledge (intercept), the average growth rate over time (slope), and the average rate of acceleration of growth over time (quadratic slope; Raudenbush & Bryk, 2002) were estimated. Fixed and random effects of intercept, slope, and quadratic terms were evaluated to capture growth in children's mathematics skills from the end of prekindergarten to the end of first grade as accurately as possible. Fixed and random effects were entered sequentially one at a time (i.e., fixed intercept, random intercept, fixed slope, random slope, fixed quadratic). A random quadratic term could not be estimated because there were only three data points for each child. Akaike information criterion (AIC) and Bayesian information criterion (BIC) values were used to determine the best fitting model. Table 2 shows the results of AIC and BIC among unconditional

Table 2. Results of Information Criterion Among Unconditional Growth Curve Models.

	AIC	BIC
Informal knowledge		
Random intercept, no slope	6,201.06	6,210.51
Random intercept, fixed slope	5,009.23	5,022.68
Random intercept, random slope	5,011.85	5,030.75
Random intercept, fixed slope, fixed quadratic	4,991.36	5,000.81
Formal knowledge		
Random intercept, no slope	5,009.74	5,019.19
Random intercept, fixed slope	4,291.96	4,301.41
Random intercept, random slope	3,910.85	3,929.74
Random intercept, random slope, fixed quadratic	3,811.87	3,830.76

Note. Inclusion of the random slope term caused issues of model nonconvergence for informal mathematics knowledge. Therefore, it was excluded from the final model. AIC = Akaike information criterion; BIC = Bayesian information criterion.

Table 3. Baseline Models for Informal and Formal Skills.

	Informal	Formal
Intercept	21.46**	4.25**
Slope	0.83**	0.31**
Var (intercept)	25.30**	5.26**
Var (slope)	—	—
Acceleration	−0.01**	0.01**

***p* < .001.

Table 4. Model 1: Growth Curve Models for Informal and Formal Skills With Effects of MD Status, Gender, and Grand-Mean Centered Age.

	Informal	Formal
Intercept	23.72**	4.87**
Slope	0.79**	0.32**
Var (intercept)	11.98**	3.31**
Var (slope)	—	—
Quadratic	−0.01**	0.01**
MD	−9.57**	−2.86**
Slope × MD	0.06*	−0.14**
Quadratic × MD	0.01*	−0.004*
Female	0.74	0.35
Slope × Female	0.03	0.07*
Quadratic × Female	−0.001	0.002*

Note. MD = mathematics difficulties.
**p* < .05. ***p* < .001.

growth curve models. A decrease in AIC and BIC values greater than 10 indicated that the less restricted model provided a better fit to the data than did the more restricted model (Kass & Raftery, 1995).

Finally, after establishing the unconditional growth curve models that provided the best fits to the data, conditional models were evaluated to examine whether MD status and gender were related to the growth parameters. We did not include child age as a covariate because child age was used as a proxy for time. Therefore, the slope term already captured the effects of child age. However, we controlled for gender because prior research suggests that there may be gender differences in informal and formal mathematics knowledge (e.g., Hyde, Fennema, & Lamon, 1990; Jordan et al., 2007; Royer, Tronsky, Chan, Jackson, & Marchant, 1999). Some studies indicate that boys retrieve arithmetic facts faster than girls during calculation processes and that numerical skills develop faster for boys than they do for girls (e.g., Jordan et al., 2007; Royer et al., 1999) while other studies report that girls have stronger calculation skills than boys in the elementary grades (Hyde et al., 1990).

Results

As can be seen by the AIC and BIC values in Table 2, the model that included a random intercept, fixed slope, and fixed quadratic term provided the best fit for informal skills.

Inclusion of the random slope term caused issues of model nonconvergence. Therefore, the random slope term was excluded from the final unconditional model. By contrast, the model that included a random intercept, random slope, and fixed quadratic term provided the best fit for formal skills.

The results of the unconditional models are presented in Table 3. As expected, the unconditional models indicated that the average raw scores for both informal and formal skills at Time 2 were significantly different from zero. The predicted average scores were 21.46 for informal skills and 4.25 for formal skills at the end of kindergarten. In addition, there were statistically significant linear increases in both informal (.83) and formal skills (.31) with every 1 month increase in child age, such that children in general showed growth in their informal and formal skills over 2 years. These numbers correspond to a 19-point increase over the course of 2 years for informal mathematics skills and a 7.44-point increase for formal mathematics skills. The rate of acceleration was also significant for both informal and formal skills; however, the pattern of acceleration was different across two skills. Whereas children's growth in formal skills accelerated over time, as indicated by the positive quadratic term, children's growth in informal skills decelerated over time, as indicated by the negative quadratic term.

