TEACHING AND LEARNING OF COLLEGE MATHEMATICS AN D STUDENT MATHEMATICAL THINKING : ARE THE LINES OF THE SAME TRACK ?

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ABSTRACT

Studies related to school mathematics have shown that students who scored well on standardized tests often are unable to successfully use memorized facts and formulae in real-life application outside the classroom. The outcome of TIMSS and PISA studies further emphasizes the importance of mathematics teaching and learning in the Malaysian education system. Various measures have been taken by the Ministry of Education to enhance the teaching and learning of mathematics in schools. However, issues related to college mathematics have yet to be addressed.In the past decade, universities have been bogged down with ranking systems(such as Times Higher Education University Rankings, QS ranking) and Quality Indicators for Learning and Teaching related to issues such as studentsgraduating on time (GOT). In this paper, we investigate the finer points of mathematics teaching and learning. Our premise is that practical knowledge (common sense) and mathematics knowledge are closely related in the learning of mathematics in college.Three case studies are discussed in this paper to highlight this premise. These studies revealed that college students gradually practices rote learning and their final grades do not reflect the development of mathematical thinking. Furthermore, the teaching approach that focuses on computation deters students from fully developing their understanding of why or when they should be applied. Teaching instructions should shift *from learning the rules for operations to understanding mathematical concepts which promotes the development of mathematical thinking. Students should be equipped with "problem solving tools" that would allow them to be accommodative to changing needs (Treffinger, 2008). It involves the acquisition and application of mathematics concepts and skills in a wide range of situations, including non-routine and real world problems to provide learning opportunities for problem solving. Hence we strongly propose mathematical problem solving as a new course central in the development of mathematical thinking at the tertiary level.*

Keywords*: Mathematics, mathematical thinking, teaching, learning, achievement*

INTRODUCTION

Researchers, educators, parents, social scientists, politicians and other stakeholders have eloquently depicted mathematics as a useful and important subject which must be mastered. Various literature reflect this as thus:

> *Mathematics is one of the most important subjects of our life. No matter to which field or profession you belong to, its use is everywhere. That is why it is necessary to have a good understanding of the subject. Imagining our lives without it is like a ship without a sail.* (Biswas, 2015)

> *Mathematics is the cradle of all creations, without which the world cannot move an inch. Be it a cook or a farmer, a carpenter or a mechanic, a shopkeeper or a doctor, an engineer or a scientist, a musician or a magician, everyone needs mathematics in their day-to-day life. Even insects use mathematics in their everyday life for existence.* (The Times of India, Aug. 3, 2015)

> *Mathematics is beautiful because it helps you discover the truth about everything, particularly about nature.* (Crean, 2015)

If Mathematics is perceived to be useful, great, timeless and beautiful, why do people abhor it too? On the contrary, there is less hatred towards Music, Art or Literature as reflected in Dudley's (1987) writing in *The American Mathematical Monthly* (note the sarcasm):

> *Mathematics is so useful that there could be no civilization without it, and it is so beautiful that some theorems and their proofs—those which cause us to gasp, or to laugh out loud with delight — Should be hanging in museums.* (p. 3)

The statement by Dudley (1987) three decades ago is definitely relevant. It is no longer justified to merely say that Mathematics is useful, especially to the 19-22 year old "Millennial" generation. They expect to be engaged in their learningand they do not do well being passive learners (Starlink, 2014). However, a majority of non-science major college graduates will testify that they do not need Mathematics beyond Arithmetic to be successful. This has raised a concern among educators who agree that Mathematics should be taught as a thinking activity and they have repeatedly called for instructors to shift their approach from the traditional computation and routine based one to a conceptual one. The former method involves teaching of rules and procedures rather than the learning of Mathematics. Is there a possibility that current traditional methodological approaches are making it difficult for graduates to see its applications in real life? In this paper, we will discuss the development of mathematical teaching and learning in college settings in relation to mathematical thinking.

Mathematics Teaching and Learning in Colleges

Research outcome on college mathematics in Malaysia has consistently depicted a dichotomous situation where on one hand, students have been obtaining good grades in their transcripts but this did not truly reflect their development of mathematical thinking. On the other hand, research has expressed near universal agreement that many students arrive unprepared for the intellectual demands and expectations after high school and struggle during their early years. The consequence are high failure rate, not graduating on time and dislike for Mathematics. The quality of college students' mathematical knowledge has always been a crucial matter. The crucial factor determining the quality of knowing is the quality of the students' experiences in constructing their knowledge in classroom instruction. Before we discuss the quality of student's mathematical knowledge in depicting their thinking capacity, we will briefly discuss the development of the teaching and learning of mathematics over the last few decades.

