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Abstract

This study aims to investigate the properties of peer tutoring of first year engineering students on a Calculus-I course. Five dyads are constructed with different properties, and eight volunteer students. The students met regularly and studied calculus together for two months in technology-rich classrooms. To identify the properties and structure of the peer dyads, a multiple case study was conducted. Data was gathered through transcribed video recordings, and two themes (tutor-tutee relationship and teaching mathematics) were constructed in order to analyze the data. The success of the dyads depends on many variables such as structure of the tutoring, previous mathematical knowledge of the tutor, the way of teaching, questions types, and how feedback is provided. According to analysis, tutors and tutees have three kinds of relationships that are dependent on the skills of the tutors and the personality of the peers. These are interdependent to each other, scaffolding one another, and tutoring. Most students were good at mathematics, but tutors experienced some problems explaining why mathematical statements were true or false. Only two students demonstrated conceptual knowledge, with others emphasizing procedural and computational knowledge.

Introduction

Mathematics is a very important subject area for engineering students. Most engineering programs involve studies of a core engineering specialty, taught alongside courses for both mathematics and the natural sciences. Understanding life through mathematics starts with Calculus; a fundamental course for engineers, therefore, the learning of calculus is fundamental to the future profession of engineering students. Effective calculus learning and teaching is a must for engineering departments; however, traditional teaching and learning models employed for many years require renewal with newer perspectives.

Alternative models of teaching and learning are being discussed to support quality in higher education, such as peer-supported learning/ peer tutoring. Requirements to improve teaching quality, while achieving more with less, has increased interest in higher education peer tutoring (Bruffee, 1978; Falchikov, 1990; Saunders, 1992; Topping, Watson, Jarvis, & Hill, 1996). Tutoring is one form of facilitating teaching and learning in higher education, as increased student numbers in larger class sizes make learning more difficult than ever; leading students to learn by themselves or seeking help from their peers. Treisman (1992) compared the failure of black students on calculus courses and their Asian counterparts who had a history of very successful calculus course achievement during the 1970s and found that most black students studied alone while Asian students sought peers with whom to collaborate. He revealed that peer study or collaborative learning produced this distinction among minority students' achievements in Berkley during that time (Treisman, 1992). In his doctoral studies (1985), he developed a program in which minority students excelled and thrived. Training in an alternate learning environment on how to learn cooperatively facilitated students replacing their skill-oriented attitudes, and their ineffective study habits with effective learning habits.

Peer tutoring is a long-established practice. Definitions of peer tutoring perceive peer tutors as backups, or as an indirect transmission of knowledge from teacher to student, via a tutor. Peer tutoring has involved several different formats of peer/tutor/tutee interaction, as developed and studied in higher education (Bruffee, 1978; Falchikov, 2001; Saunders, 1992; Topping, 1996). For instance, cross-age (Allen & Feldman, 1976), same-age (Annis, 1983), and reciprocal tutoring (Fantuzzo, Riggio, Connelly, & Dimeff, 1989). In same-level tutoring, student peers are placed together in similar academic settings, based on similar levels of academic achievement so that student tutors and tutees in the same class share similar levels of knowledge, skill, and expertise (Falchikov, 2001). Tutee/tutor interaction is effective where the more skilled/experienced learners assist and guide lesser-skilled learners. Based on the idea of mutual exchange producing academic gain, Fantuzzo et al. (1989) created the Reciprocal Peer Tutoring (RPT) strategy for students preparing to tutor other students (Pigott, Fantuzzo, & Clement, 1986; Wolfe, Fantuzzo, & Wolfe, 1986). RPT specifically addresses the fundamentals responsible for increased learning, with each student playing tutor and tutee (Fantuzzo, King, & Heller, 1992; Pigott et al., 1986; Fantuzzo et al., 1989). Overall, RPT increases academic cognitive gain, decreases exam stress, and improves course satisfaction over other experimental approaches (Fantuzzo et al., 1989).

Peer tutoring is an effective learning model, positively affecting group goals or interdependence, student responsibility, and both individual and group rewards (Falchikov & Goldfinch, 2000; Hattie, 2009; Johnson, Johnson, & Smith, 2007; Prince 2004; Rohrbeck, Ginsburg-Block, Fantuzzo, & Miller, 2003; Slavin, 1996; Springer, Stanne, & Donovan, 1999; Topping, 1996). Some research studies also show that tutoring support increases students' self-esteem and confidence (Gaustad, 1992; Kalkowski, 1995) and helps social skills through the removal of barriers and the building of new friendships (Kalkowski, 1995; Miller, Kohler, Ezell, Hoel, & Strain, 1993). Moreover, peer tutoring provides emotional support and positive role models (Martino, 1993). Peer learning is considered very effective, but there are exceptions. Usually, tutors should be high-achievers and considered as role models, but sizeable ability differences can prove under-stimulating for tutors. For average ability student tutors, both tutor and tutee need cognitive challenges in their teaching-learning. Tutoring itself involves cognitive challenges, particularly involving simplification, clarification and exemplification. Peer features are important for effective peer-assisted learning. In the literature, to indicate the relational importance of peer relation interdependence, scaffolding, and tutoring are the terms used. Moreover, prior social relationships between peers are also examined as another dimension of the peer relations (Boud & Lee, 2005; Johnson et al., 2007; Topping, 2005; Tosey & Gregory, 1998).

Besides the social benefits of peer interaction, there are studies that show that peer interaction positively affects calculus achievement. Malm, Bryngfors, and Mörner (2010) state that participation in peer tutoring markedly improved engineering students' success on a first year calculus course. Additionally, clear indications show the program created a positive social introduction to engineering, concluding tutoring as positive for both first year course examinations and general student performance. Moreover, Reinholz (2015) conducted a quasi-experimental research design for two semesters to show that calculus success rate can be improved by peer-assisted reflection (PAR) activities. These activities provided students with the opportunities to analyze, explain and discuss the work of their peers. Along with the calculus courses, Crouch and Mazur (2001) report on ten years of teaching with Peer Instruction (PI) on physics courses. In Peer Instruction sessions, topics for the lesson are taught in a series of parts which consist of a short presentation on a concept, a conceptual question, individual student responses to the questions, peer discussions in which students explain ideas and try to convince each other they are correct, another poll of student responses, and a short explanation of the correct response by the instructor. In these parts the key concept is "explanation" in order to promote learning. Their results indicate increased student mastery of both conceptual reasoning and quantitative problem solving. In his study, Reinholz (2015) investigated helping students through peer-review activity and PAR. In a college calculus class, the researcher apply PAR cycle in which students were required to (1) complete a draft solution, (2) reflect on their work, (3) analyze peers' work and exchanges feedback, and (4) revise their own work (Reinholz, 2015). The core point of Reinholz's (2015) study is similar to Crouch and Mazur's (2001) study, where students need to explain the math in a way that their peer could understand.

Moreover, Bénéteau et al. (2016) conducted a study at the University of South Florida on a Peer-Led Guided Inquiry (PLGI) program which included Process Oriented Guided Inquiry Learning and Peer-Led Team Learning to teach calculus. Fifty minutes of weekly lecture in Calculus for Engineering and Life Sciences was replaced by a PLGI program curriculum, where students worked in groups with peer leaders as instructors. According to the results of the study, higher pass rates were observed in both department's Calculus courses. Additionally, withdrawal rates declined for Calculus courses of both departments. According to a survey conducted by the researchers during the study, students in both departments preferred teaching through peer groups. Peer learning within a group was also studied by Reisel, Jablonski, Munson, and Hosseini (2014), who named peer group learning as Peer-led Team Learning (PLTL). According to result of the research (Munson & Hosseini, 2014), increased participation in the PLTL groups correlates to better course performance and the achievement of students in PLTL groups achieved in Calculus I courses. This educational technique, developed by Gosser (2011) for Chemistry courses, is designed to improve student learning of subject matter by fostering interaction between students on the course as they help each other to learn. Similarly Laskey and Hetzel (2011) suggest that students often feel more comfortable asking questions of a tutor as the tutor has no direct influence over their grades. The comfort in asking a tutor questions exists in individual sessions or in small group tutoring sessions where tutors direct the learning experiences. In a small group model, group discussions provide an open environment for discussing work with peers (Solomon, Croft, & Lawson, 2010) where tutors can lead discussions and intentionally scaffold conceptual knowledge. Tutors can help students improve their self-efficacy, confidence, and the ability to do well in school, which can help students connect to university life (Tinto, 1999). Hoops, Yu, Wang, and Hollyer (2016) investigate what types of instructional practices are developed in a college precalculus classroom that could influence students' self-regulated learning for the course. Participants were a university mathematics instructor, students enrolled in two sections of the researcher's undergraduate precalculus course, and eight peer tutors at a large public research university. According to the results of this study, the majority of help-seeking references encouraged students to find assistance or involved students engaging in help-seeking activities during class, and most in-class help-seeking involved peer assistance rather than students seeking help from the instructor (Hoops et al., 2016).

