ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume 10, No. 1, January 2019, pp. 21-36

STUDENTS' COGNITIVE PROCESSES IN SOLVING PROBLEM RELATED TO THE CONCEPT OF AREA CONSERVATION

Rooselyna Ekawati, Ahmad Wachidul Kohar, Elly Matul Imah, Siti Maghfirotun Amin, Shofan Fiangga

Universitas Negeri Surabaya, Jl. Ketintang Gedung D1 Surabaya 60231, Indonesia
Email: rooselynaekawati@unesa.ac.id

Abstract

This study aimed to determine the cognitive process employed in problem-solving related to the concept of area conservation for seventh graders. Two students with different mathematical ability were chosen to be the subjects of this research. Each of them was the representative of high achievers and low achievers based on a set of area conservation test. Results indicate that both samples performed more cyclic processes on formulating solution planning, regulating solution part and detecting and correcting error during the problem-solving. However, it was found that the high achiever student performed some processes than those of low achiever. Also, while the high achiever student did not predict any outcomes of his formulated strategies, the low achiever did not carry out the thought process after detecting errors of the initial solution gained. About the concept of area conservation, the finding also reveals that within the samples' cognitive processes, the use of area formula come first before students decided to look for another strategy such as doing 'cut-rotate-paste' for the curved planes, which do not have any direct formula. The possible causes of the results were discussed to derive some recommendation for future studies.

Keywords: Students' cognitive processes, Area conservation, Problem-solving

Abstrak

Penelitian ini bertujuan untuk menentukan proses kognitif yang digunakan siswa dalam memecahkan masalah yang berkaitan dengan konsep konservasi luas. Dua siswa kelas VII dengan kemampuan matematika yang berbeda, yang masing-masing merupakan perwakilan dari kelompok berkemampuan matematika tinggi dan rendah berdasarkan seperangkat tes konservasi luas dipilih untuk menjadi subyek penelitian ini. Hasil menunjukkan bahwa kedua sampel melakukan lebih banyak proses siklik dalam merumuskan perencanaan solusi, melaksanakan rencana solusi, serta mendeteksi dan mengoreksi kesalahan selama menyelesaikan masalah. Namun, ditemukan bahwa, siswa berkemampuan tinggi melakukan lebih banyak proses siklik daripada yang berkemampuan rendah. Juga, sementara siswa berkemampuan tinggi tidak memprediksi hasil dari strategi yang diformulasikan, siswa berkemampuan rendah tidak melakukan proses pemikiran lanjut *(thought process)* setelah mendeteksi kesalahan dari solusi awal yang diperoleh. Dalam kaitannya dengan konsep konservasi luas, temuan ini juga mengungkapkan bahwa dalam proses kognitif subjek yang diteliti, penggunaan rumus luas datang terlebih dahulu sebelum siswa memutuskan untuk mencari strategi lain seperti melakukan 'potong-putar-tempel' untuk bangun datar melengkung, yang tidak memiliki rumus yang tetap. Kemungkinan penyebab hasil dalam penelitian ini didiskusikan untuk memperoleh beberapa rekomendasi untuk penelitian selanjutnya.

Kata kunci: Proses kognitif siswa, Konservasi luas, Pemecahan masalah

How to Cite: Ekawati, R., Kohar, A.W., Imah, E.M., Amin, S.M., & Fiangga, S. (2019). Students' Cognitive Processes in Solving Problem Related to the Concept of Area Conservation. *Journal on Mathematics Education, 10*(1), 21-36.

Area measurement is one of fundamental topics in mathematics. The measurement of the area of plane figure invite to the study of further mathematics and its application. The measurement becomes the essential competence that build upon scientific knowledge for all fields and careers (John et. al. 2011). Therefore, the idea of area measurement should be taught carefully such that the students could gain a well conceptual understanding of it. Kordaki (2003) asserts that in understanding the concept of area, students need to integrate three interrelated aspects: concept of area measurement, area formulae, and area conservation. However, the latter concept is often isolated from the first two concepts when students solve an area-related problem (Kordaki, 2003). In addition, in the teaching of area measurement topic, the teacher tend to focus only on the use of formula. Unfortunately, most of the area measurement teaching gives the area formula too early for the students (e.g. Kordaki & Balomenou, 2006; Kospentaris et al., 2011; Papadopoulos, 2010). In fact, the idea of area conservation is deeper than finding the relation of area formula. The use of the formula in measuring the area of plane figure is considered as procedural algorithm only (Fauzan, 2002). In fact, relating to the area conservation skills, most of the pupils have difficulties in decomposing problems (Kordaki & Balomenou, 2006). They are unable to see that decomposing shape into another form would make the area of the figure invariant. Therefore, the students decide to the shortcut by only interested in the formula from which a non-meaningful learning is resulted. This fact results that the students understanding on area measurement is limited to procedural only. In fact, remembering the formula is becoming the main problem on students learning not only in mathematics subject but also other science subjects. Therefore, the topic of area conservation plays an important role in the development of students reasoning on area measurement.

Area conservation can be defined as quantitative value of a certain area of figure remains unchanged after the figure is altered (Smith et al., 2011). Piaget, Inhelder & Szeminska (1960) stated that the term “conservation” means the invariance of the quantity value of the area of a plane while the plane may be transformed into a qualitatively different one. For example, students need to understand that when a shape is divided into several parts and these parts are re-arranged, the area remains the same. To state how wide the area of the figure is, a unit is selected and integrated until shape of the figure is fully covered. When arranging units into rows and columns, students can understand the area depending on the number of rows and the number of columns that there is a multiplicative relationship between these numbers. The studies of students’ performance regarding the concept of area conservation have been reported by previous research with regard to some point of interest, such as students’ error and misconception (Sisman & Aksu, 2016), students’ solution strategies (Kospentaris, 2011), links between students’ performance on the problems related to non-measurement and calculation tasks in area measurement (Tumová & Vondrová, 2017). The findings of studies is considered by many scholars as the preliminary step in understanding students’ adequate mastering of area measurement (Clements & Stephans, 2004; Kospentaris et al, 2011). However, limited studies found to concern on how students perform their cognitive processes when solving area conservation-related problem. Therefore, In this study, we stress the need for the investigation into the nature of students’ abilities by exploring their cognitive processes required for the improvement of students’ performance on the topic of area conservation.