The development of mathematics teaching and learning has been dramatic, where in the 90s, the focus has been on computation and applying procedures in solving problems. In later stages of the 19th century and early 20th century, the conception of mathematics learning tilted from emphasis on computation towards understanding abstract concepts and relationships. This shift relied heavily on formulas to solve problems, i.e. the teaching of *what* and *why* on the conceptualization of the problems given. Then early this century, mathematical thinking has been the focus of attention. Mathematical thinking is defined as a thinking style that is guided by cognitive activities (Karadag, 2009). Ridgway, Nicholson and McCusker (2011) asserts, "*thinking mathematically is about developing habits of mind that are always there when you need them - not in a book you can look up later (p. 311).* It is a pre-built thinking of mathematical thinking in the mind of an individual when solving problems.The question to ask is whetherthe philosophical stance of mathematical learning has shifted in tandem with the evolutionary shifts in the nature of mathematics teaching and learning at higher education institutions in Malaysia.

One of the major aims of mathematical learning is the development of mathematical thinking. The common misconception is that "doing mathematics" is the same as getting involved in "mathematical thinking". This misconception stems from the pedantic mathematics education in our systems that highlight the mastery of mathematics through rote memorization of formulaic structures. The consequential impact is negatively felt when such approach is no longer viable at the higher level of tertiary education. As the focus of education shifted from repetitive impractical exercises to critical production and innovation, a more authentic and creative manner of solving problems is needed by professional mathematicians in resolving real life problems be it theoretical, mechanical, industrial or philosophical. These observations seem to point to the fact that there is a disparity between school mathematics, where success is guaranteed in conformist formulaic approach, and true mathematical thinking that requires "thinking outside-the-box", which would be more valuable to university students and professionals.

Three published research cases in literature is discussed in the following sections to support the need for promoting students' mathematical thinking. These literatures examined both quantitative and qualitative methodological approaches to learn about students' mathematical thinking and to examine their interpretive practices. Analysis of interactions between students and instructors and students' reflective writing revealed changes in the patterns of their interpretations. We characterized these as changes in the focus of interpretation, from correctness to meaning, and in the interpretive approach, from quick and conclusive to thoughtful and tentative. We will also discuss factors associated with these interpretive turns.

STUDY 1 (2012)

Parmjit, Singh and White, Allan (2012). Unpacking First Year University Students' Mathematical Content Knowledge Through Problem Solving. Asian Journal of University Education, 2, 1, 33-56.

This mix method study conducted in 2012 involved a total of 536 homogeneous groups of first year college students majoring in engineering. The researchers investigated the use of problem solving for students to "unpack" previously learned mathematics, assess understanding, reconstruct understandings, and connect mathematical concepts for deeper understanding. The researchers took into consideration the students' national examination grades (SPM) in teaching college mathematics. From this group, 84.5% obtained grade 'A'(1A and 2A) while another 15.5% obtained a 'B' (3B and 4B) in their SPM Mathematics. It is not surprising to see a large number of students with 'A' in Mathematics because one of the pre-requisites for entering college is to have a minimum of 6Cs in SPM Mathematics. Since these students were from the engineering faculty, their mathematical background is deemed to play an important role in their academic pursuit of becoming engineers. These students have been formally taught the fundamental mathematical concepts in high school and this research enabled the researchers to assess the students' quality of understanding. They also investigated if there was a relationship between SPM Mathematics grades and the Problem Solving test scores.

The results obtained from the written assessment depicted a low mean score of 24.63 (Max score=48) with a SD of 3.16. In other words, these students obtained a percentage score of 51.3% (24.63/48 x 100) in the written assessment test. The findings seem to indicate that these students have an instrumental understanding rather than a relational understanding where the data shows that they were not able to apply knowledge to new contextual situations.

To support this assertion, consider students' response to Item 8 which stated: *"If it takes 9 workers to mow a certain lawn, how long would it take 6 workers to mow the same lawn?"* (Assume that workers are all performing at the same rate and are all working for the entire time)

Here, 78.7% of the students failed to see an inverse proportion relationship and solved the question by utilizing a cross multiplicative procedure. They applied the "rules without reason" as shown below and failed to realize that the resulting answer was unreasonable

9 workers – 5 hours
6 workers – X hours;
So, X/5 = 6/9 -------- 9X = 30 ; and X = 30/9 = 3 1/3 hours.

