

Pre-Service Teachers Learning To Engage All Students, Including English Language Learners, In Productive Struggle

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Abstract

Engaging in productive struggle—grappling with challenging problems—is conducive to understanding mathematics. This paper examines five pre-service teachers as they learned to engage English language learners in productive struggle. The context of this descriptive case study was an elementary mathematics methods course in the Northeastern United States. Data collected included surveys, interviews, lesson plan reflections, and homework reflections. After selective coding, data were analyzed individually before using cross-case analysis. A possible trajectory to productive struggle is presented. The trajectory requires refinement, but it offers teacher educators insight for teaching pre-service teachers to engage all students in productive struggle.

Keywords: English language learner, ELLs, mathematics education, pre-service teaching, and productive struggle.

Introduction

Students learn mathematics with deeper meaning when they engage in productive struggle—grappling to make sense of problems within their zone of proximal development (Dixon et al. 2015; Hiebert & Grouws, 2007; Vygotsky, 1978). Teachers should increase the cognitive load by providing students less help when solving problems (Hiebert & Grouws). Warshauer (2014) added teachers can engage students in productive struggle by asking them questions instead of telling them the answers; questioning helps students organize their thoughts as they struggle to make sense of problems.

In 2005, Zeichner called for additional research on preparing pre-service teachers (PSTs) to meet the needs of English language learners (ELLs). Faltis and Valdes (2016) echoed Zeichner and spoke of the necessity of current empirical research to prepare PSTs to meet ELLs' needs. This paper, a descriptive multiple case study, is in response to these authors' calls. The study examined ways in which PSTs learned to facilitate all students, including ELLs, to engage in productive struggle.

Research Framework

Current research related to ELLs and mathematics instruction was reviewed. Murrey (2008) explained it may be necessary to provide ELLs with access to content by scaffolding the English language; however, mathematics must remain rigorous. In his research, Zahner (2012) observed a bilingual teacher who facilitated students' access to algebra by reading the questions in two languages. He found that by allowing students ample wait time, a safe environment, and opportunities to work in small groups, the students were able to solve challenging problems. Zahner added monolingual teachers could still engage ELLs to solve challenging problems by offering a supportive environment and opportunities to talk about mathematics in their own language.

Research is sparse on how to prepare PSTs to engage all students, including ELLs, in productive struggle. Four themes revealed in current research relating to such engagement in mathematics follow.

Connections to mathematics. Students are more likely to persevere at solving problems if teachers tie mathematical content to students' personal lives (Koestler et al., 2013), but teachers tend to have different backgrounds than ELLs, so it is often challenging for them to connect learning (Howard, 2006). Teacher preparation educators must prepare PSTs, including the many who have had limited experiences with ELLs, to connect with all students—a challenging task for the educators who are also predominately white and monolingual (Faltis & Valdes, 2016). To make a connection, it is not enough simply to insert ELLs' names into word problems. Teachers need to understand the students' culture (Kersaint et al., 2009).

Providing access. Many students find access to mathematics challenging because the language is denser in mathematics than in other subjects (Schleppegrell, 2010). Therefore, providing students with word problems having multiple entry points may help all students make sense of problems (Koestler et al., 2013). Maldonado et al. (2009) explained using manipulatives is another method for providing access to all students, including ELLs. The authors stated that using manipulatives not only facilitates ELLs to justify their answers verbally in English, but also provides them better access to content when their peers justify their arguments.

Productive disposition. Kilpatrick et al. (2001) defined productive disposition as “habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy” (p. 5). Students enjoy mathematics, according to these authors, at a young age, but our educational system results in many students losing this disposition. These authors argued for teachers to help students develop productive dispositions by making mathematics more engaging. If teachers have productive dispositions themselves, Kilpatrick et al. argued, they can better make sense of the ways students solve problems. Just as teachers must foster productive dispositions,

teacher preparation educators must encourage PSTs to view themselves as capable mathematicians (National Council of Teachers of Mathematics, 2014).