Cognitive processes may be described as online mental activities that are proactive in nature (the “to do” strategies) (Montague, Krawec, Enders, & Dietz, 2014). In a similar vein, cognitive processes are defined as the mental processes of an individual, with particular relation to a view that argues that the mind has internal mental states (such as beliefs, desires and intentions) and can be understood in terms of information processing, especially when a lot of abstraction or concretization is involved, or processes such as involving knowledge, expertise or learning.

Some scholars have derived some stages of cognitive processes. For example, Montague, Warger and

Morgan (2000) through the cognitive strategy instruction: *Solve It!* Believes that cognitive processes incorporate the activities of reading (identifying relevant/ irrelevant information), paraphrasing (rewording the information of the problem without changing the problem meaning), visualizing (transforming problem information to a representation that shows the relationships among problem parts), hypothesizing (setting up a plan to solve the problem by deciding on the type and order of operations), estimating (predicting the outcome based on the question/goal), computing (conducting the basic operations needed for solution), and checking (reviewing the accuracy of the process, procedures, and computation). Another cognitive process was offered by Montague (2002). The processes incorporates some stages: comprehending linguistic and numerical information in the problem, translating and transforming that information into mathematical notations, algorithms, and equations, observing relationships among the elements of the problem, formulating a plan to solve the problem, predicting the outcome, regulating the solution path as it is executed, and detecting and correcting errors during problem solution.

In this regard, the cognitive processes can be traced along the way how a learner process his/her thinking based on the types of reasoning mainly demanded by the tasks, i.e. non-measurement reasoning or measurement reasoning. Since in this study, we focus on measurement reasoning, the cognitive processes were measured following the stages of Battista (2007) from the use of numbers which not connected to unit iteration, the employment of unit iteration and enumeration which includes units properly located only along the sides/edges, the operation of numerical measurement, and the integration of measurement and non-measurement reasoning, such as understanding formulas for non-rectangular or composite shapes or determining the value of particular shapes based on a quantitative context inherent in the problem being solved . In fact, according to Montague (2002), students simply may not know "what" to do or even "how" to think about beginning the problem. In addition, if students are not asked how they solve a particular problem and if the work and explanations that accompany their answers are not observed properly, a researcher learns a little about students' understanding and misunderstanding of mathematical ideas (Stylianou et.al, 2000).

Thus, this study took a part of carrying out an in-depth investigation of what students were thinking while they performed their cognitive processes on the problem related to area conservation. The cognitive processes model guiding this investigation is based on modification of Montugue's model of cognitive process, in which thought process and extending problem of Mason's (2015) model are added in the model.

METHOD

Sample of Research

Prior to selecting the student interviewees participating in the interview session, as many as 25 seventh graders with various background in terms of gender, mathematical ability, and verbal communication from a private junior high school in Surabaya city participated a test consisting of five items examining their mathematical ability particularly around the concept of area conservation on two dimensional figure. They were asked to do the test in 45 minutes. They were also informed that their work would not be graded so that they could use their own methods to solve the tasks.

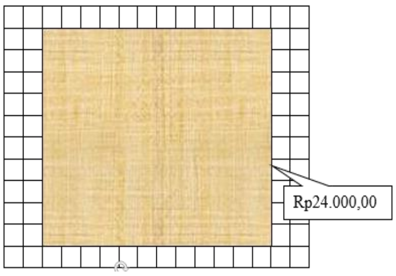
The result of the test informed that approximately half of them were in the group of high achievers (score >60 out of 100) and half in the group of low achievers (score ≤ 60 out of 100) based on their written test performance. As many as two samples were recruited from each of those two groups as the representatives by considering same gender as the control variable, ease of verbal communication based on information from their mathematics teacher and willingness to participate. Beside, to ensure the subject, we also confirm with data of students' mathematics performance. Thus, we had one male student having good score/High Achiever Student (code as HAS) and the other one male student having low score/Low Achiever Student (code as LAS). The data were analyzed qualitatively.

Instrument and Procedures

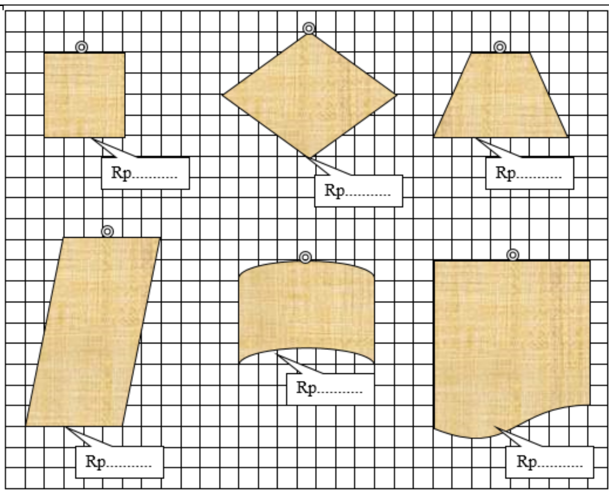
Data were collected from the samples' work on written test which is different from the test given in the initial stage of selecting samples and follow-up interviews. First, students worked on two area conservation-related tasks in 30 minutes. The first task was arranged by the authors in quantitative approach in which the real-world situation was embedded in the tasks, while the second task was developed by the authors relying on students' quantitative approach without any real-world situation. Furthermore, those two tasks were developed around the view of Euclid's elements, in which the practice of measuring area is the use of "additivity axiom", i.e. dividing one figure into some parts which rearranged would form another figure, in order to prove the area equivalence of the figures (Freudenthal, 1986). Thus, instead of only focusing *static* perspective of area measurements, the tasks also focus a *dynamic* perspective where the qualitative approach: emphasizing the conservation of area without the use of numbers (Hiebert, 1981). Those tasks were then validated by experts in terms of content, construct, and language as well as by learners, i.e. students' aside the samples to examine the practicality such as the ease of and presentation of picture and table. See those two task at Figure 1.