Many studies have been conducted on Peer-led Team Learning methods and the considerable performance improvement of participating students (e.g., Liou-Mark, Dreyfuss, & Younge, 2010; Loui & Robbins, 2008). In the aforementioned studies, students worked together in order to solve appropriate mathematics problems under the guidance of peer facilitators, who are upper-level engineering students. On the other hand, research by Chi, Siler, Jeong, Yamauchi, and Hausmann (2001) addressed one-to-one tutoring as a most effective form of instruction. Moreover, tutees' learning and skill development through interactive-styled tutoring was associated with deeper levels of inquiry and increased scaffolding for the tutees. Additionally, tutees made greater efforts to take control of their learning. Similarly, Merkel and Brania (2015) investigated peer-led team learning in Calculus-I courses with an experimental research approach during a five year period in an all-male college. The peer-led team is a student-centered instructional approach in which students work collaboratively in small groups of five to eight tutees facilitated by a peer leader. Although the aim of the research was not to explore the achievement of peer-led team learning, but achievement retention, the researchers found that peer-led team learning did not support the retention of achievement.

In another study, Parkinson (2009) carried out a carefully controlled study of the effects of peer-assisted learning of first year students led second year students. Prior to tutoring, the tutored and non-tutored groups were very much evenly matched. However, after just one semester of tutoring, there were substantial and significant differences seen between the tutored and non-tutored students. The tutored students had progressively increased their performance in calculus compared to the non-tutored students, with their examination marks in chemistry and calculus substantially improved and failure rates cut dramatically. Student progression overall had substantially improved. Besides observing the achievement and achievement retention of students subjected to peer tutoring, with a different aim, Hannah (2008) examined peer tutoring of a high school mathematics course as a potential enhancement to the learning process. The investigation focused on the attitudes of tutees following their interaction with peer tutors during after school tutorial sessions. The results of the study showed participation in peer tutorials appeared to have a predictive effect on increased achievement, but not on student attitudes toward learning mathematics. Therefore, it may be concluded that tutors benefit the most from peer tutoring (King, 2002; Falchikov, 2001), and studies have shown that the attitudes of tutors may also be influenced by their tutees during the interaction (Cohen & Kulik, 1982; Topping, Campbell, Douglas, & Smith, 2003).

Studies mostly underlined the achievement of peer study groups for calculus, with a few also interested in affective domain, but less investigations into what happens among peers during peer studying. One study by Reinholz (2016b) considered the nature of peer conversations between pairs of calculus students. The conversations took place in the context of peer-assisted reflection, an activity structure that has been shown to have a significant impact on student success. In another study, Neubert, Khavanin, Worley, and Kaabouch (2014) tried to understand the communication among the calculus peer groups. The researchers conducted a method for allowing calculus taught by mathematics faculty to be augmented with real-world engineering problems. The engineering students completed modules out of class and then discussed them in mentor-led sessions. Results showed that discussion session interested the students, make them to drill calculus, and produce opportunity to talk about calculus among their peers.

From this point of view, the literature afforded a general perspective regarding peer tutoring on calculus courses. Mostly the focus has been on achievement and social interaction. Since Klaus (1975) asserted that tutoring needs more naturalistic, in-depth qualitative description, the current study aims to conduct naturalistically-set case studies in tutoring higher education calculus, through observing the structure of the peer tutoring environment and identifying its dynamics and components through engineering students' dyads. In this study, the properties of tutors are investigated, and the relational structure between tutors and tutees examined through socio-cultural theories. The overall question guiding this study is "What is the nature of the peer tutoring dyads teaching-learning sessions on Calculus?" The specific research questions addressed in this study were as follows:

1. What is the behavior of tutors and tutees to each other?
2. What are the differences and similarities of tutors and tutees among the tutoring dyads within the mathematics teaching-learning process?

Theoretical Framework

This study is broadly framed within the sociocultural view of learning. Interaction in peer learning and process contributions of peer relations are investigated, requiring an approach considering interactional patterns and changes in practice. Underlying theoretical assumptions are therefore socio-cultural understanding of the relationship between knowledge, individuals and their context (Vygotsky, 1978; Wertsch, 1991), and how individuals produce meaning in their interactions with each other and the environment, and the learning process of interactions between individuals and their context (Lave & Wenger, 1991; Wenger, 1998).

Theoretical foundation for peer tutoring can be traced back to Vygotsky's (1978) social constructivist learning theory, a learning process in which humans learn new ideas and concepts based on experience, either directly or indirectly, in the social environment. Peer interaction is an integral part of peer tutoring, allowing students to take self-responsibility for their learning (Topping & Ehly, 1998) and share construct knowledge with society. However, Vygotsky (1978) stated that learning does not simply take place in a social context, but through social interaction as the basis of learning. Constructing knowledge among peers usually involves one who is more skilled than the other; leading to educational communication that helps the less skilled peer improve. This kind of scaffolding is key to Vygotsky's (1978) theory. According to Wood, Bruner, and Ross (1976), there are six key scaffolding elements: (1) recruitment – gathering learner interest in the task's requirements; (2) reduction in degrees of freedom – simplifying the task; (3) direction maintenance – keeping the learner on a particular objective; (4) marking critical features – confirming/checking; (5) frustration control – managing the learner's emotional state; and (6) demonstration – task-based solution (p. 98).

Another building block of social constructivism is zone of proximal development (ZPD), defined as 'the distance between the actual development level as determined through by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with other capable peer' (Vygotsky, 1978, p. 57). ZPD is integral to peer tutoring, with tutoring interactions occurring as tutor-tutee collaborations. For scaffolding to improve learning, tutors must deliberately link new information to already-known information (Valkenburg & Dzuback, 2009). Valkenburg and Dzuback (2009) suggest that tutors work as translators by changing the language into one that students can understand, in this manner deliberately producing contexts for constructing new ideas. Furthermore, tutors can help clarify content by explaining knowledge in a different context with his/her own language where students can easily ask questions and explain himself/herself (Laskey & Hetzel, 2011).

Method

This study employs a qualitative design, aiming to provide in-depth information about undergraduate students' peer learning in five dyads, based on extensive data collection (Creswell, 2005). Qualitative or 'naturalistic research paradigm' focuses on discovery, insight, and understanding (Merriam, 1998, p. 3), involving an interpretive theoretical stance as the design's framework as a multiple case study. The use of cases allowed the researcher to investigate differences and similarities of the five dyads in respect to their teaching and learning calculus to/from each other.

According to Yin (2002, p. 13), 'case study is an empirical inquiry that investigates a contemporary phenomenon within its real life context especially when the boundaries between phenomenon and context are not clearly evident'. Since in the current study the aim is investigating the peer dyads' teaching and learning sessions on calculus out of the class settings which can be regarded as a naturalistic environment for peer learning context, the case study design is chosen. According to Marshall and Rossman (1999), Studies focusing on society and culture, whether a group, a program, or an organization, typically espouse some form of case study as an overall strategy; this entails immersion in the setting and rest on both the researcher's and the participants' worldviews (p. 61). The unit of analysis is dyads of study group peers. This multiple case study has five dyads for comparison within and between cases, and each case serving a specific purpose within the overall scope of inquiry (Yin, 2002). Multiple cross-case analysis was conducted between four mathematics tutors (peer-tutor) in order to document and identify patterns of pedagogical practice. This qualitative format was chosen to examine and interpret underlying patterns and themes from observations of tutoring session interactions.