For these college freshmen, the word proportion seemed to be equated with direct proportion. During the interviews, students were asked questions such as: "Why do you cross multiply?"; "Why cross multiplication can be used here?", "Can cross multiplication always be used in this type of problem?", "What is the meaning of cross-multiplication?", and the responses from the students were: "We were taught this way" or " I don't know".

The second outcome of this study indicates that there was no difference in the performance of the "A Math" students and "Non-Math" students in the Problem Solving Test. One would expect these college freshmen, especially those with 'A's from the SPM mathematics paper, to be excellent problem solvers. But sadly, this was not the case. The performance of the 'A' students was unsatisfactory. The question which arises here is: "How well do the current national examination grades reflect the mathematical knowledge of students?"

The conclusion from this research depicts that students have an instrumental understanding rather than a relational understanding (Skemp, 1976), where they were not able to unpack their mathematical content knowledge and apply it to new contextual situations.

STUDY 2 (2015)

Teoh, S.H, Kor, L.K, & Parmjit, S. (2015). Prototyping Routine Practices in Solving Non-routine Mathematics Problems. In Zuraimi Zakaria, Badrul Isa & Sumardianshah Silah (Eds). 21st Century Skills: Language and Arts Education through the Lens of Culture and Heritage. Malaysia: ITBM.

This qualitative study conducted in 2015 involved a sample of 25 math undergraduate students who were pre-service teachers from a public university in Malaysia. The researchers investigated learners' perspectives by providing them with an environment to solve non-routine problems and not just equipping them with skills and processes. The data from the investigation was collected via reflective writing journals and discussions after each lesson (a total of five lessons were conducted). The results show that students learned to use heuristics approaches in solving non-routine problems. From the reflective thinking, it was evident that Draw a Diagram, Systematic Listing, and Guess and Check were attempted by the respondents and these heuristics have been proven to be successful insolving non-routine problems. Further justifications in the solutions were made based on the diagram interpretation. Besides the types of heuristics identified as mentioned above, this study also found that peer-support learning played a very important role in successful non-routine problem solving.

The findings indicate that the pre-service teachers experienced positive cognitive growth in the context of thinking processes in solving non-routine problem solving. Students were empowered to process skills in a problem solving class, such as reasoning, connecting, communicating and representing mathematical ideas. They stressed that the thinking skills could be built during classroom practice. They could be trained to provide ways to contextualize the communication process in assisting learners to solve mathematical problems.The findings serve as a basis for the development of pedagogy in the teaching of a mathematical problem-solving course.

STUDY 3 (2016)

Parmjit Singh, Syazwani Rasid, Nurul Akmal, Teoh Sian Hoon, Cheong Tau Han(2017, in press). How Well Do University Level Courses Prepare Students To Be Mathematical Thinkers? Accepted for publication in The Social Sciences

This study conducted in 2016 investigated how well university level courses prepared students to be mathematical thinkers. The focal of study was based on the premise that university students, especially in the field of sciences, take various Mathematics courses throughout their degree programs (such as Calculus 1, Calculus 2, Algebra etc.) in order to graduate. The quality of students' mathematics knowledge is always a crucial. Thus the researchers used problem solving as an assessment tool because it is the means by which mathematics can be applied to a variety of unfamiliar situations to assess students' mathematical thinking. Using a descriptive design method, a paper and pencil test comprising 16 items was administered to 120 students (majoring in Mathematics, Physics and Engineering) among semester 5-6 in a college in Klang Valley. All these students have taken courses such as Calculus 1, Calculus 2 and Algebra as the requirement of their respective courses. The overall means score obtained by the students was 10.50 (SD=7.72) from a maximum score of 49. The types of errors made by university students were similar as the types made by lower secondary students based on previous research. For example:

Task 3

Eva and Alex want to paint the door of their garage. They first mix 2 cans of white paint and 3 cans of black paint to get a particular shade of grey. They add one more can of each. Will the new shade of gray be lighter, darker or are they the same?

Approximately 85.9% of the students answered this item wrongly with approximately 59.4% (n = 71) reasoning it as the same. The data from the interview reflected their problem solving skills. In fact, the students (45.3 %) used primitive additive reasoning. The reasoning is that if an equal number of cans for each type of paint is added to the mixture, the shade will remain the same. They were unable to see the proportion of white paint to the black paint before and after the addition of two cans of paint.

S_{F3} : In my opinion, it is the same if you add one can of white paint and one can of black paint as the differences are the same. If we intend the outcome to be lighter, we should put in more white paint and if we want to have a darker effect, we put in more black paint. (Pada pendapat saya, sama, jika ditambah satu tin cat putih dan satu tin cat hitam, kerana bezanya sama. Jika ingin mendapatkan yang lebih terang, kita akan menambahkan lebih banyak cat putih dari cat hitam dan jika ingin mendapatkan yang lebih gelap, kita akant ambah lebih banyak cat hitam dari cat putih).