Task 1

Eko gets an assignment from his father to give the suitable price tag on the piece of wood he will sell. His father gave a standard price for a rectangular piece of wood measuring 12 cm x 10 cm, which is IDR 24,000

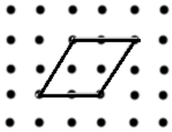


In each of the following pieces of wood, give the suitable price in the available price column.



Task 2

Look at the parallelogram below.



which of the figures below having the same area with the above parallelogram? Explain your reason.

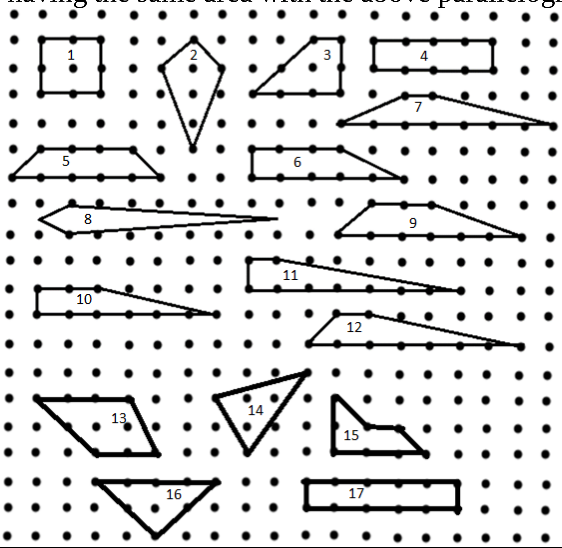


Figure 1. The dynamic area conservation tasks used in this study

In the following day, we interviewed and videotaped the two samples. Table 1 describes the interview protocol that guided the interviewer to collect data. However, this protocol does not mean to guide the interviewers used all the question items too rigidly. Rather, it plays role as the tool to confirm some particular subject's responses. This is to keep the subject reveal their thinking processes as naturally as possible. Thus, when the responses of the subject did not indicate particular cognitive process to occur during the interview, the interviewers did not ask such processes further.

Table 1. Protocol for interviewing the subjects

Cognitive process	Examples of item questions
1. Comprehending linguistic and numerical information in the problem	Please read the question to me. If you don't know certain word, say it. Tell me what the question is asking you to do. How do you understand the meaning of the particular information of this question? Which words / sentences / parts of graphics that makes you difficult to understand have you not yet identified from this problem?
2. Translating and transforming that information into mathematical notations, algorithms, and equations	Have you ever encountered this context before? In what ways? What kind of mathematics do you usually find in relation to this context?
3. Observing relationships among the elements of the problem	Which information from the sentence in the question that you think is important to use in the process of finding answers? Is there any missing information?, Can you recognize the pattern / relationship of the information provided by the problem ?
4. Formulating a plan to solve the problem, predicting the outcome	Tell me how you are going to find the answer
5. Predicting outcome	What might happen if you carry out your plan?
6. Regulating the solution path as it is executed	Show me what to do to get the answer
7. Detecting and correcting errors during problem solution	Tell me how do you convince yourself about your answer Asking about the relationship between the mathematical results obtained by the questions on the question, such as the question: "Are you sure your answer makes sense to answer the question of the problem?, Is there an image or the like that you made to strengthen your answer?
8. Thought process	Do you think there are other possible ways to find your solution?, Ask about the possibility of alternative solutions that students think about, such as through questions: do you think there are other ways to solve this problem? What is your idea? If there are, try showing how you use
9. Extending problem	Is your method of solution applicable for any cases of problem similar to this problem?

Data Analysis

Data of interview were analysed by firstly reducing data, displaying data, and finally drawing conclusions and verification (Miles & Huberman, 1994). The conclusion was sought to understand the most dominant pattern of cognitive processes performed by samples within their problem-solving activities on the tasks. To analyze data interview, we employed a modification of cognitive processes from Montugue (2002). The modification regards to the addition of one more stage as the last stage following the recommendation of Mason in which in the last stage, a solver should not only accentuate an analysis of answers, but also carry out the thought process and problem extension (Mason, 2015). Figure 2 shows the stages which possibly occur during solving a mathematics problem. The arrow direction indicated in figure 2 points out that a solver may follow a cyclic process where the solver moves back and forth, perhaps getting stuck and having to take steps back along the way (Mason, 2015). For instance, there is a possibility that a solver moves back to the stage of observing relationships among information when he/she gets stuck in formulating a solution. In addition, the cycling process can occur for more than two times depending on the degree of his/her confidence and plausibly of solution strategies obtained. Furthermore, the arrows presented in the stages in figure 2 indicate the logical progression from one process to another although it is possible for a student to skip any of these processes, or they can just jump from one process to another process when they change their solution process. For example, when trying to regulate the solution using a plan the student has derived, he/she may be directly arrive at the stage of regulating the solution path as it is executed. Thus, he/she skip any activities indicated in the stage of predicting any outcome. The model of using arrows in analysing the stages that might occur on student's cognitive process are proven as a helpful tool for keep track student's behaviors (Yeo & Yeap, 2010). While the model of analysing students' cognitive processes employing Montugue's (2002) model has been used by Jones (2006) to track the existence of the Montugue's stages of cognitive processes, there are still lack findings reporting the Montugue's stages which consider both the existence and the order of process of the stages. Thus, in this study, the modified Montugue's model in terms of the dynamic processes which might occur during student solution process indicated by the arrows of the stages were used as a tool of analysis shown in Figure 2.