Participants

Convenience sampling was conducted. The study was explained to the students at the beginning of the Calculus-I course of the researcher's own class, with students asked to join the project in the 2015-2016 fall semester. Eight out of 50 Calculus-I freshman students volunteered. They were told that participation in the study would not contribute to their grades or garner extra credits. After explaining the schedule, teaming up into pairs was left to the participants as the researcher did not want to force students be team up with someone they did not know for two months of working side-by-side. Some students preferred someone they already knew as their partner, whereas some put no such condition on who could be their partner. In the current study, the researcher encouraged students to study together outside of the class as extra hours. One participant was assigned as tutor for two tutees, e.g. Steven had two tutees, one of whom was Sudanese and spoke little Turkish. Moreover, one tutee was chosen by two different tutors, constituting a different type of pairing. This resulted in three types of pairs; (1) tutor and tutee pairing of the same gender (male), (2) one tutor teaching two same gender tutees (male), and (3) two different tutors (one male, one female) with a male tutee. Overall, there are seven male and one female voluntary participants for the study. Their ages ranged from 18 to 20, with a mean age of 19.25 years. The names used are pseudonyms assigned by the researcher.

Settings

The study took place with engineering students at an urban Turkish university. Although the university mostly educates in Turkish, all Engineering Department courses are instructed in English, with most students attending a one-year English language preparatory course. The class consists of ten international students from Sudan, Azerbaijan, and Afghanistan. From the eight voluntary participants, four tutors and four tutees were assigned, with one Sudanese and seven Turks. Sessions were conducted outside of classes in a technology-rich classroom, selected due to the nature of the research. Three cameras recorded the sessions from different perspectives (one capturing the whole scene, one for hand movements, and one for facial movements). In each tutoring session, peers studied a Calculus-I course topic discussed in the classroom that week. Topics were not studied beforehand, outside of these sessions. After the first sessions were held, the researcher interviewed both tutors and tutees separately in order to understand what they expected and what they found. At the end of the study, the researcher held a single-session group interview to discuss the sessions with all participants at the same time. Interviews were video recorded and later transcribed. The researcher took field notes in both interview sessions and while watching recordings of the dyads' 14 teaching sessions.

Data Collection

Data was collected through conversations of five dyads studying calculus together, their pre and final interviews of participants, and researcher's observations (as indicated Table 1). Video recordings of the teaching sessions were watched by the researcher twice: first each dyads' sessions were watched and field notes made.

Table 1.Instrument details

Instrument	When	Why
Pre-unstructured interview	Before video recordings	To understand participants' feelings about peer learning
Video	One hour/week (two months for each dyad)	To characterize peer learning sessions
Post-unstructured interview	After video recordings	To understand participants' feelings and thoughts about the process
Field notes	While watching each record	To describe what could not be seen or heard, or from only reading transcripts; e.g. gestures, voice intonations, hand movements, etc.

During the semester, in each dyad, peers gathered in four separate one-hour sessions, studying calculus for a period of two months. Table 2 presents the topics that they studied.

Table 2. Details of dyads, names and sessions

Tutor	Tutee	Session 1	Session 2	Session 3	Session 4
Michael	Daniel	Transformation of functions	n/a	Limits of the functions	n/a
Sandra	Daniel	n/a	Average Rate of change	n/a	Definition of derivative
Kevin	Mark	Transformation of functions	Average Rate of change	Limit of the functions	Definition of derivative
Steven	Edward	Transformation of functions	Derivatives	n/a	Question solving
Steven	Ronald	Transformation of functions	Derivatives	Question solving	n/a

Data Analysis

Four data sources were gathered over a two month period. The data was analyzed by using the code matrix (Table 3) which is produced before starting to watch each recording. The matrix is produced based on the concepts of ZPD and teaching learning steps. For example, tutor tutee relationship is formed based on ZPD; mathematical knowledge of tutor and tutee, preparing for sessions, teaching techniques and teaching levels are formed based on the teaching-learning concept. On the other hand, some extra themes are also identified while watching the videos like tutor and tutee question. Since the codes and themes are formed before the investigating the data, descriptive analysis is conducted. Each interview was transcribed and analyzed thematically. The same procedure was conducted by another researcher qualified in qualitative analysis. Coding lists were compared and unmatched codes discussed, continuing until all codes were agreed and matched. The intercoder reliability was found to be 80% at first, but after discussion of the unmatched codes, full consistency was obtained. The two main themes, five subthemes, and the related codes for analyzing the study and presenting the findings are shown in Table 3.

Table 3.Themes and Codes Matrix

Themes/subthemes	**Codes**
Tutor and Tutee Relationship	Interdependence Scaffolding Tutoring
Teaching Mathematics	
a. Mathematical Knowledge of Tutor and Tutee	Poor Moderate Good
b. Preparation for sessions	Reading the topic beforehand Notetaking before session
c. Teaching techniques	Memorizing Understanding Application Imitating instructor’s teaching
d. Teaching level	Procedural Conceptual
e. Tutor and Tutee Questions	Closed Comprehended

Results are supported with detailed descriptions of participants' activities, behaviors, actions and conversational excerpts.

Role of the Researcher

The researcher is the instructor teaching the Calculus-I course in this study. The researcher clearly explained that this study will not affect participants' grade when she offered the idea of peer study group to the class. She was not in the study room when eight volunteering students were studying, and she did not interrupt any of the sessions to prevent students got affected from authority. Her familiarity with the participant students did not affect her decisions or interpretations during data analysis. During data analysis, a colleague of the author coded one of the randomly selected recordings and transcribed the teaching session. In order to obtain intercoder reliability, the researcher and her colleague discussed both the theme matrix and the coded dataset. The second coder also helped during interpretation of the data and in producing the conclusion of the study. Discussions with the second coder also helped prevent researcher bias of familiarity. Although familiarity with students may help in understanding the background of the students towards their calculus course, to ensure appropriate interpretation validity was maintained (Maxwell, 1992), the researcher discussed each step of the analysis with her colleague.

Trustworthiness and Credibility

According to Lincoln and Guba (1985), there are four criteria for establishing trustworthiness: credibility, transferability, dependability, and confirmability. Merriam (2002) claims that credibility deals with the question "How congruent are the findings with reality?" For the current study, to answer this question, each participant volunteered frankly to be a participant in the study, acted naturally, and the data gathered from sessions presented honest and real situations. Researcher's "reflective commentary" (Shenton, 2004) in the field notes also contributed to this perspective. Another important tool to establish credibility has been thick description of the phenomenon under examination. The researcher described each dyad's communication and important situations in detail based on socio-constructivist and ZPD concepts.

The second criterion for trustworthiness is transferability, which implies that findings from a study can be applied to another. Yet, since findings of the current study are specific to eight engineering students in particular, it is not possible to show applicability of the findings and conclusions to other situations and populations. However, since contextual information about the field of the study is clarified openly, readers may decide on some degree of transferability. Moreover, the study provided detailed information on data collection methods, and the number and length of the data collection sessions.

Dependability is another criterion for trustworthiness, meaning that when a study is repeated within same context, with same methods and participants, the same result would be obtained. This can be fulfilled with a thick description of the site, operational details of data gathering process and reflective point of views.

The last criterion is conformability, which refers to objectivity of instruments. In the current study, video recording was used during sessions, and the researcher had a chance to watch and re-watch each session many times. Moreover, she took field notes while watching the recordings, and conducted interviews both before and after the sessions with the students to ensure triangulation. The researcher focused on student interaction and mathematical communication among the peer dyads, and she constructed a framework based on social constructivist theory and ZPD concepts. Therefore, her detailed methodological description enables the reader to determine trustworthiness of the study.

Results and Discussion

Findings are presented separately first for each dyad to explain the peer groups dynamics and then a comparison is given as summary to identify differences and similarities among the dyads. This gives a clear understanding of the peer dyads teaching learning sessions' dynamics.

Case 1: Michael and Daniel

At the semester end, Michael and Daniel achieved the same CB grade (top grade: AA) from Calculus-I, showing same level performance. Out of class they are friends too, having known each other before the study.