R : Therefore, you believe that if you add another can of white paint and another can of black paint, the color will be...?
(Jadi anda berpendapat jika ditambah lagi satu tin cat putih dan satu tin cat hitam, warna adalah)

S_{F3} : Same. (Sama.)

In short, these students failed to construct a coordination of two ratios simultaneously as: 2 white to 3 black and 3 white to 4 black. Their thinking was based on the primitive additive reasoning and not the expected multiplicative thinking. The findings indicate that the university level mathematics courses taken by students did not match the level of mathematical thinking expected of them. It seems to indicate that the current university mathematics courses are based almost exclusively on formal mathematical procedures and concepts.

DISCUSSION AND CONCLUSION

The three studies cited above supports, firstly, the nature of college students' understanding of basic mathematical concepts and some critical factors to be taken into account in facilitating their mathematical thinking. Secondly, the grades obtained in their transcripts for mathematics do not reflect their mathematical knowledge in problem solving. The studies signify that the current modes of teaching mathematics at colleges are not only ineffective but also seriously stunt the growth of students' mathematical thinking and problem-solving skills.

The fundamental Mathematics courses taught in colleges today for students (major and non-major requirement) include Calculus, Algebra (modern and linear), Number Theory, Topology, Logic, Geometry, Probability etc. In the study by Parmjit and Teoh (2015), they elucidated that college students have learnt how to do numerical computation at the expense of learning how to think mathematically. The clinical interviews findings indicate that these students have an instrumental understanding rather than a relational understanding due to their emphasis on procedure rather than the process of learning. These courses have been taught throughout the years by instructors and students have been obtaining good grades in their transcripts (based on the number of students graduating with honours, Stuart & Christopher, 2012; Catherine, 2011; Parmjit, 2009). However, these grades in their transcript are not being translated into the development of their mathematical thinking (Devlin, 2013; Parmjit & Allan, 2006; Liu, & Niess, 2006).

Students in college need to 'unpack' their mathematical knowledge which they bring from school to allow them to examine the undergirding and interconnections of college mathematics with other relevant areas of mathematical application such as in Physics, Chemistry, Biology, Engineering and other related areas (Parmjit, 2009). The success of a well-prepared college student is built upon a foundation of key cognitive strategies that enable them to learn content from a range of disciplines. Unfortunately, the development of these key cognitive strategies in college (as shown in the studies cited) is often overshadowed by an instructional focus on decontextualized content and the imparting of facts necessary to pass semester-end examinations.

Several studies in both local (Intan, 2016; Aida, 2015; Parmjit & Teoh, 2016) and international contexts (Camera, 2016; Borsuk, 2016; Adams, 2014; Conley, 2003) have expressed near universal agreement that most students arrive unprepared for the intellectual demands and expectations after high school and struggle during their early years in college. Thus, these struggling students at college can quickly develop a strong negative attitude towards mathematics. Without early intervention and successful practices, the students are lost to a revolving door of remedial programs -

most of which we know do not work. These interventions have not worked for over the last two decades but we still continue to rely on them for the mathematical salvation of most of our struggling student's population in higher institutions of learning. A lot of debates have taken place in blaming schools for this problem but researchers need to ask what can be done to resolve the issue.

On solution is to teach mathematics as a thinking activity (Devlin, 2013; Liu & Niess, 2006). Another solution is to encourage the transition is by providing students with "problem solving tools" that would allow them to be accommodative to changing needs (Treffinger, Selby, & Isaksen, 2008). The teaching of such "tools" constitutes an important step towards developing problem solving and reasoning skills. Simulating classroom practices with non-routine mathematics tasks is indeed important to equip learners with the heuristics required to teach non-routine problem solving in their future mathematics classroom. Another salient approach will be toencouragethem tothink deeply about the mechanics and process of the mathematical thinking upon completing problem solving exercises. We are conjecturing that there is a dire need to introduce a Mathematical Thinking Model application in enhancing student's cognitive growth in mathematics learning. They will participate in a variety of exercises, problems, and investigations as they explore mathematics concepts from a problem solving perspective in an interactive manner. The emphasis will be on exploration of various mathematics contexts to learn mathematics, to pose problems and problem extensions, to solve problems, and to communicate mathematical demonstrations. To operationalize this development, instructors will have to shift their approach from the traditional computation and routine based one to a conceptual one, which is by getting students to think about mathematics and representing topics in ways other than procedures.

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