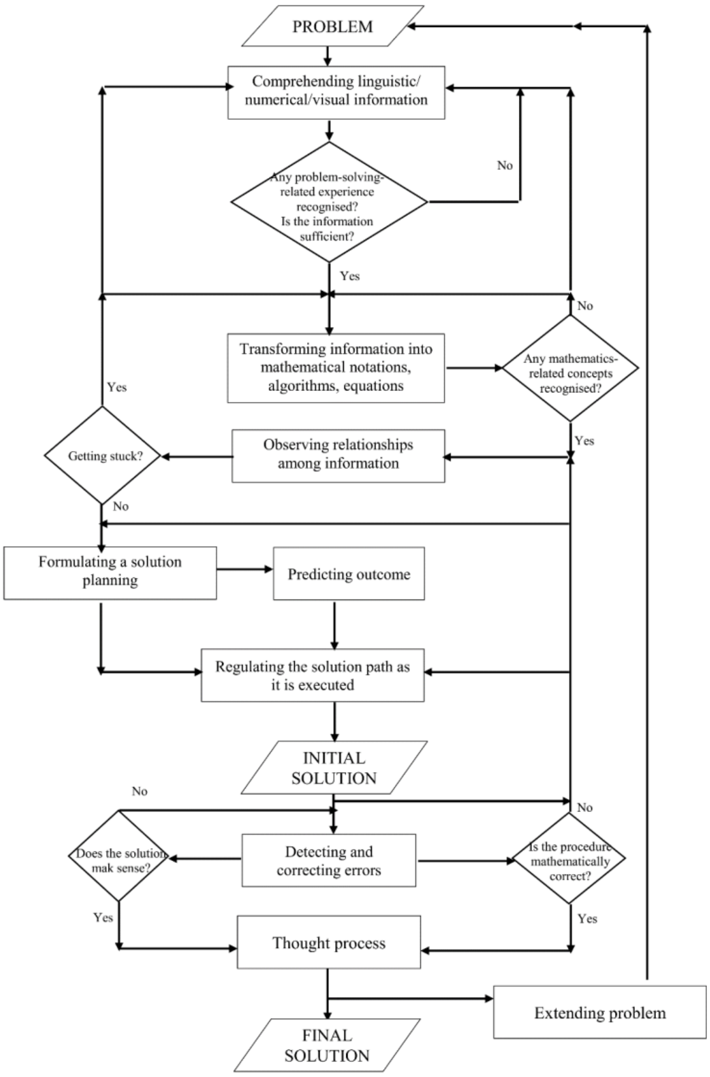


Figure 2. Framework of analysing students' cognitive processes

RESULT AND DISCUSSION

The Cognitive Processes of HAS

On the first task, HAS begun his cognitive processes by claiming to have encountered similar problems, namely about the area of land in various forms of two dimensional figures. He then mentioned information that is known: rectangular pieces of wood whose size is limited and called unknown but necessary information: no price per ‘cm’. These all activities are included in the process of comprehending linguistic, numerical, a spatial information. In the stage of translating and transforming that information into mathematical notations, algorithms, and equations, HAS redrew the figure in the problem to show the intended part per ‘cm^2', which refers to the area of the rectangle. He then observed relationships among the elements of the problem by mentioning that to find the price of the piece of wood in the problem, he needs to find the price per cm^2 or the price per square cm first. He continued his processes by starting to formulate a plan to solve the problem. He said, “To find prices per cm^2, it is necessary to divide the total price by the area of the rectangle. Then, I need to determine the price of each piece of wood by multiplying the price per ‘cm^2’ with the area of each of planes that has been calculated using the formula of plane area I ever studied”. When asked which planes he worked at first, he intended to determine the order of plane (i.e. (1) square, (2) trapezoid, and (3) rhombus) based on the formula of area. However, he had no idea for the curved planes and skipped the process of predicting outcome and continued with regulating the solution path as it is executed. Within this process, he calculated the price per ‘cm^2', obtaining 24000: (12x10) = 24000: 120 = 2000 rupiahs, then calculated the price of square pieces of wood, namely 4x4x2000 = 32,000 rupiahs.

Furthermore, detecting and correcting errors were executed by being aware of his incorrect calculation, namely the price per cm^2 should be 200 rupiahs, instead of 2000 rupiahs. That is why he repeated the process of regulating the solution path as it is executed by recalculating the price of the square–shaped wood, finding 4x4x200= 3,200 rupiahs. Likewise, on the trapezoid-shaped wood, he found the price is ((7+3) x 4)/2 x 200 = 4000 rupiahs. However, he was not sure of the formula he used for finding the price of parallelogram-shaped wood because he forgot whether to use the base x high formula to determine the area of the parallelogram although he finally used the formulate. Therefore, he reformulated his plan to solve the problem by reviewing the method he used. According to him, he needs to find other ways that do not rely on memorization of the plane area formula. In this regard, he said, “I start thinking of looking back at the parallelogram-shaped wood, then drawing line (altitude) from the upper left corner of the parallelogram. Thus, I found a triangular shape that if moved and pasted to the right side will form a rectangle.” In this case, he used ‘cut-paste’ technique. Since he thought this technique was successful, he then used it to solve other quadrilateral-shaped woods. He got stuck on using this technique for rhombus-shaped wood. He observed, “I need to get what I call as ‘a target plane’, that is a plane that becomes final object after cut-paste method is employed, so it was necessary to find out how to cut the existing plane and change it to the target plane.” Such an observation led him reformulated his plan by rotating the cut parts of initial plane, then uniting all such parts become the target plane (cut-rotate-paste). This method was then used for the rhombus-shaped wood and got success. When he applied this method for the last curved plane, however, he got stuck. He said, "It's difficult to apply this method for this plane, when the cut parts are rotated, they do not match each other."

Before completing his processes, HAS he evaluated his steps by executing thought process by comparing formula he remembered and the methods he used. In summary, on the basis of the framework of analyzing students' cognitive process in figure 1, the cognitive process in solving area conservation problem of HAS can be derived logically in figure 3a.