Tutor and Tutee Relationship

They are interdependent tutor/tutees. Mostly, Michael explained topics to Daniel, with Daniel adding extra information to make explanations clearer. In the sessions, Michael showed dominance over Daniel; however, Daniel does not like to be dominated, and struggled with Michael's behavior. Michael enjoys the power that comes from being the tutor. However, when Daniel shows he is more knowledgeable than Michael in some specific topic, Michael stops teaching and become a tutee of Daniel. In this dyad, the roles are interchangeable much of the time, so the type of tutoring is reciprocal. In this dyad the tutor was not well prepared, and subsequently was not that confident while teaching. Therefore, although he was the 'tutor' for this study, in actuality, Daniel could have been the tutor instead and taught Michael. In the following excerpt Daniel taught to Michael.

> Daniel: When a function has a limit and a value, at the point where the values equal each other, then we call that a continuous function.
> Michael: Is it?
> Daniel: Yes.

This role-changing happens when tutors are not prepared, or lack confidence in the topic. Sometimes Daniel applied his own method to solve the question, not accepting Michael's. In one sense, Daniel is trying to show himself as knowledgeable as Michael. When Michael asked a question to Daniel, he does not always let him solve the question, wanting instead to show he can solve it. Daniel explained a shortcut to Michael about drawing the graph without raising your hand, which implies continuity of the function. Michael seemed to admire Daniel, who felt proud in himself teaching Michael something.

Teaching Mathematics

Mathematical Knowledge of Tutor and Tutee. While teaching Calculus, Michael had some misconceptions. In the transformations of functions and limit topics. For example Michael said that "The inverse of the functions are symmetric in origin", but actually it is symmetric about y=x. Moreover he was confused with the horizontal line test, which shows that a function is one-to-one and vertical line test, which shows if a graph represents a function. In the limit topic, Michael insisted that "limit is an approach and a value at a specific point is not a limit value".

Similarly, Daniel has also some misunderstandings; for example, as the inverse of cotx function (arccotx), he said "it is sinx/cosx, isn't it?" But Michael did not confirm or reject this, showing that he too was confused. Both thought that inverse of the secx (other representation 1/cosx) and cosecx (other representation 1/sinx) is the reciprocal of the rational version of these functions, meaning cosx and sinx, respectively. Both have similar mathematical knowledge, and also misunderstandings. They also had some controversial conversations. For example, in the limit topic, while investigating whether one of the oscillating functions has a limit; both agreed it was not a function. Daniel drew a horizontal line with an imaginary pencil on the oscillating function graph to prove that it was not a function. Agreeing, Michael added:

> Michael: If it is given an interval, then it would be a function. After that, from right... it approaches to zero and same on the left.

Here, Michael confused the idea with restricted domain, discussed in class to explain one-to-oneness of trigonometric functions. Both said that "since it's not a function, there's no limit". Therefore, both had some problems defining function and limit.

Preparation for the Sessions. Michael confessed he did not prepare, and regretted it when experiencing problems remembering some concepts. Sometimes, due to poor preparation, he taught incorrect relations to Daniel, realizing afterwards. For example, the inverse of trigonometric function secx would be cosx.

Teaching Techniques. Sometimes Michael referred to high school mathematics courses, but with his own way of teaching; not imitating the course instructor. He could be a bit sharp; sometimes getting upset when Daniel didn't agree, stating it was that was the rule and they have to know it. For example Michael stated "$\lim_{x \to 0} \frac{sinx}{x}$ is 1 and this is a rule; you have to know it". Michael taught topics depending on procedural knowledge, more than conceptual. For example, for transformations of the functions, rather than explaining the rules and ideas behind them, he preferred to give values of both the original and transformed function to show the differences, instead of explaining each rule by reasoning. He didn't add information, meaning Michael read his notes line-by-line, trying to explain topics from his notebooks. First explaining the instructor's statements, he then gave his own strategies saying "I used it this way, so I'll explain to you my way".

Teaching Level. Michael applied rule-based teaching. Since he doesn't know enough about drawing the graphs of the functions, he stated "the important thing is learning the logic; the drawings you can do later". He attempts to make Daniel memorize the rules. Therefore the level of teaching is memorization, instructing Daniel to memorize the rules.

Tutor and Tutee Questions. In their dyads, Daniel (tutee) asks more questions than Michael (tutor), posing some difficult questions to Michael. For example, when transforming the function -f(-x), Daniel asked "Does this mean taking symmetry about the origin?" Michael wasn't sure, and checked his notebook. They checked the definition and they resolve it through an exchange of ideas and building the information together.

Although the peer who asked questions the most was Daniel, Michael mostly used yes/no questioning, e.g. "Is that ok for you? Is it clear for you? Is there any topic that you want me to explain?" and gave corrective feedback like "That's right, no it is not true". Michael is unskilled in questioning and giving substantial feedback when teaching mathematics.

Case 2: Sandra and Daniel

Sandra passed Calculus-I with grade BB. A good listener in class, she takes notes and asks questions. She also taught mathematics to her friend since middle school, even teaching from the board like a teacher, saying she likes and is very confident teaching.

Tutor and Tutee Relationship

Sandra acts like a teacher. She is supportive to Daniel and scaffolds his learning during tutoring. She portrays self-confidence, even if not knowing some part of a topic, she comfortably admits when she does not know. In this dyad, Daniel is very much the student compared to Daniel's other dyad. Where Daniel and Michael had reciprocal tutoring, with Sandra, Daniel is a tutee who tries to learn and understand the topic from Sandra. In this dyad, Sandra scaffolds by giving effective explanations and asking effective questions. However, sometimes they changed roles seamlessly. Sandra became tutee when she didn't know a concept. For example, Daniel helped Sandra to understand why the unit is $\frac{(inch)^2}{inch}$during the solution of one problem.

Daniel asked some points to Sandra that were discussed with Michael but had not originally been understood. For example, finding the inverse function and one-to-one function by looking at the graph of the function – they discussed and achieved a result by working on it together. In this dyad, the tutor is scaffolding the tutee most of the time, but occasionally roles were reversed, or they both have the same level of knowledge and work together. Therefore, their relationship is both scaffolding and interdependent. She presented scaffolding elements in her teaching, such as gaining Daniel's interest through helping him solve specific questions. She directs the coaching, confirming and checking his answers, responding to Daniel's emotional state by motivating him, and finally, she is modelling the solution.

Teaching Mathematics

Mathematical Knowledge and Tutee. Sandra is very good at mathematics. She can transfer knowledge from one discipline to another; for instance, she constructed a similarity between physics problems with average rate of change, and transferred the physics knowledge to mathematics with the formula "v=a.t". However, she experienced some problems in the new topic, for example, the average rate of change. In an average rate of change problem, they conducted ($\frac{(inch)^2}{inch} x \frac{1}{inch} = ?$) and found the result without any units which was meaningless for both of them.

Similarly, Daniel was confused about limit and continuity. But Sandra wrote an example, drawing a graph from memory, explaining the relationship between continuity and limit. When Daniel said "How does this happen, when it has a limit but is not continuous?", Sandra drew a graph of the function (with jump discontinuity given in Figure 1) to explain.

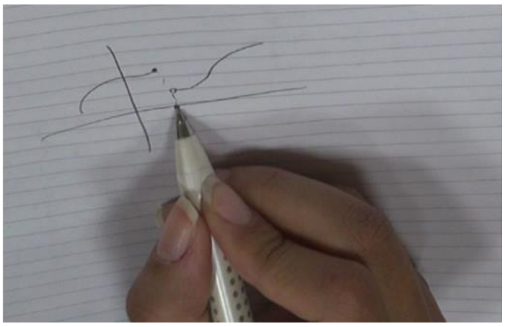

Figure 1. Sandra's example for discontinuity

Preparation for the Sessions. Sandra was very enthusiastic about the peer-assisted learning sessions. She enjoys responsibility, stating she studied at home for these sessions, looking for problems and solutions for her tutee.