Table 4 shows the results of the conditional growth curve models with the effects of risk status and gender on initial level and rate of growth of informal and formal mathematics skills. For informal skills, children with MD had significantly lower scores at the end of kindergarten than did TYP

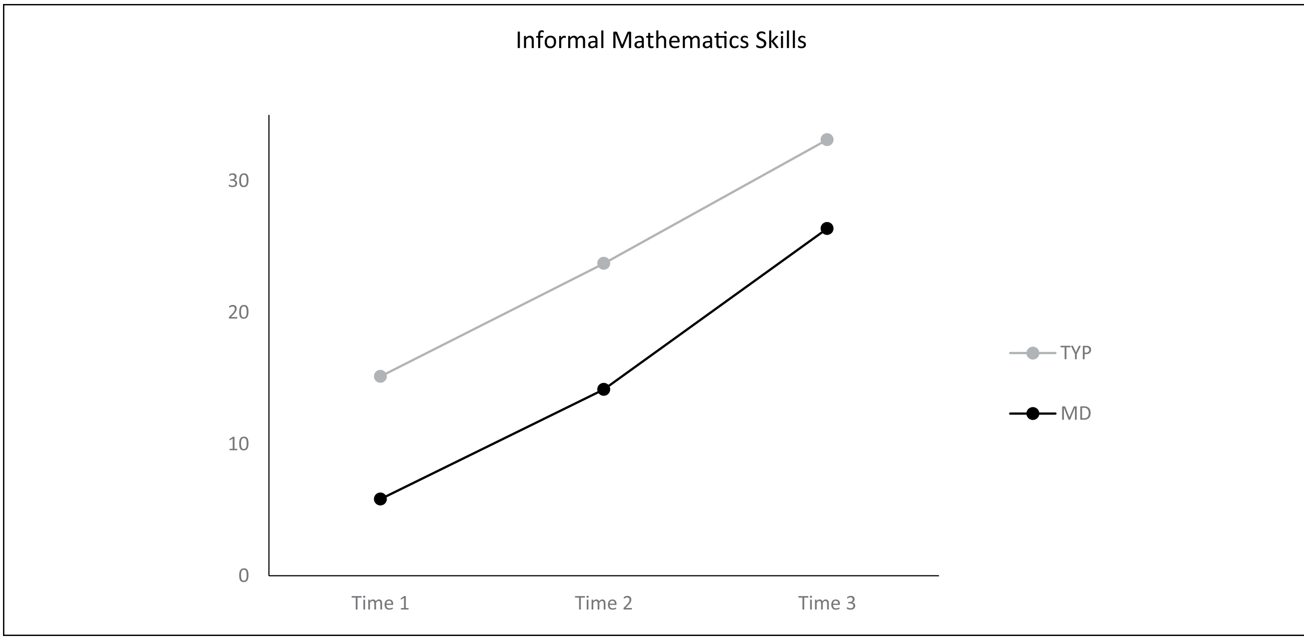

Figure 1. Fitted growth trajectories for informal skills.
Note. TYP = typically developing; MD = mathematics difficulties.

children, as indicated by the negative effect of risk status on intercept. However, children with MD grew at a significantly faster rate than did TYP children, as indicated by the positive interaction between slope and MD status. The rate of acceleration was also significantly faster for children with MD than it was for TYP children, as indicated by the positive interaction between quadratic growth and MD status. There were no gender differences in initial level or rate of growth or acceleration for informal skills.

As was the case for informal knowledge, children with MD had significantly lower scores on formal skills at the end of kindergarten than did TYP children. In contrast to the results for informal skills, children with MD grew at a significantly slower rate than did the TYP children, as indicated by the negative interaction between slope and MD status. The rate of acceleration was also significantly slower for children with MD than it was for TYP children. Whereas there were no gender differences in the initial level for formal skills, significant gender differences were found in rate of growth and acceleration over time. Specifically, rates of growth and acceleration of formal mathematics skills among male children were significantly faster compared with those of female children. The predicted growth trajectories for informal and formal mathematics skills by MD status after controlling for gender are shown in Figures 1 and 2, respectively.