On the second task, HAS started his cognitive process by observing the shape of the two-dimensional plane at the figure given as the information of the task. He said, "I never found this kind of task before, the task that ask me to compare the area without any numerical information given. But, I know that I need to find the area of the parallelogram in this figure and find which figures among these choices [the two-dimensional figures presented in the task]" having the same area with the parallelogram." In this regard, HAS identified the crucial thing of the task that need to be found for the subsequent steps of solving the task. To that, he translated the missing information about the area of the parallelogram by introducing the formula of finding area of a parallelogram, which is base x height, and obtained the area is 2 x 2 = 4. When observing the relationship among the elements of the problem, he admit the some figures have similar shapes, in which the shapes consist of at least three groups: figures without hypotenuse, figures with one hypotenuse, and figures with more than one hypotenuses. He said, "I found similar shapes like a group of 1, 14, and 17, then a group of 3, 6, and 11, and maybe a group of 7, 9, and 14, the very likely difficult ones since the shape looks have more hypotenuses." This observation led him to formulate a plan to solve the task by working out the group of figures he thought was the easiest first, namely group of figures without hypotenuse. The plan, we observed, was around the use of formula for the group of figures without any hypotenuse, while keeping no idea for the other two groups.

Without predicting any outcomes of the plan he formulated, he directly executed his idea of using quadrilateral formula to find the area of figures no 1, 4, 17 and simply found that the area of figure no 1 and 4 is same with the area of the parallelogram, while the area of figure no 17 is not same with the area of the parallelogram since he said, "the area of this is 5 x 1 = 5, not same with the parallelogram, which is 4." Being aware of the weakness of the method of using plane area formula he experienced in the first task, he then tried to use his 'cut-rotate-paste' method to solve the second and the third group of figures. Thus, he reformulated his plane. Interestingly, he did not put the parallelogram figure in the information as the target plane. Rather, he put the square (figure no 1) as the target one. He argued, "I think I need to bring all the remaining figures into the shape of this square instead of the parallelogram because this is likely easier". In this regard, he predicted the more likely easier method, instead of any outcome resulted from such a method. He regulated the plan by firstly choosing figure no 3 for the first trial. In this case, he said, "I imagine this part is cut off and put the cut part into this part, I get a square".

With the same way, he get a square for figure no 10 and 11. For the third group of figures, when he tried examining figure no 7 and 9, he found it was easy and decided that these two figures also have the same area with. He argued, "Same with figure no 6, I cut off these two parts and paste them so that the figure becomes a square. "However, when he examined figure no 2, which is a kite-shaped plane, he got stuck. It was observed that he found difficulties in determining the place where he should cut the figure in his mind. After for more than 1 minute, he finally revealed that he need to cut the plane twice, rotated the cut plane, and paste it so that it forms a square. The only figure that he did not any idea to solve is figure no 14. He said, "It is very difficult to find where to cut off this plane, it might have another method, and I don't know."

To complete his solution process, he was asked to compare which method he should use when finding a similar problem in the future. He argued, "I found some difficulties when using a formula since I sometimes forget with the formula. Therefore, I have to use another method such as by cutting particular parts of the figure and move the cut parts to the other part of the figure so that it becomes a square." In brief, his cognitive processes are illustrated in Figure 3b.

Figure 3 compares the cognitive processes of HAS when solving two area conservation tasks. It indicates that there are some repeating processes done for the two tasks, primarily from the process of determining relationship among element of problems to detecting and correcting errors of the solution resulted.