High expectations. Finally, the last theme revealed how important it is for teachers and PSTs to have high expectations for students. The assumption that ELLs have not mastered English leads teachers to exhibit low expectations for ELLs in mathematics, which causes teachers to provide ELLs with problems below their zone of proximal development (National Council of Teachers of Mathematics, 2014). Another obstacle associated with low expectations is teachers tend to allow struggling students less time to make sense of problems before telling them the answers (Kilpatrick et al., 2001). According to Murrey (2008), teachers may provide access for ELLs, but they should still have high expectations for ELLs' mathematical abilities. It is imperative for teacher educators to prepare PSTs to have high expectations for ELLs' abilities in mathematics (Turner et al., 2012). This review of literature provided the foundation for the descriptive case study.

Methodology

Context. In a northeast research university in the United States, the researcher taught an elementary mathematics methods course which was the context of this study. Of the 22 PSTs enrolled in the course, 17 were sophomores, and five were juniors. There was one male, and the rest were female. The researcher and participants met for fifteen sessions with each class lasting about three hours. Before taking the mathematics methods course, the PSTs had taken at least two mathematics content courses.

At the beginning of the semester, in a homework assignment, the researcher learned how the PSTs had been taught mathematics by asking them to write their experiences in mathematics. Towards the middle of the course, the researcher emphasized the importance of engaging all students, including ELLs, in mathematics. For one class, the researcher delivered content in Spanish and purposely did not use visuals or gestures. PSTs were not allowed to converse in English during this lesson. The researcher recorded their small group discussions after the lesson. Except for one PST who understood Spanish, all mentioned they did not have access to the content. In a homework reflection, the PSTs were asked to discuss how the instructor could have made the lesson content more accessible to them.

Throughout the course, the researcher discussed the importance of engaging all students in productive struggle. As a course requirement, the PSTs were to teach in the field a unit of three consecutive mathematics lessons. University supervisors observed PSTs teach their mathematics lessons in elementary schools with urban or suburban settings. To solidify their understanding of productive struggle, the researcher requested the PSTs to include in their lesson plans how they were going to engage all students in rigorous mathematics. After teaching their lessons, the PSTs reflected on productive struggle.

Participant selection. From the class of 22 PSTs, ten were purposefully selected: five who appeared to have connections with ELLs, and five who had none. The five who had ELL connections included two who had ELLs in their placement, two who were bilingual, and a PST who had 11 ELLs in her placement before taking the mathematics methods course.

Eight of ten PSTs agreed to an interview after the course grades were posted. The PSTs field host mentor teachers and university supervisors were invited to participate in the separate interviews. All eight of these PSTs were included in a larger study, but the researcher reports here on the five who have data most relevant to this paper. Two obvious participants were Abigail and Kim (pseudonyms), the ones who had ELLs in their placement; the other three were selected because their cases varied widely. Of these, Charles (pseudonym) was bilingual; Amanda (pseudonym) mentioned in interviews and homework assignments that she valued challenging her students, which is said to be related to productive struggle (Hiebert & Grouws, 2007); and Sarah's comments about the course were the most negative the researcher received during the initial interviews and homework reflections.

Data collection. Since the researcher was the instructor for the mathematics course, a colleague passed out the consent forms to the PSTs on the first day, and the researcher did not know who had given permission to use their data until 48 hours after the grades were posted. The researcher collected open response surveys (pre- and post-) during which the researcher asked the PSTs to describe past experiences with ELLs, whether they spoke another language, and to imagine how they might accommodate ELLs in mathematics. For each participant, the researcher collected data from 15 homework assignments based on readings and group discussions. PSTs also prepared guided lesson plans, which incorporated reflections on use of productive struggle for their teaching experiences in the field. University supervisors and host teachers completed forms describing how the PSTs addressed issues of diversity, language, and productive struggle in mathematics lessons. The researcher implemented, to the invited PSTs semi-structured interviews at the end of the course, asking indirect questions aimed at determining what resources they drew upon to help all students in mathematics. All the above interviews were recorded and generally lasted an hour.