Teaching Techniques. Since Sandra had been teaching mathematics to her friends, she acted professionally. She first explained the topic, solving one example, and then continued with other examples with: "It's your turn now and you're going to solve this." She categorized questions as easy and difficult, and let the tutee solve the easiest, while she solved the more difficult. In doing so, she organized her teaching and presented teaching skills. Moreover, she encouraged and motivated Daniel when he solved questions, with "Good. Nice. Good for you". These are the two important properties of scaffolding (Wood et al., 1976), therefore Sandra was scaffolding Daniel. Sandra emphasized conceptual knowledge by asking questions about concepts rather than procedural knowledge or computation, giving suggestive feedback to deepen Daniel's understanding. For example, Sandra asked "What is the value? How we are going to find it? What does it mean?" within an average-value problem, to make the problem clearer.

Upon realizing she had made a mistake, Sandra stopped and restarted correctly from the beginning. She appears to have no ego, or shame about being wrong. For example, when explaining the y=cf(x) transformation, she was confused if the function was going to stretch or shrink, but after two unsuccessful attempts, with help from Daniel, she reached the correct explanation. This is an important characteristic, showing she persevered; not giving up with understanding and learning.

Teaching Level. Sandra follows the course notebook, but tries to explain the main ideas behind the topics, giving importance to concept rather than computation, whilst also adding some personal information. This shows a strong mathematical background and ability in mathematics. She demonstrated the teacher's skills in most sessions, often trying to scaffold him.

Tutor and Tutee Questions. Sandra asked explanatory questions. When Daniel responded, she complemented with suggestive feedback. For example, when Daniel asked about continuity and limit, she drew a graph and asked whether that graph was continuous, moving step-by-step to the idea of limit after that. Mostly Sandra asked questions to make Daniel think about concepts and to understand key points.

Case 3: Kevin and Mark

In this dyad, the peers' mathematical competence is very distinctive. Kevin is very good at mathematics, passing the course with BA; the highest grade among the study group, but Mark scored DC, the lowest grade of the group. They are also not friends outside of this study group. Mark finished high school through distance education, and he said that explained his difficulties in understanding mathematics. His other difficulty was the language, since lectures are conducted in English. On the other hand, Kevin is very hard working and a good listener in the classroom who takes notes, asks questions and solves additional questions on topics. However, Kevin also has problems understanding English, but opted not to tell the researcher.

None of the peers studied their topics before the sessions. However, after the first session, Mark said he should repeat what they talked about at home, because he understood but afterwards forgot and lost any understanding.
In the interview, Kevin said he liked peer study sessions, because studying in a quiet environment was better than studying at home, adding;

> Kevin: If we study at home, we always make small talk, eating or drinking something all the time. But here it is more serious and we study without distraction. We study really well. If there are some places like this we would go and study - there should be a place like this for every course. We know this experience is very good; if we hadn't come here, we would never have known how effective it is.

Tutor and Tutee Relationship

Kevin is very confident teaching, and a skilled tutor at scaffolding Mark's learning. The relationship is purely tutor-tutee. Kevin presented skilled teaching properties, including scaffolding his peer whilst they learned calculus. Although Kevin is authoritarian, he gave importance to Mark's questions/answers, and where incorrect, he would immediately correct him. Kevin satisfies the six elements of scaffolding; such as trying to fulfill Mark's expectations (teaching shortcuts and solving exam-like questions), he reduced degrees of freedom by directing him to solve some conceptual questions, and also checks and confirms Mark's answers with feedback.

Teaching Mathematics

Mathematical Knowledge of Tutor and Tutee. Kevin has a strong mathematical background. However, from both his previous educational life and current education, he has picked up some misunderstandings. For example, in the one-to-one function, Kevin was confused about the $\sqrt{x}$ function, mixing it with the horizontal line test. He drew horizontal lines on the whole Cartesian coordinate system, saying 'It is not one-to-one, look here, the function is not cut by lines' (pointing out the fourth quadrant) (see Figure 2). But if he checked the domain of the function, he would have seen it as a one-to-one function.

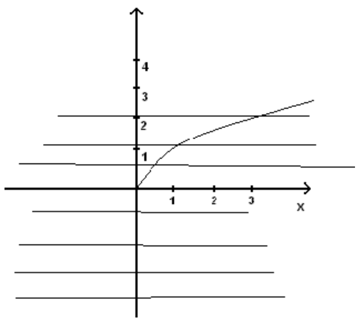

Figure 2. Graph of $y = \sqrt{x}$ function with horizontal lines

Kevin's mathematical problems are mostly related to his poor command of English. For example, some terms were totally misunderstood and therefore he conducted the wrong mathematical application (e.g., mile, inch, tangent line). In an example of drawing a horizontal tangent line of a given curve, since 'horizontal' was first discussed for horizontal asymptote, he mixed them and thought the question was about finding an asymptote.

Preparation for the Sessions. Kevin confessed he didn't prepare, giving an interesting explanation that "I believe this is more natural; you said it should be natural. I thought if I prepared it would not be natural teaching". Next session he said "This time I prepared, but not much. I listened to the course very carefully and took notes; rewriting some important points at home and producing summary tables". According to Kevin, a good teacher should be always ready; not needing to 'prepare'. But after the 3rd session, Kevin realized he too should prepare in order to explain topics in detail. He stated his "responsibilities are increased" and feels responsible for Mark's learning, hence he prepared examples and strategies to help Mark understand better.

Teaching Techniques. Kevin emphasized conceptual knowledge like an instructor; actually imitating his instructor and trying to teach the same. He used the same examples in his session, following his course notes.

At the beginning Kevin tried to give conceptual knowledge and make Mark understand the rule. But he did not receive positive feedback from Mark, who wanted to solve the questions, and continuously asked "Is she [the instructor] going to ask this in the exam?" After that, Kevin abandoned concepts, trying only to teach rules without explanation, and in doing so, Kevin didn't pursue discussion or questioning with Mark.

Kevin used multiple representations to explain the transformation of the function. First he explained using graphs, then algebra; trying to explain there are different y values for every changed x value. For sinx and cosx functions they preferred to apply algebraic explanations rather than graphical. However, for x^2, Kevin immediately drew the graph and used algebraic explanations in tables (see Figure 3).

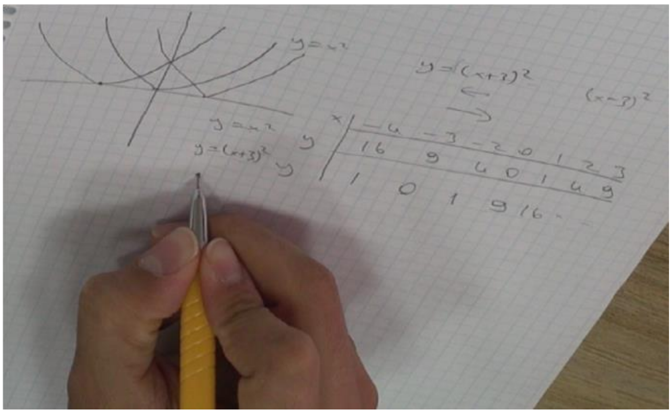

Figure 3. Kevin's graphical and algebraic explanations

Kevin can transfer knowledge to new topics. For example, in the limit topic and derivative, he gave an example to explain limit, drawing graph f(x)=x+1, requiring Mark remember that transformation:

> Kevin: Do you remember y=x graph? To draw y=x+1 we can apply shifting to y=x. What do you think this 1 affects?
> Mark: It affects y. For example, on y=x since 1 affects y this graph moves on y axis, meaning it becomes x+1, making this graph. 1 unit moves on y axis and 1 unit moves on x axis.

As a skilled tutor, after explaining the topic, Kevin asks Mark to solve it, thereby practicing the topic. As Mark worked on the questions, Kevin observed if Mark had understood.

Teaching Level. Kevin tried teaching the conceptual level, but not for long. He attempted to explain topics within conceptual borders, but Mark required explanation or examples. The teaching level was actually decided by the tutee's expectations and understanding level, not by the tutor.

Tutor and Tutee Questions. Sometimes Kevin asked probing questions; for example, "How can you find the y-intercept point when you transform $y = x^2$ to $y = (x-3)^2$ and then waited a little. Since Mark didn't answer, Kevin answers the question and continues. Another time, Kevin started to teach another transformation of a function. Before explaining, he draw a graph and then asked Mark to explain this transformation verbally. He wanted Mark's reasoning on the transformation, but Mark couldn't. After these unsuccessful attempts, Kevin preferred didactic questions, answering just yes or no.