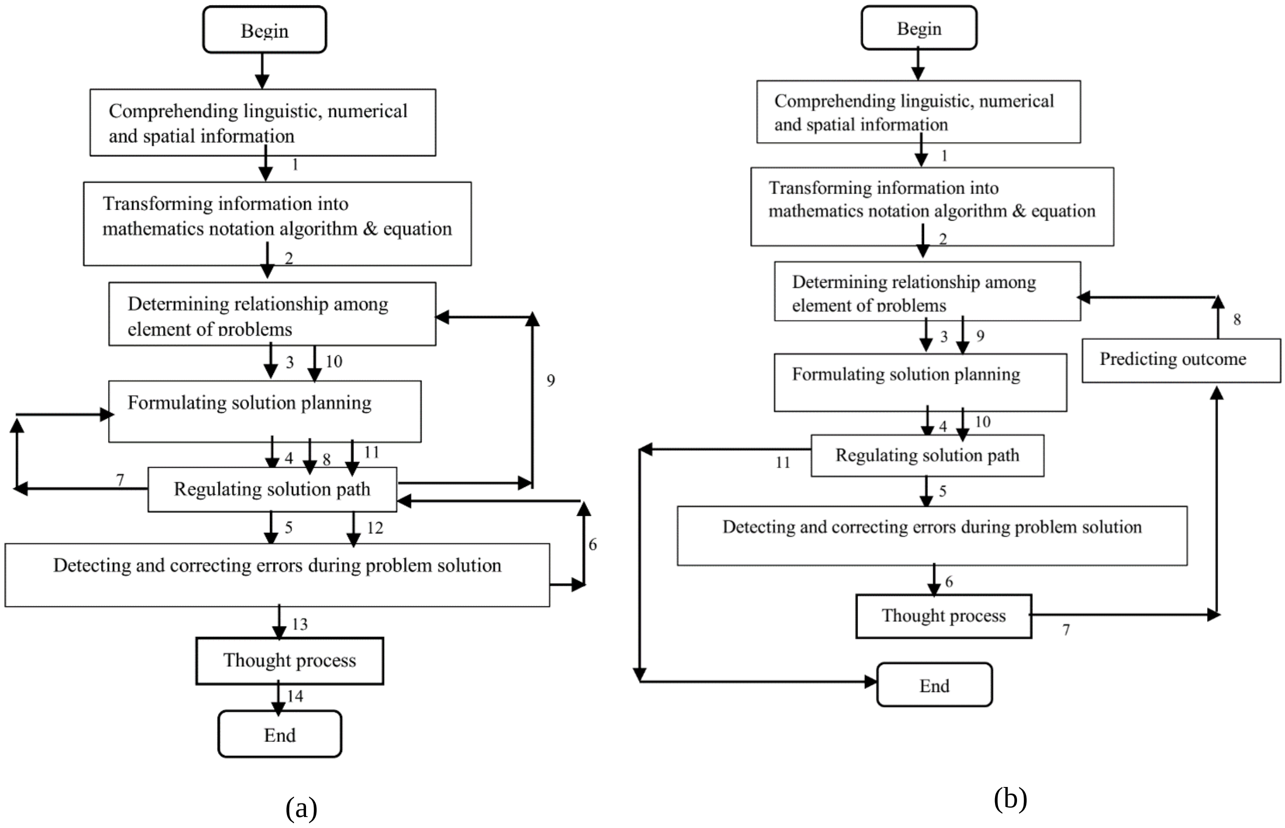


Figure 3. The cognitive process of HAS

Interestingly, HAS carried out the thought process on both the two tasks, which are proven as important processes to convince the correctness and the most effective method to derive the solution. Also, for HAS, the process of predicting any outcome does not seem likely becomes a crucial process in the initial cognitive processes. As evidence, this stage was not carried out in the first task, while this was carried out in the second task, but after the thought process, instead of between the process of determining relationship among information and formulating solution planning.

The Cognitive Processes of LAS

On the first task, LAS began his cognitive processes by claiming that he ever faced similar problem and mentioned the information either known or unknown. He then re-drew the information from the problem to show the meaning of magnitude notation as circumference of rectangle as part of the process of 'transforming information into mathematics notation, algorithm and equation. Afterwards, he observed the relationship among element of the problem by finding the unit price as basis to find total price. He continued with formulating solution planning. He determined three plans such as 1) divided the given price of pieces of wood to explore the price for the unit magnitude, 2) calculating the circumference of each plane by counting the number of square that were covered by the explored plane, 3) determining the price of each piece of wood by multiplying the price of unit magnitude with the circumference of each plane. He skipped doing predicting outcome and continued with regulating the solution path as is it executed by calculating the price of each 'cm'. However, LAS could not sure about the result of his calculation but he did not prove his prediction. This process was coded as predicting outcome process. After he did prediction, he formulated a plan again to solve the problem by doing revision on the plan of determining unit price and delete his initial idea of calculating the rectangle circumference. He tend to doing multiplication 2 (12x10) = 240 cm. Furthermore, another solution path was regulated such as by calculating the unit price of every 'cm', though he did a little error in calculation. In determining the price of unit piece of woods, a unique order of work on plane was as follows: square – rhombus – parallelogram – trapezoid –rectangle with arc modification – trapezoid with arc modification. For all planes, he counted the number of 'box/square' that cover the planes.

The cognitive process continued with comparing the price of wood in the form of parallelogram and rectangle. By this, he shared his uncertainty and did checking since he found for bigger form of wood is cheaper than those smaller one. This process was coded as detecting and correcting error during problem solution. Afterwards, another activity of formulating a plan to solve the problem was executed by revising the unit price. As a result, regulating the solution path as it is executed process appeared again by repeating the calculation for the price of each. However, LAS found difficulties in determining the price for rectangle with arc modification and trapezoid with arc modification. He felt uncertain with his strategy to solve the problem. Therefore, the following process were not performed. By considering the framework of analyzing students' cognitive process, the cognitive process in solving are conservation problem of LAS can be derived logically in figure 4a.

On the second task, LAS started his cognitive processes by admitting that he never experience with the task. He claimed that the types of mathematical questions related to area he ever work out is finding the area of a particular plane with some numerical information given. Nevertheless, he tried to understand what the task actually ask him to do. He said, “I need to find the area of the parallelogram first, then find which of the figures [figures in the options] having the same area with the parallelogram. Thus, I found there are three dots for the base and three other dots for the height.” Subsequently, he argued that the area of the parallelogram can be found by using the plane area formula for parallelogram. He continued his explanation, “because the area of parallelogram is base x height, then I found the area is 3x3 = 9”. In this stage, LAS transformed the information by recognizing relevant formula of area of parallelogram, although he selected irrelevant information, which is the number of dots, instead of the length of the height and the base of the parallelogram. This irrelevant information then become one of the causes of LAS’ failure in the subsequent stages of his problem solution process. As the evidence, when he formulated his plan to solve the problem of the task, he used the area of the parallelogram he found, which is 9, as the criterion for finding the planes which also have the area of 9 square units.

Such the above mistakes continues until the stage of regulating solution path. For example, when he examined whether figure no 4, he explained, “this rectangle has the area of 10, because it comes from 5x2 [five dots for the length and 2 dots for the width]”, even though the actual area of figure no 4 is same with the area of the parallelogram, which is 4 square units. Another example was indicated by figure no 1, in which this figure was assumed to have the same area with the parallelogram, i.e. 9 square units. Interestingly, LAS remember all the relevant formula to find the area of each of quadrilateral-shaped figures given in the task. However, he found no formula fit with the non-quadrilateral-shaped figure such as figure no 14, 15, and 16. Thus, he got stuck on these figures and did not continue his work. Finally, he also did not carry out the crucial stages of cognitive processes, i.e. detecting errors and thought process within his problem solution process. In summary, LAS’ cognitive processes in task 2 is given in Figure 4b.

Figure 4 illustrates the comparison of the cognitive processes of LAS on task 1 and task 2. While these figures points out that LAS carried out the first five cognitive processes of Montague (2002), LAS did not perform the process of detecting and correcting errors. Also, he did not perform any thought process within his solution process. In this case, it is clear that the difference between the cognitive process of HAS and LAS is the existence of thought process during their solution processes. The same characteristics of HAS and LAS, however, is indicated from the dynamic process proven by the repeated processes primarily within the process of formulating solution planning to regulating solution path done by them.