Data analysis. The focus was not to evaluate the researcher's teaching, but to describe the PSTs' experiences as they were learning how to teach mathematics to all students. Audio data for each individual PST was transcribed and analyzed before using a cross case analysis (Miles & Huberman, 1994). Selective coding was implemented and permitted the researcher to identify the sources of collected data. Two of the PSTs described the importance of guiding students rather than telling them the answers, and a code for "productive struggle" was used. Unexpected themes were embraced (Miles & Huberman), and when two PSTs said that they would reduce the cognitive load for ELLs, a code for "expectations" was included. In addition, the concepts outlined in the themes provided the foundation of the content taught in the mathematics methods course.

Results

Four themes emerged from the data analysis suggesting a trajectory supporting productive struggle: connecting content with students, providing access, high expectations, and productive dispositions. A description of these themes follows.

Connect content with students. Data analysis revealed all of the participants had experienced times when mathematics was connected to their personal lives, and they reflected on these experiences throughout the semester. One common thread from the PSTs was "connection" meaning teachers had to implement strategies focused on

showing meaning and relevancy. The PSTs reflected in their lesson plan about how they had connected their students to the content when they taught in the field (Table 1).

Table 1
Examples from lesson plans of PSTs connecting content for their students

Pseudonym	Excerpt from lesson plan
Abigail	I incorporated being culturally relevant into my lesson by making real world connections with my students so that they may be able to see the importance of learning the material for their personal benefit.
Amanda	It was a riddle about fractions, and there was a person pretending not to understand fractions who asked for their help with learning fractions, so basically I called them fraction experts. We need to learn this so we can teach someone else... it was pretty motivational..
Charles	I will be telling you a story to help us understand how math can be used in the real world...What are some things that you guys do during the summer?
Kim	I will start with a story about how I baked brownies for the kids and gain his interest in the delicious dessert and to be relevant to real life.
Sarah	At my center, there are no word problems. I write a problem vertically and visually, and they do the same. There is no relating to life because they are just working with numbers and equations.

Throughout the semester the PSTs used an inquiry-based lesson plan called Launch, Explore, Summary (Lappan et al., 2002). Both Abigail and Charles mentioned how they would connect their students to the lesson through real world connections to personal experiences. The school where Sarah was placed followed a program that used scripted lessons, and her host teacher asked her to use them too. Thus, unlike the other PSTs, she did not write her own lesson plans; they were provided to her. Sarah discussed connecting with students in the methods course; however, she felt that she was unable to apply this in the field.

Provide access for students. The PSTs were in an inclusion program and learning how to provide access for all students was infused throughout. In the mathematics methods course, the PSTs reflected on how access was provided to them in homework reflections. For example, Amanda pointed out that when the researcher taught a lesson in the course in Spanish, they were denied access to the content. She states, “In the end, we discovered the struggle ELLs face when teachers do not have the training to modify their lessons.”

Table 2 exemplifies how the PSTs, except for Sarah, applied what they had learned by making accommodations for their students, thereby providing their students access to mathematics lessons. Unlike the first four PSTs, Sarah felt she had been restricted from providing access to her students because she was given scripted lessons and could not veer from the lesson plan to meet her students’ individual needs.

Table 2

Excerpts from lesson plans in which PSTs provided access for their students

Pseudonym	Excerpt from lesson plan
Abigail	I learned to make learning math more accessible to ELL students which were simplify language, use manipulatives and provide a sufficient amount of time.
Amanda	The kids aren't going to be able to do this [said the teachers as they were planning.] That's a difference, that's a huge difference I saw with the collaboration between the teachers and the collaboration between Kim and me. We'd be like, "Okay well, how can we get them to do this?"
Charles	To accommodate for my students' needs, I made sure to make my material challenging for high achieving students.
Kim	I made the lesson accessible for all students—culturally relevant.
Sarah	I didn't use them [accommodations] so much because I couldn't come up with my own lessons so I couldn't include students' interests or put in manipulatives.