Discussion

The purpose of this study was to explore the growth trajectories of informal versus formal mathematics knowledge among children with MD in comparison with their TYP peers. Our findings indicate that children with MD have difficulties in both informal and formal mathematics skills compared with TYP children. However, patterns of rate of growth and acceleration differed. For informal mathematic skills, children with MD grew at a significantly faster rate and demonstrated a faster rate of acceleration than did TYP children. This finding suggests that the achievement gap in informal mathematics skills between the two groups of children should narrow over time. By contrast, for formal mathematics skills, children with MD not only grew at a significantly slower rate than did TYP children, but they also demonstrated a slower rate of acceleration, suggesting that the achievement gap between the two groups should widen over time.

First, our findings provide evidence that children identified with MD in prekindergarten continue to have difficulties in informal knowledge as well as formal knowledge in kindergarten and first grade. Although children with MD showed initial delays in developing informal knowledge, across the study period, their growth trajectories suggest they will catch up to their TYP peers. This was not the case with formal knowledge, as the growth trajectories suggest that the gap between children with MD and TYP children will widen across the study period. These findings are consistent with those of Desoete et al. (2012) who also reported that children with MD had difficulties with both informal and formal mathematic skills at kindergarten, but children with MD outgrew the difficulties with informal skills by second grade. Studies with older children (e.g., second and

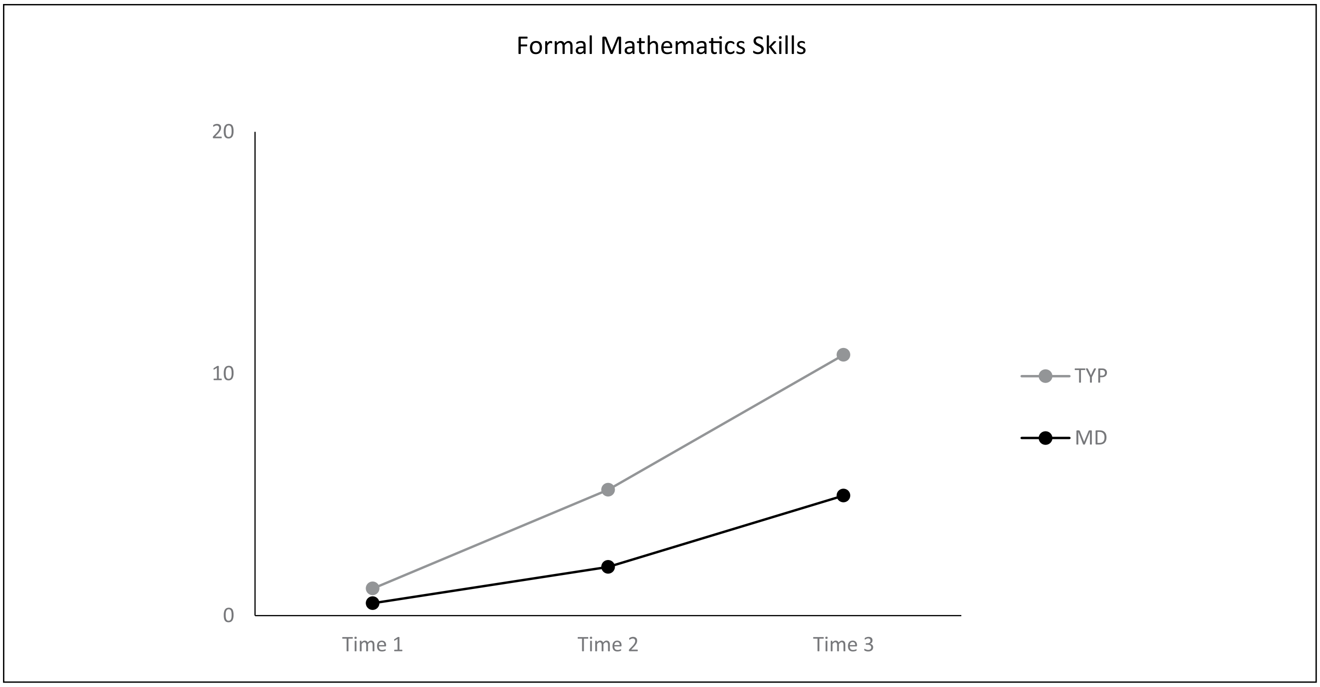

Figure 2. Fitted growth trajectories for formal skills.
Note. TYP = typically developing; MD = mathematics difficulties.

fourth graders in Rousselle & Noël, 2007; Russell & Ginsburg, 1984) also found that children with MD had no difficulties in informal mathematics knowledge but performance scores reflected continuing difficulties in formal knowledge.