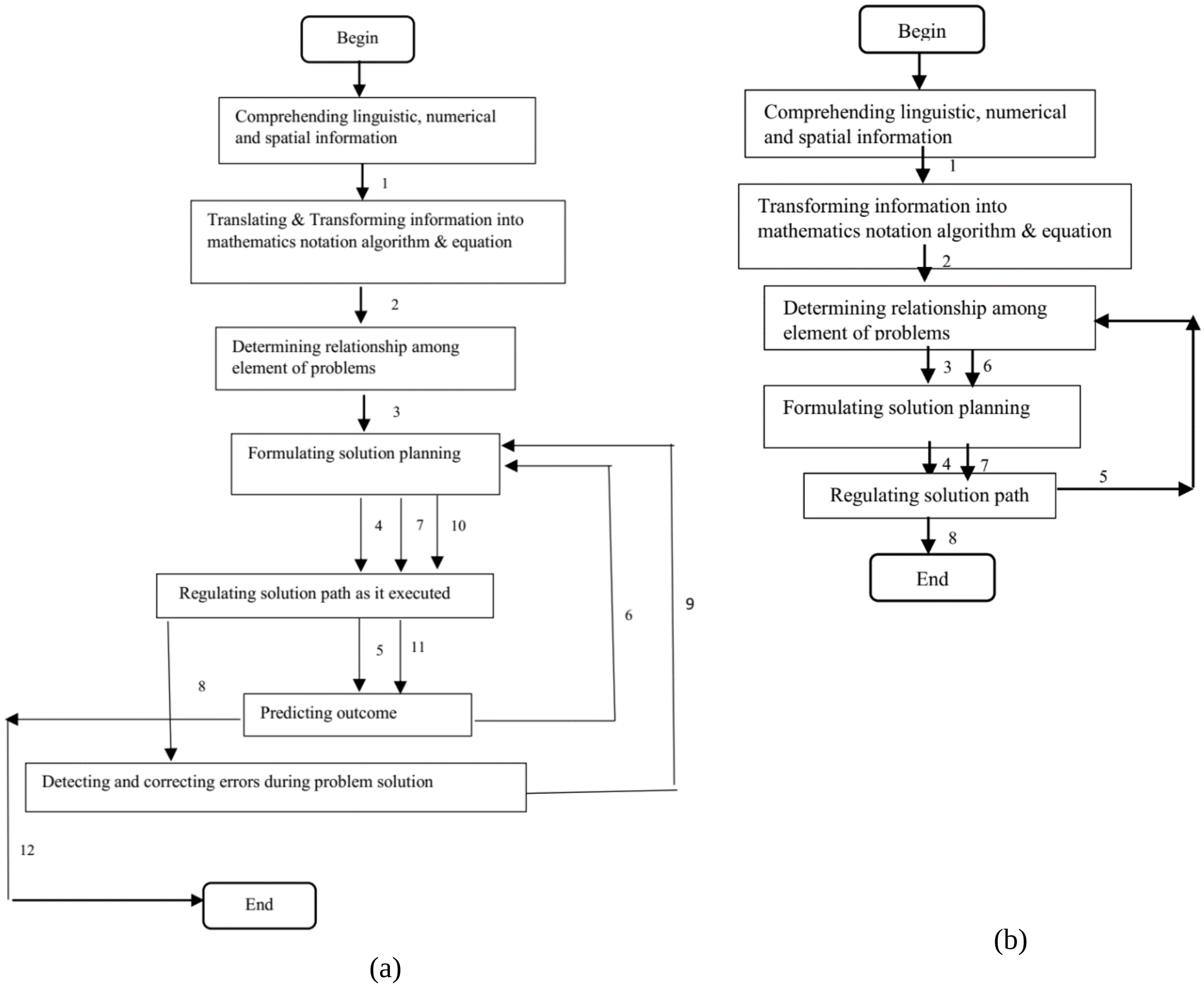


Figure 4. The cognitive process of LAS

Discussion

The present study investigated the two students' with different mathematics ability cognitive processes in solving problem of area conservation. With regard a modification of cognitive processes from Montague (2002), the two students managed to obtain mathematics' solution and describing their process. The common feature finding was those students follow the hierarchical step of cognitive process except *the thought* that was not elaborated by Low Achiever Students (LAS) . The initial strategy used to find the price was by elaborating 'unit price'. Cramer et.al (1993) suggested that unit rate approach was the most popular strategy and responsible for the largest number of correct answers. However, LAS got stuck after detected errors and repeating the cyclic of formulating solution planning and regulating solution path. He still fell uncertain with his used strategy. On the contrary, High Achiever Students (HAS) performed more cyclic cognitive process on the task with quantitative approach. The cyclic started after he detected errors and he tried to revise his errors. He turned back the process until he re-determined several elements of the problem, re-formulating solution planning and re-regulating solution path. In addition, the thought process were elaborated by HAS to arrive to his final solution. These findings in line with previous research that suggested that high academic achievers and low academic achievers have significant differences in the integration of cognitive structures and the usage of information processing strategies. (Bischoff & Anderson, 1998, 2001; Tsai, 1998, 1999; Tsai & Huang, 2001). As has been illustrated, the difficulty of LAS in determining the solution was caused by the inability of students to establish the crucial relationship between the representation in the problem and the data he found. To be more specific, Stillman (1996) hypothesized the contributing factors to an unsuccessful solution were unsatisfactory comprehension skills, lack of understanding of mathematical concept and inhibit impulsive responses to the problem.

CONCLUSION

In performing analysis of students' cognitive process in solving area conservation problem quantitatively, three crucial findings must be taken into account. The first two cognitive process namely comprehending linguistic, numerical and spatial information; Transforming information into mathematics notation algorithm & equation; and determining relationship among element of problems were two crucial starting process in solving mathematics problem related to area conservation in this study. More specifically, the primarily process which encountered by students as a continuous cyclic were formulating solution planning, regulating solution part and detecting and correcting error during problem solution. Furthermore, the predicting outcome process was elaborated by low achiever student but not for high achiever student. On the contrary, the thought process was elaborated by high achiever student but not for low achiever student.