Case 4: Steven and Ronald

Steven was the second group member to achieve BB from Calculus, having strong high school mathematics. He depends on this knowledge, and carries an exercise book, using this for both tutees. Ronald is Sudanese, knowing little Turkish. Communication was mostly in Turkish, and sometimes in English. Ronald completed a very similar high school mathematics curriculum, and is very hard working and a good listener. Since joining this peer tutoring group, he attends class more often, now sitting at the front and asking questions. He was gained confidence in class, and achieved CC from Calculus.

Steven said "While explaining something, I also understand that topic more deeply". This peer tutoring study group advantages both tutor and tutee. When Steven was asked what he thought about the tutoring, he answered:

> Steven: The only negative is these sessions take lots of time, but being positive, I couldn't study outside because other courses give lots of homework, therefore I have time for calculus here.

Similarly Ronald explained his feelings:

> Ronald: I benefit from many things. For example, I wasn't good at mathematics, but since studying you can see how I'm more active in the classroom. I want to improve in mathematics, but I have other subject homework and exams every week, so benefit a great deal. Steven is the best because we don't embarrass each other. There are no other students and I can ask anything I want. He repeats until I understand. The majority of information comes from Steven, but sometimes I also add something.

Tutor and Tutee Relationship

Steven is self-confident in the group. Steven and Ronald have a good friendship and their own peer study sessions outside of this peer tutoring research, and also study other courses together. Ronald stated he joined other courses since starting this research study. Their relationship in the sessions is at the tutoring level. Ronald has respect for Steven, saying "Steven explains every detail to me. He doesn't get bored and explains even the most stupidly small things".

Teaching Mathematics

Mathematical Knowledge of Tutor and Tutee. Steven has strong high school mathematics knowledge about functions. Because of this, he starts his lessons with basic information about functions. Even with topics not in Calculus, he took the initiative and explained each detail to Ronald. When the researcher asked why, Steven said "I believe they are the basics and everybody should know them". He used his high school test book, saying "Many shortcuts are in that book. The topics are clearly explained". Since he used the high school test book, at the beginning he gave more importance to procedural than conceptual information.

Preparation for the Sessions. At the beginning, Steven didn't prepare, because, according to him, he knew all the information. But once the topics diverged from high school knowledge, he started studying before the sessions.

Teaching Techniques. Steven emphasized procedural knowledge. He conducted a lecture and asked very few questions to Ronald. Generally Steven did the talking and Ronald listened. When Ronald asked questions, Steven answered, then continued lecturing. He didn't check for Ronald's understanding. Because Ronald didn't ask questions, Steven didn't know if his tutee had understood. Probably he was waiting for questions like "I don't understand this part", and since there were no such questions, he continue lecturing.

Teaching level. The teaching level was very low and depending on memorization. Steven used either exercise book or notebook definitions, without additional explanations, thereby failing to make them internalized. He made topics clear by giving examples, such as trying to explain one-to-one function. After reading the definition from the notebook, he said "I mean, let's write a function… $f(x) = 2x^2 + 1$ and for this set... this is domain... each number has a value... this set is range..." But when Ronald asked "Why don't we have inverse function when it is not one-to-one?" Steven answered depending on the graphical representation and horizontal line test definition, with no deeper explanation; so, whilst Steven knows mathematics, he couldn't explain it appropriately. While selecting tutors; knowledge is one thing, but explaining it to someone is something else. This teaching skill is "knowing how to teach mathematics", or professionally, 'pedagogical content knowledge' (Shulman, 1986).

Tutor and Tutee Questions. Steven usually asked closed questions, and giving didactic feedback. He didn't ask explanatory questions or provide suggestive feedback. But Ronald asked some critical questions which could lead to effective understanding of the concept, but Steven couldn't give qualified answers, which could have led to other questions. Ronald asked "and sometimes that c is inside of the function, right?" in the transformation of the function topic; but Steven simply answered "yes".

Case 5: Steven and Edward

In this dyad, Edward is also very confident, and achieved CC in Calculus. He is quiet in class; not asking questions either during or after lessons, preferring to solve questions by himself. Although he prefers to be alone while learning, he volunteered to join this study group.

Tutor and Tutee Relationship

Steven and Edward are friends outside of these study sessions, but in the dyad, Edward is very confident and tries to show he is as knowledgeable as Steven. Because of this, Edward rarely accepted Steven as the teacher. Their relationship is interdependent, with roles changing from time to time. They have reciprocal peer learning style.

Teaching Mathematics

Mathematical Knowledge of Tutor and Tutee. Steven has good mathematical background and could answer questions, yet sometimes couldn't give reasonable explanations. For example, Edward asked "How are we going to decide the sign of the trigonometric function?" Steven drew a unit circle and tried to name the axes and said all trigonometric functions signs by looking at the coordinate system, but didn't explain how. Similarly, when explaining transformation of the function on x-axis with y=f(x-c), he couldn't explain why the function shifted right when 'c' is subtracted from the function.

Teaching Techniques. Edward questioned why topics didn't match course content. Steven responded "these are starting points; the beginning of functions". Actually Steven insisted on applying his own method, giving procedural knowledge, then explaining the topic, but not asking Edward any questions.

Steven's high school textbook gives some graphs, so he asked what the functions were when explaining the horizontal line test. But he only gave the definition of the horizontal line test as "If the lines cut the graph at one point then it's a function, otherwise it's not a function". Edward has some problems understanding, yet Steven didn't explain further. Edward repeated the definition, as if learning by himself, but actually he had memorized.

Teaching Level. Steven's teaching is at the memorization level. However, in the transformation of the function on the x-axis, Steven warned Edward when he repeated the rule.

> Steven: You don't memorize the rule like that. I asked the instructor who said that…for example, our function is y=f(x-c) to make it equal to zero…I mean to make it intersect the x axis you have to add c to the function right?'
> Edward: Why do we need to make it equal to zero?
> Steven: We have y=f(x) and it first cuts the x axis for example, here (showing a point on the x axis). For another function to make the same thing… for example, how I can intersect it…if you add plus c, it means shifting left, if you add minus c, it means shifting right.

Although Steven wouldn't make Edward memorize this rule, he could not explain why it is, so at the end he confirmed his memorization.

Tutor and Tutee Questions. Steven preferred not to ask explanatory questions to either Ronald or Edward. But as with Ronald, Edward asked questions. In one example, although Steven gives a rhyme about the sign of the trigonometric functions, Edward asked "How do we specify the sign of the trigonometric functions on the coordinate system?" Edward drew a coordinate system and a unit circle with the identifying intercept points. Then Steven assigns cosine and sine names to the axes, but couldn't provide clear explanation to Edward about the sign of the functions with these names. He went back to his rhyme, saying "It is simple, keep it in your mind".

Comparison of these overall findings are shown as indicated in Table 4 as a summarization of the five dyads, based on themes.

Table 4. Cases and themes-codes comparisons

Case	Tutor & Tutee Relationship	Teaching Mathematics
One: Michael-Daniel	Interdependent RPT	Some misunderstandings, Teaches procedural knowledge, Unprepared, Dictated, Moderate questioning, Corrective feedback
Two: Sandra-Daniel	Scaffolding RPT	Good mathematical background, Teaches conceptual knowledge, Prepared, Explanatory approach, Conceptual knowledge, Suggestive feedback, Comprehended questioning.
Three: Kevin-Mark	Scaffolding	Good mathematical background, Teaches what tutee ask for, Unprepared, Explanatory, Suggestive feedback, Comprehended questioning.
Four: Steven-Ronald	Tutoring	Strong high school mathematics knowledge, Strong procedural knowledge, Prepared, Stuck to high school knowledge, Memorized notebook definitions; couldn't explain, Tutee asked questions, but couldn't clearly explain.
Five: Steven-Edward	Interdependent RPT	Strong high school mathematics knowledge, Having difficulty to explain, Didactic explanations, No questions.

Conclusion and Recommendation

The purpose of this study has been to identify and understand the behavior of tutor and tutee to each other during peer tutoring study sessions on a Calculus-1 course for engineering students. The five dyads were investigated based on socio-cultural theory, zone of proximal development concepts and teaching-learning mathematics context. This study has attempted to provide details on the relationships of peers during tutoring, plus some properties of the tutors and tutees in playing their roles in effective tutoring.