High expectations. Whereas four of the five PSTs had learned to connect content and provide access to their students, only three appeared to have acquired high expectations for ELLs.

PSTs with high expectations. Amanda, Charles, and Kim reflected on the importance of having high expectations for all students. In an interview Amanda stated how she had high expectations for her students when she had taught them, "This is why I decided to call my students mathematicians at the beginning of my lesson. It's something so small but can make a huge difference when you presume competence in your learners." Amanda's allusion to "presumed competence" indicates she assumed all of her students were capable of solving her mathematics problems. Charles also used the term "presumed competence" and argued teachers need to have high expectations for ELLs' abilities in mathematics regardless of their English proficiencies. Furthermore, Kim stated in a reflection that "it is not okay to assume the ELL student is behind in math because of lack of proper schooling." These three PSTs appeared to have had high expectations for all of their students.

PSTs with lower expectations. Abigail and Sarah displayed lower expectations for ELLs. When asked how she would teach mathematics to ELLs, Abigail said, "You can have the same worksheets but you can say that they can do two or one or something like that so they won't feel pressure to do the same thing as everyone else." This quote exemplified Abigail implied she needed to lower the standards for the ELLs by giving them less work. Despite having discussed "presumed competence" in another methods course, Abigail did not apply this concept to ELLs in the interview when she referred to them as "hopeless," "lesser," and "language difficult." She had low expectations for ELLs and failed to recognize them for being language experts. Not only did Abigail imply that the ELLs would perform lower than non-ELLs because of the language barrier in monolingual classrooms, she also said that the ELLs might not be able to solve the problems in their own language.

Although Sarah did not use derogatory language, her reflections suggested she might have low expectations for ELLs. In an interview Sarah said, “I just can’t imagine a student who doesn’t understand any English being able to learn in a classroom. I just can’t visualize it. I can’t understand how they can learn math and understand it the same as everyone else.” Despite discussing in the mathematics methods course strategies for providing access for ELLs so they could engage in solving rigorous problems, Sarah could not picture ELLs achieving the same results as non-ELLs. Her view focused on the ELLs’ lack of proficiency in English rather than their strengths and mathematical abilities.

Acquire productive dispositions. This section is divided into two—PSTs who demonstrate signs of having productive dispositions and those who do not.

PSTs with productive dispositions. Amanda appeared to have a productive disposition as she was learning mathematics growing up. In the methods class she reflected about her past mathematical experiences, “I really enjoyed solving problems and learning new strategies. In elementary school, I tested into the gifted and talented program for math. I was definitely challenged by the curriculum.” Amanda also stated she had struggled in calculus, but she worked hard to get a good grade, implying that her efforts were worthwhile.

In a homework reflection Charles was critical of the way he was taught mathematics as a student by expressing, “You just learn one way. You’re taught, ‘Here is the way we solve 16×15 .’ And there is one specific way.” Charles appeared to initially have had an unproductive disposition because he said he had negative experiences and despite his struggles, he did not tend to retain the concepts. However, by reflecting deeply in the methods course about his teachers’ ineffective pedagogies, his disposition shifted from unproductive to more productive. After teaching a lesson, Charles reflected he allowed his students ample time to make sense of problems because he was denied these opportunities as a student. Therefore, despite Charles’ negative experiences as a student, his deep reflections allowed him to improve his disposition.

PSTs with unproductive dispositions. Despite Abigail mentioning at times she was successful in mathematics, she frequently associated mathematics with assessments as this quote exemplifies: “As long as the teacher is actually teaching what is supposed to be taught and will be tested on, I am pretty much fine with taking any math course. When I get confused or stumbled, I will ask for help.” Abigail reflected she wanted her teachers to cover the material in which she was tested. However, Kilpatrick et al. (2001) maintained that students with productive dispositions enjoy mathematics not simply because they succeed in assessments but because they find the process enjoyable. Abigail recognized there were a lot of assessments, but unlike Charles, she was not critical of her teachers’ approaches. Indeed, Abigail was overly-focused on getting the right answers and would ask her teachers for help, rather than enjoying the process and making sense of problems on her own.