Thus, the entry and goal setting phase of problem solving model by Mason et.al (1985) play crucial rule in solving problem. The following cognitive process phase were influenced by students' ability in activating their mathematics' thinking. The first implication of this study findings is that the characterization and the activation of cognitive process may inform teachers on the teaching strategy that can be applied. The second implication is to inform research that the seven cognitive process are elaborated by students with continuous cyclic for several processes. The process may be useful to lead teachers develop process for their students. This study focus on the solving problem quantitatively, thus, it is still possible to explore more on the cognitive process in the eyes of visually.

ACKNOWLEDGMENTS

We would like to thank the Ministry of higher education and research on PDUPT grant in the second year 2018, the Rector of Universitas Negeri Surabaya (Unesa), and all the participants involved in this study. Our thanks also to the reviewers for the useful feedback.

REFERENCES

Battista, M. T. (2007). The development of geometric and spatial thinking. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 843–908). Charlotte, NC: Information Age Publishing Inc.

Bischoff, P. J., & Anderson, O. R. (1998). A case study analysis of the development of knowledge schema, ideational network, and higher cognitive operations among high school students who studied ecology. *School Science and Mathematics, 98*(5), 228-237.

Cramer, K., Post, T., & Currier, S. (1993). Learning and Teaching Ratio and Proportion: Research Implications. In D. Owens (Ed.), *Research Ideas For the Classroom* (pp. 159-178). NY: Macmillan Publishing Company

Freudenthal, H. (1986). *Didactical phenomenology of mathematical structures* (Vol. 1). Springer Science & Business Media. John P. Smith III, Marja van den Heuvel-Panhuizen, Anne R. Teppo. (2011). Learning, teaching, and using measurement: introduction to the issue. ZDM Mathematics Education (2011) 43:617–620 DOI 10.1007/s11858-011-0369-7 FIZ Karlsruhe

Hiebert, J. (1981). Units of measure: Results and implications from national assessment. *The Arithmetic Teacher*, *28*(6), 38-43.

Jones, V. O. (2006). *Cognitive processes during problem solving of middle school students with different levels of mathematics anxiety and self-esteem: case studies*. A Published dissertation of Florida State University

Kordaki, M., & Balomenou, A. (2006). Challenging students to view the concept of area in triangles in a broad context: Exploiting the features of Cabri-II. *International Journal of Computers for Mathematical Learning*, *11*(1), 99-135.

Kospentaris, G., Spyrou, P., & Lappas, D. (2011). Exploring students' strategies in area conservation geometrical tasks. *Educational Studies in Mathematics*, *77*(1), 105-127.

Kordaki, M., & Potari, D. (2002). The effect of tools of area measurement on students strategies: The case of a computer micro world. *International Journal of Computers for Mathematical Learning, 7*(1), 65-100.

Mason, J., Burton, L., & Stacey, K. (1985). *Thinking mathematically* (Rev. Ed.). Wokingham, UK: Addison-Wesley

Mason, J. (2015). On being stuck on a mathematical problem: What does it mean to have something come-to-mind?. *LUMAT (2013-2015 Issues)*, *3*(1), 101-121.

Miles & Huberman. (1994). *Qualitative Data Analysis: An Exposed Sourcebook 2nd*. London: SAGE Publication Ltd.

Montague, M. (2002). Mathematical problem solving instruction: Components, procedures, and materials. In M. Montague, & C. Warger (Eds.), *Afterschool extensions: Including students with disabilities in afterschool programs.* Reston, Va.: Exceptional Innovations.

Montague, M., Krawec, J., Enders, C., & Dietz, S. (2014). The effects of cognitive strategy instruction on math problem solving of middle-school students of varying ability. *Journal of Educational Psychology*, *106*(2), 469.

Montague, M., Warger, C., & Morgan, T. H. (2000). Solve it! Strategy instruction to improve mathematical problem solving. *Learning Disabilities Research & Practice*, *15*(2), 110-116.

Papadopoulous, I. (2010). Irregular Plane Figures: From the Eighteenth Century to the Modern Classroom. *International Journal of Science and Mathematics Education, 8,* 869-890.

Sisman, G. T., & Aksu, M. (2016). A study on sixth grade students' misconceptions and errors in spatial measurement: length, area, and volume. *International Journal of Science and Mathematics Education*, *14*(7), 1293-1319.

Stillman, G. (1996). Mathematical Prospective and Cognitive Demand in Problem Solving. *Mathematics Education Research Journal, 8*(2), 174-197.

Stylianou, D. A., Kenney, P. A., Silver, E.A., & Alacaci, C. (2000). Gaining Insight into Students' Thinking through Assessment Tasks. *Mathematics Teaching in the Middle School, 6*(2), 136-144

Tsai, C.-C. (1998). An analysis of Taiwanese eighth graders' science achievement, scientific epistemological beliefs and cognitive structure outcomes after learning basic atomic theory. *International Journal of Science Education, 20*, 413-425

Tsai, C.-C. (1999). Content analysis of Taiwanese 14 year olds' information processing operations shown in cognitive structures following physics instruction, with relations to science attainment and scientific epistemological beliefs. *Research in Science & Technological Education, 17,* 125–138

Tsai, C.-C., & Huang, C.-M. (2001). Development of cognitive structures and information processing strategies of elementary school students learning about biological reproduction. *Journal of Biological Education, 36*, 21-26.

Tumová, V., & Vondrová, N. (2017). Links between Success in Non-Measurement and Calculation Tasks in Area and Volume Measurement and Pupils' Problems. *Scientia in Education*, *8*(2), 100-129.

Yeo, J. B., & Yeap, B. H. (2010). Characterizing the cognitive processes in mathematical investigation. Accessed from http://www.cimt.plymouth.ac.uk/%20journal/jbwyeo.pdf, 10 October 2018.