It was observed that tutors and tutees have three kinds of relationships. These are interdependence to each other, scaffolding one another, and tutoring. These relationships are dependent on the skills of the tutors and personality of the peers. Tutoring effectiveness has traditionally been attributed solely to tutors, even where the tutor is unskilled (Chi, 1996). For example, Sandra's character, based on teaching and believing in herself to do the right thing, is helpful for a good tutor. Moreover, not only are tutor characteristics important, but also tutee characteristics are important. Aligned with the results of the studies of Cohen and Kulik (1982) and Topping et al. (2003), in the current study it is observed that tutors' behaviors and attitudes can be effected by tutees' during the intervention. For example, with Kevin and Mark, although Kevin tries to teach conceptual knowledge, Mark looked for only procedural knowledge and shortcuts, making teaching less effective and less qualified. Therefore, each peers' personality and even their interactions is important for a successful learning environment. Although Reinholz (2015) claimed that "the superficial conversations took place between...students who worked with the same partners repeatedly" (p. 243), according to Chi (1996), the more interactions between tutor and tutee, the more learning within the peer environment. The quality of the discussion is also important. Like Leseman and Sijsling (1996), the current study's tutors may ignore the learner's expectations, views or feelings. Steven was the best example, having ignored tutee needs and only concentrating on his teaching. However, students' active responses are crucial to enhancing learning. Chi et al. (2001) also stated that tutors tend to give unnecessary information that tutees do not need, e.g. when Steven gave unnecessary extensive explanations, making topics overly-complicated and distant to the learner. Although, Leinhardt (2001) claims that good explanations are those aimed at the students' lack of understanding, explanations should be correctly measured.

Another important construct for peer learning is peer matching, with personality and skills the most important elements in the matching decision. The decision-maker must know the subjects, their capabilities, and their personalities. Well-matched pairings may increase academic achievement. In this matching the leader is also important. Merkel and Brania (2015) claimed their unsuccessful peer-led team learning process was due to a lack of leadership qualities exhibited by some peer leaders. Although leaderless peer groups like Reciprocal Peer Tutoring, as created by Fantuzzo et al. (1989), are claimed as being very effective types of tutoring since tutor and tutee roles are interchangeable at times, the mathematics achievement, enthusiasm and teaching experiences should be generally close to each other. Therefore, the personality of each peer affects the success of the peer tutoring. From this point of view, the most successful dyad is Sandra and Daniel, since the tutor has teaching skills, self-confidence in teaching, and also demonstrates scaffolding skills; on the other side, the tutee respects his tutor and completes her deficiencies. The less successful dyads are seen as Michael and Daniel, and Steven and Edward. In both these dyads, their relationships are interdependent of each other; that is, neither scaffolding nor tutoring. For Michael and Daniel, the tutor has neither teaching skills nor a strong mathematics background; however, for Steven and Edward, although the tutor has a strong mathematics background, he was not able to display his knowledge during teaching, so he does not possess teaching skills. Because of these differences found among the reciprocal peer tutor type relationships, the first and the fifth cases are deemed to be the least successful type of peer learning.

Success in pairing calls for monitoring, assessment, and modification at numerous steps throughout (Wlodkowski, 1999). During tutoring, peers should be observed for some reasonable level of change. Two cases in the current study presented this very clearly. Daniel acted quite differently with the two tutors. With one he was interdependent, and with the other it was reciprocal peer tutoring, with the tutor actually scaffolding to Daniel. In the other, the tutor with two tutees acted similarly with both, but the tutees had very different personalities. With one, Steven demonstrated strong teaching skills, but with the other he was weaker as the tutee questioned his teaching and Steven wasn't good at explaining. As Fantuzzo et al. (1992) suggested, reciprocal peer tutoring is developed mutual assistance and social support, which is similar to Michael and Daniel, Sandra and Daniel, and Steven and Edward. Only Sandra and Daniel had scaffolding properties, and because of this, this dyad's peers were more socially supportive to each other. Ronald took an active role and benefitted from the sessions with Steven, just as Korner and Hopf (2013) revealed when they suggested that both tutor and tutee should be active contributors to gain more during peer-assisted tutoring. Moreover, a peer learning environment helped Ronald in being a more sociable student, although he is a foreign student among his peers. This observation aligns with the results of studies by both Miller et al. (1993), where social benefits of peer tutoring are discussed, and Kalkowski (1995) who claimed that peer tutoring can increase the formation of friendship bonds between partners; like the Steven and Ronald dyad in the current study.

Scaffolding was observed in two dyads (Sandra-Daniel, Kevin-Mark), yet King, Stafferi, and Adelgais (1998) found that peer assistance creates 'scaffolds for learning', which has tutors sometimes playing the role of tutee and sometimes tutor. However with Kevin and Mark, reciprocal peer tutoring was not observed, yet they had fixed tutoring where Kevin helped Mark to construct mathematical knowledge. Fixed tutoring has to do with acknowledging a skill differential between tutor (as expert) and tutee (as learner). Kevin achieved this by tutoring to Mark, but with Sandra and Daniel, they sometimes swapped roles, scaffolding each other. This is known as maintaining 'continuity of the role of tutor or tutee' (King et al., 1998, p. 152). Reciprocal tutoring on the other hand, portrays the tutor as working peer partner, taking a reciprocal role while sharing their knowledge (Duran & Monereo, 2005).

Besides peers' different personalities and skills, they also differed in mathematical knowledge. Most students were good at mathematics, but tutors had problems explaining why mathematical statements were true or false. For example, Steven is very good at mathematics, but he could not provide reasonable explanations to his tutees when they asked about conceptual knowledge. Having purely a good mathematical knowledge is insufficient for teaching as there is also the need for effective communication through mathematics. One important finding often overlooked, is that tutors also learned the subject matter they tutored. Among the four tutors, some had strong mathematical knowledge, and some had difficulties with some points. The dyads with mathematically less knowledgeable tutors created problems such as creating misconceptions and inability to provide reasonable explanations, leading tutees to memorize rules rather than comprehension. Ideal mutual learning comes from students working together, each taking turns at demonstrating solutions, co-constructing knowledge, negotiating meanings, and sharing responsibility for learning.

Teaching of mathematics decomposes into four subparts. Preparation by tutors gave important emphasis to this study, with some students concerned that preparation would make for unnatural recording (e.g. Kevin's first session was unprepared). Teaching techniques for only two students (Sandra and Kevin) demonstrated conceptual knowledge, with others emphasizing procedural and computational knowledge, and so tutoring sessions varied, with some teaching conceptual knowledge and some computational facts or shortcuts. This maybe resultant of these first year students' prior educational experiences, having recently sat Turkish university entrance exams where computational skills take precedent over conceptual information. Based on Kevin and Sandra's results; it can be deduced that expert or more experienced tutors know many instructional practices and strategies, and routines, so are better adapted to guiding tutees' actions and thinking. This observation is supported by previous research results like Webb (1991) and Fuchs, Fuchs, Bentz, Phillips, and Hamlett (1994). Moreover, according to Reinholz (2016a), explaining mathematics and giving detailed feedback using the student's mathematical solutions improves both the quality of the explanation and the student's understanding. Conversely, unskilled tutors like Steven and Michael, are typically limited to superficial teaching strategies, asking tutees closed questions such as checking tutees' understand of particular problems or concepts. Aligned with Chi's (1996) study in which tutors and tutees' questions are examined, some questions required didactic explanations, closed questioning needed corrective feedback, and explanatory questions needed suggestive feedback. Kevin and Sandra tried using explanatory questions needing suggestive feedback, whilst the others preferred didactic questioning and mostly corrective feedback and closed questions. All tutors used corrective answers at least once, seeing this as easier to answer as right or wrong. Only Kevin used didactic explanations. He usually tried to give additional information to help tutees understand the topic more deeply. Sometimes long explanations caused confusion, so tutors should limit to providing just enough explanation for the tutee, dependent on their understanding, background knowledge, and motivation. Suggestive feedback was seen rarely as if tutee's give incomplete answers, then the tutor has to give indirect hints to alert the tutee to provide the complete answer; seen as giving hints or non-specific query to redirect the tutee's thinking. Kevin usually preferred suggestive feedback; however, Mark isn't sufficiently competent to use this feedback to reach the correct answer. This result may lead to questions such as: what should the tutee's role be in successful peer learning? Also asked by Chi (1996), she stated that tutees are given the greatest opportunities in answering or explaining to the tutor. Answering questions can be as effective as generating self-explanations, and an effective means of learning. Other than Steven, the tutors asked comprehending questions, with Kevin doing so the most; together creating enhanced opportunities for tutees to engage in meaningful discussion and increase comprehension of the course topics presented in traditional lecture formats.