Sarah also appeared to have an unproductive disposition for mathematics. She wrote in a homework reflection that she liked mathematics at times, “When I understood math, I loved it but when I was confused and lost in a class, it was my absolute worst subject.” This reflection from Sarah suggested she likes mathematics as long as it was not confusing; yet Hiebert, Morris, & Glass (2003) argued that working through confusion can help students develop meaning of mathematical concepts. Unlike Charles who

shifted from an unproductive to a more productive disposition by reflecting on better teaching practices, Sarah was not critical of the pedagogy her teachers had employed with her.

Productive struggle. The next section will discuss PSTs who facilitated productive struggle for all students and those who did not.

PSTs who facilitated productive struggle for all students. One PST who focused on productive struggle throughout the semester was Amanda. She commented she was challenged as a student and wanted all of her students to be challenged as well.

In an interview Amanda disagreed with how her host teacher “would tell students the answers.” Similar to Amanda, Kim was disappointed with the way her host teacher instructed her students. Kim tried to engage the students in mathematics so they would make sense and persevere at solving problems themselves. In an interview Kim described how she observed students were bored with her teacher, so she attempted to make the students more excited about mathematics through engaging students in group activities. Kim believed if she made the content more exciting, then her students would persevere to solve them with less teacher guidance.

PSTs who did not appear to facilitate productive struggle for all students. There are two incidents when Abigail appeared to have reduced the cognitive load for her students. In her lesson plan reflection Abigail wrote: “I also had to stop at the second center to do a mini lesson on how to correctly make equal parts.” Unlike Amanda who shared how she had not interfered as her students made mistakes, Abigail corrected the students’ mistakes. This action prevented students from developing a deeper understanding of the mathematical problem through productive struggle. Furthermore, for students who finished assignments early, Abigail would allow the students to read a book instead of challenging them with problems that facilitated productive struggle.

Sarah was another PST who did not appear to challenge her students to make sense of problems when she taught in the field. In her description of how she taught her students in centers, she did not encourage the students to make sense of problems themselves: “As a group, we understand the problem before solving it.” In the methods class we discussed how students should make sense of problems on their own and persevere to solve them, but Sarah’s interpretation of these concepts was shallow. She said after understanding the problem, they would solve more problems on their own. However, by discussing how to solve the first problem, Sarah reduced the cognitive load for the remaining problems—if they had already understood, the struggle would be reduced.

Discussion

Four of the PSTs connected the content with their students’ backgrounds and provided access for their students to make sense of the problems. These two themes were infused throughout the program, and the fifth PST, Sarah, discussed these themes in her methods classes but felt that she did not have opportunities to apply them in the field. Abigail and Sarah did not appear to have high expectations for all students, have positive dispositions, or facilitate productive struggle for all students, while Amanda and Kim showed signs of all of these.

One explanation for the differences among the PSTs’ progress was the themes of having high expectations for ELLs and productive dispositions were not infused

throughout the program. In one methods course, the PSTs were introduced to the concept of “presumed competence” in which they discussed the importance of having high expectations for all. However, Abigail and Sarah did not apply this concept to ELLs.

A second possible explanation for why some PSTs engaged all of their students in productive struggle while others did not could be that their dispositions towards mathematics varied. Amanda and Kim had positive dispositions; both engaged their students in productive struggle. Abigail and Sarah did not appear to have productive dispositions when they were students of mathematics, nor did they engage their students in productive struggle. Charles, in contrast, shared he had negative experiences towards mathematics as a student, yet he showed signs of engaging his students in productive struggle. One of the differences between Charles and the other two PSTs is that he was critical of the way he had been taught mathematics. He developed a plan during the methods course of how mathematics should be taught. Abigail and Sarah did not reflect deeply on how they could have been taught differently. Additional studies could analyze the process of how PSTs can adopt positive dispositions in methods courses after having negative experiences as students of mathematics.