Our findings may help to explain the inconsistencies in results found in the individual differences literature examining the predictive relations between informal and formal mathematics skills, and mathematics achievement (e.g., Desoete et al., 2012; Landerl et al., 2004; Mazzocco & Thompson, 2005; Rousselle & Noël, 2007; Russell & Ginsburg, 1984). Our findings suggest that the predictive value between informal mathematics knowledge and mathematics achievement changes over the course of development, resulting in differential predictive power for informal mathematics scores depending on the age of assessment. Thus, informal mathematics knowledge may not accurately discriminate children with MD from TYP children at older ages whereas formal mathematics knowledge continues to build a stronger link to individual differences in mathematics achievement.

Both informal and formal mathematics skills are important precursors of MD in early childhood. Our findings have implications for identifying children at risk for MD and for providing early interventions. With respect to informal mathematics knowledge, children develop informal knowledge through everyday experiences with quantitative concepts, such as hearing adults using numbers to count and comparing sizes (Ginsburg & Baroody, 2003; Libertus et al., 2013), recognizing numerals, and creating sets. Thus, the development of informal knowledge is facilitated by individual experiences in the environment, indicating that home environments and other care environments (such as day care and early childhood education programs) play a critical role in fostering informal mathematics knowledge of young children prior to formal schooling.

Unfortunately, the experiences young children have in their environments vary greatly. Parent–child interaction in the home environment around mathematics affects children's mathematics knowledge, but such interactions may be underutilized as parents of young children focus more on supporting literacy activities, particularly book-reading, than on specific mathematics activities, such as counting (Barbarin et al., 2008). Prekindergarten teachers also vary in their understanding of and feelings of efficacy in teaching early mathematics (Ginsburg, Lee, & Boyd, 2008; Greenfield et al., 2009). Furthermore, prekindergarten teachers report preferring to focus on development of socio-emotional skills over literacy and mathematics skills (Ginsburg et al., 2008).

Taken together, research indicates that parents' and prekindergarten teachers' engagement in fostering an environment supporting the development of informal mathematics knowledge is uncertain. However, prior studies (e.g., Galindo & Sonnenschein, 2015; LeFevre, Polyzoi, Skwarchuk, Fast, & Sowinski, 2010; Ramani & Siegler, 2008; Skwarchuk, Sowinski, & LeFevre, 2014) report that parents' and teachers' awareness of the early mathematical competencies can be increased by providing more information on home activities including home–school partnership activities, such as those targeting mathematics-specific (e.g., counting, adding) and mathematics-related (e.g., playing board games, card games, cooking) competencies. Such activities that can foster development of informal mathematics competencies may better prepare children for formal mathematics instruction and have a positive impact on children's learning trajectories in mathematics.

With respect to formal mathematics knowledge, our findings confirm that young children with MD continue to show persistent and substantial poor mathematics achievement. These difficulties with formal mathematics worsen over time as children who lack foundational formal mathematics knowledge have difficulty acquiring more advanced formal mathematical concepts via general classroom instruction alone. Indeed, Galindo and Sonnenschein (2015) claimed that children need to acquire a certain level of proficiency with early mathematical skills to profit from classroom instruction at kindergarten and that starting kindergarten proficient in mathematics is critical to subsequent growth in mathematics achievement.

The children with MD in this study did acquire adequate, although delayed, informal knowledge, upon which they could build formal knowledge. However, the widening gap in achievement of formal mathematics knowledge indicates that by the time children with MD catch up to their TYP peers and have acquired the foundational mathematics skills to develop formal mathematics knowledge, classroom instruction may be already too far beyond the basic formal mathematics skills children with MD are ready to build. As noted by Bauer (2009), learning takes time because it involves storing what is learned in memory and requires consolidation and reconsolidations as new learning occurs. Thus, children with MD are building their mathematical learning in an environment of constant catch-up when they begin their learning at a level behind their TYP peers and try to build knowledge from classroom instruction. However, further study is needed to determine whether there is a threshold of informal and formal mathematics knowledge that needs to exist to benefit from classroom instruction. Our findings suggest the need for early interventions to prevent children from falling so far behind that opportunities to learn from the systems of related mathematical concepts and processes that characterize good mathematics instruction across grades are lost.