A third possible explanation of why there is a variation in mastering the themes is examining whether there is a sequence of learning them. A possible trajectory is depicted in Figure 1. In this trajectory, when a PST learns to connect math to the students personally, provides students access to mathematics, and has high expectations for students before acquiring a productive disposition, these in turn may lead to facilitating productive struggle for all students.

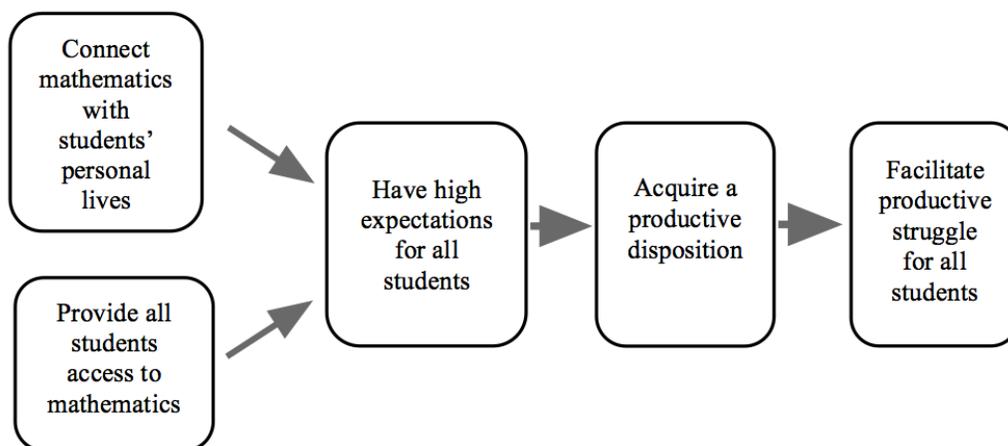


Fig. 1

Possible trajectory for pre-service teachers to facilitate all students in productive struggle

Sarah may be at the beginning of the trajectory. She discussed making content relevant and providing access for students, but she did not apply these themes in the field, whereas the other four did. Abigail moved further along the trajectory—she showed signs of teaching the first two themes in the field, but did not have high expectations for ELLs, did not have a productive disposition or engage all of her students in productive struggle. Amanda and Kim were far along the trajectory as they showed signs of all these traits. At the beginning of the semester Charles was in the middle of the trajectory

because he did not have a productive disposition, but after deep reflection in the mathematics methods course, he was able to advance along the trajectory and engage his students in productive struggle.

Conclusions

Further research is needed to test and refine this trajectory, but it offers teacher educators suggestions for how they might teach PSTs to engage all students in productive struggle. Daro et al. (2011) maintained that trajectories provide opportunities to examine both how students learn and the order of their learning. They explained that each student may have slight variations, but using trajectories can help educators observe student-learning trends. Each PST will come to teacher education programs with a variety of experiences and will be at different stages along the trajectory.

The trajectory is linear; however, as with trajectories discussed by Daro et al. (2011), this is not meant to imply that PSTs are unable to engage in the previous themes after advancing along the trajectory. For example, PSTs might acquire productive dispositions and high expectations for all students at the same time or in a circular fashion (Turner et al., 2012). Nevertheless, this trajectory may afford useful insights into how PSTs learn productive struggle.

The teacher education program at this university seemed effective at infusing the themes of connecting students with the content and providing access to the content. PSTs needed more opportunities to reflect on the importance of having high expectations for all students, including ELLs. Lastly, many elementary PSTs have negative dispositions towards mathematics and should be given opportunities to reflect on this in methods courses. Charles had a negative disposition, yet his disposition improved as he reflected on best practices, and later he engaged his students in productive struggle. As suggested in the trajectory, infusing these themes throughout teacher education programs may help PSTs move along the path of facilitating all students to engage in productive struggle.

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