It is also interesting that gender was a significant predictor of the rate of growth and acceleration for formal mathematics skills, but not for informal mathematics skills. Such gender differences appear to emerge later given that there were not substantial gender differences on formal mathematics skills at the end of kindergarten. Some studies found that boys retrieve arithmetic facts faster than girls during calculation processes and that numerical skills develop faster for boys than they do for girls (e.g., Jordan et al., 2007; Royer et al., 1999). In addition, items on the TEMA-3 that assess formal mathematics skills include many basic arithmetic fact fluency and calculation skills. Thus, findings from prior research, the structure of items on the TEMA-3, and the fact that there is more focus on formal mathematics skills at first grade may all help explain why boys showed faster rates of growth and acceleration for formal mathematics skills. However, other studies have reported that girls have stronger calculation skills than boys in the elementary grades (Hyde et al., 1990). Moreover, some studies have reported that gender does not differentially affect mathematics skills in the early elementary grades (e.g., Friedman, 1989; Hyde, Lindberg, Linn, Ellis, & Williams, 2008). Given the mixed findings of this study and prior research, the relations between gender and early mathematics development warrant further study.

Limitations

These findings should be considered in light of several limitations. First, children in this study came from low socioeconomic status (SES) backgrounds. Although many children (approximately 74%) achieved "normal" mathematics competencies in terms of their TEMA-3 percentile scores, their performance may be still lower than that of children from middle or upper SES backgrounds. As such, the findings from this study are generalizable to other children from low SES backgrounds.

Given that all of the children in this sample came from low SES backgrounds, we do not expect that controlling for SES would result in significant changes to our findings. Nevertheless, we did not have the SES data (e.g., parental education level, income) to account for the potential differences in SES. Future studies should explore whether controlling for SES result in different patterns of findings.

Second, this study was exploratory in nature given that we did not have an independent measure to identify children with MD; scores from the TEMA-3 were the only scores available in the data set used for this study. However, the data were coded such that intercept in the growth models was centered at a different time point from the time point used to identify children as having MD to minimize any potential confounding effect of the intercept on the relation between MD status and rate of growth of informal and formal mathematics skills. In addition, the initial group differences may account for the faster rate of growth in informal mathematics skills for children with MD. However, the opposite pattern of results was obtained for formal mathematics skills, suggesting that children with MD show a distinct pattern of growth and are not simply regressing to the mean. Nevertheless, future research should use an independent mathematics assessment to establish MD status when evaluating growth in mathematics skills to confirm the results of this study.

Implications for Practice

Despite some limitations, our findings provide implications for early educators. Early intervention in prekindergarten classrooms, Head Start programs, and other care environments that focus on developing informal and formal mathematics skills may prepare children with adequate mathematics knowledge for entering kindergarten and maximizing the positive impacts of classroom instruction. However, there is generally a lack of emphasis and instruction on early mathematical development compared with literacy development at prekindergarten. Mathematics instruction rarely occurs at or prior to 3 to 4 years of age, and when it does, it is often taught incidentally through play, which may not be sufficient for mathematics learning by children with MD (Clements, Fuson, & Sarama, 2017; NRC, 2009).

We recommend that early childhood educators provide a mathematically stimulating environment through the use of mathematics language (e.g., more, less, taller, shorter) and incorporating mathematics experiences in everyday activities (e.g., counting forks and spoons at lunch, comparing quantities of snacks). They should also provide explicit mathematics instruction in comparing set sizes, mastering counting words, and understanding essential counting principles (e.g., one-to-one correspondence, cardinality), separate from gross-motor or literacy activities, and adopt evidence-based practices for at-risk children. Prior research has successfully demonstrated that mathematics achievement improves with effective interventions that incorporate explicit instruction, conceptual understanding, procedural fluency, and frequent and cumulative review with corrective feedback for children with MD (e.g., Carnine, 1997; L. S. Fuchs & Fuchs, 2001; L. S. Fuchs et al., 2008; Gersten et al., 2009). Drawing upon prior research on effective instructional practices for children with MD, *early* explicit interventions in informal and formal mathematics skills prior to kindergarten may have a positive impact on children with MD's learning trajectories and help them overcome initial disadvantages prior to entering kindergarten.

Upon entering kindergarten, children need to be screened and progress monitored for their proficiency with early numerical skills. Currently, kindergarten teachers rarely screen children for early numerical difficulties whereas most kindergarten teachers screen for early literacy difficulties (Jordan, 2010). Teachers should be trained in understanding the developmental trajectories of early numerical competencies, recognizing the early signs of difficulties based on the trajectories, and providing early interventions immediately, to prevent children from falling behind.

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