As a final point, there are different strategies in the learning and teaching of mathematics. One of these, peer tutoring, is in fact very old. In educational life, every student experiences peer tutoring from one side or the other (as tutee or tutor). Most of the time, students have no formal training for tutoring their peers. In the absence of any training, peers can construct either ill-designed or well-designed dyads. In the current study it was tried to examine dyads constructed by the peers themselves, without imposing restrictions or any intervention from the tutor. With the exception of two less successful dyads, three successful combinations were observed. One of the successful dyads presented purely tutoring, while another presented scaffolding, and the most successful dyad displayed reciprocal peer tutoring with scaffolding. Besides the relationships among the peers as tutor and tutee, mathematical knowledge and how they used their knowledge in their teaching affected the structure and success of the dyads.

The results of this study will help instructors who want to take the opportunity to extend their repertoire of classroom instructional options, or to create a different learning environment outside of the classroom for their students. The use of peer tutoring offers several significant advantages. Despite realizing the benefits of peer tutoring, the constructing of a dyad, either by instructor assignment or with no conditions imposed, there is a need for formal training, monitoring, assessment and modification where necessary in order to engender more successful results.

Implications

According to results of the current study, it is observed that personality of the peers is important to successful peer tutoring. Studies conducted about peer tutoring should concentrate on peer personality. Additionally, tutors' enthusiasm is vital to any successful study. Peer personalities should be investigated upfront, seeking out personalities that teachers should ideally possess.

Other implications are that peer selection should not only consider grades, but also testing attitude towards teaching and learning, with specific weight given to mathematics. Instructors should pre-check tutors' misconceptions because students may sometimes have high grades, but considerable misconceptions which they could pass on unwittingly. The students volunteered for the study, themselves deciding who was tutor and tutee. Although reciprocal peer tutoring is considered more effective than other tutoring models, our reciprocal peer tutoring cases shared the same moderate level of mathematics achievement, so conversations and discussions were not that effective mathematically. Peer tutors should be selected by the instructor from the more mathematically skilled. Similarly, deciding who would be tutor and tutee maybe an instructor decision.

For future studies, it is suggested that personality of skilled tutors be investigated. Matched language ability of tutors and tutees should also be investigated in order to ensure mutually beneficial naturalistic learning environment among peers.

References

Allen, V. L., & Feldman, R. S. (1976). Studies on the role of tutor. In V. L. Allen (Ed.), *Children as teachers: Theory and research on tutoring (*pp. 113-129). Academic Press, New York.

Annis, L. (1983). The processes and effects of peer tutoring. *Human Learning, 2*(1), 39-47.

Bénéteau , C., Fox, G., Xu, X., Lewis, J. E., Ramachandran, K., Campbell, S., & Holcomb, J. (2016). Peer-Led Guided Inquiry in Calculus at the University of South Florida. *Journal of STEM Education, 17*(2), 5-13.

Boud, D., & Lee, A. (2005). Peer learning as a pedagogic discourse for research education. *Studies in Higher Education, 24*(4), 413-426.

Bruffee, K. A. (1978). The Brooklyn plan: attaining intellectual growth through peer-group tutoring. *Liberal Education, 64*(4*)*, 447-468.

Chi, M. T. H. (1996). Constructing self-explanations in tutoring. *Applied Cognitive Psychology, 10*, 33-49.

Chi, M. T. H., Siler, S. A., Jeong, H., Yamauchi, T., & Hausmann, R. G. (2001). Learning from human tutoring. *Cognitive Science, 25*, 471-533.

Cohen, P. & Kulik, C. (1982). Educational outcomes of tutoring: A meta-analysis of findings. *American Educational Research Journal, 19*(2), 237-248.

Creswell, J. W. (2005). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research*. Upper Saddle River, NJ: Merrill.

Crouch, C. H., & Mazur, E. (2001). Peer instruction: Ten years of experience and results. *American Journal of Physics, 69*(9), 970-977.

Duran, D., & Monereo, C. (2005). Styles and sequences of cooperative interaction in fixed and reciprocal peer tutoring. *Learning and Instruction, 15*(3), 179-199.

Falchikov, N. (1990). An experiment in same-age peer tutoring in higher education: some observations concerning the repeated experience of tutoring or being tutored. In S. Goodlad, & B. Hirst (Eds.), *Explorations in Peer Tutoring* (pp. 120-142). Oxford: Blackwell.

Falchikov, N. (2001). *Learning together: peer tutoring in higher education*. Florence, KY: Routledge.

Falchikov, N., & Goldfinch, J. (2000). Student peer assessment in higher education: A meta-analysis comparing peer and teacher marks. *Review of Educational Research, 70*(3), 287-323.

Fantuzzo, J. W., King, J. A., & Heller, L. A. (1992). Effects of reciprocal peer tutoring on mathematics and school adjustment: a component analysis. *Journal of Educational Psychology, 84*(3), 331-339.

Fantuzzo, J. W., Riggio, R. E., Connelly, S., & Dimeff, L. A. (1989). Effects of reciprocal peer tutoring on academic achievement and psychological adjustment: A componential analysis. *Journal of Educational Psychology, 81*(2), 173-177.

Fuchs, L. S., Fuchs, D., Bentz, J., Phillips, N. B., & Hamlett, C. L. (1994). The nature of student interactions during peer tutoring with and without prior training and experience. *American Educational Research Journal, 31*(1), 75-103.

Gaustad, J. (1992). Tutoring for at-risk students. *Oregon School Study Council, 36*(3), 3-66. Retrieved from http://files.eric.ed.gov/fulltext/ED353642.pdf

Gosser, D. K. (2011). The PLTL boost: A critical review of research. *Progressions: Journal of PLTL*, *14*(1), 4-19.

Hannah, D. C. (2008). *Attitudinal Study: The Interaction of Students Taking Calculus and Prerequisite Courses while Participating in Peer Tutorials* (Doctoral dissertation). Georgia State University. Retrieved from http://scholarworks.gsu.edu/cgi/viewcontent.cgi?article=1034&context=msit_diss

Hattie, J. A. C. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement.* London, UK: Routledge.

Hoops, L. D., Yu, S. L., Wang, Q., & Hollyer, V. L. (2016). Investigating Postsecondary Self-Regulated Learning Instructional Practices: The Development of the Self-Regulated Learning Observation Protocol. *International Journal of Teaching and Learning in Higher Education, 28*(1), 75-93.

Johnson, D. W., Johnson, R. T., & Smith, K. (2007). The State of Cooperative Learning in Postsecondary and Professional Settings. *Educational Psychological Review 19*(1), 15-29.

Kalkowski, P. (1995). Peer and cross-age tutoring. *School Improvement Research Series, 18*, 1-7.

King, A. (2002). Structuring Peer Interaction to Promote High-Level Cognitive Processing. *Theory into Practice 41*(1), 33-39.

King, A., Stafferi, A., & Adelgais, A. (1998). Mutual peer-tutoring: Effects of structuring tutorial interaction to scaffold peer learning. *Journal of Educational Psychology, 90*(1), 134-152.

Klaus, D. J. (1975, March). Patterns of peer tutoring. Paper presented at the *Annual Meeting of the American Education Research Association,* Washington, D.C. Abstract retrieved from http://files.eric.ed.gov/fulltext/ED103356.pdf

Korner, M., & Hopf, M. (2013). Cross-age peer tutoring in physics: tutors, tutees and achievement in electricity. *International Journal of Science and Mathematics Education, 13*(5), 1039-1063.

Laskey, M. L., & Hetzel, C. J. (2011). Investigating Factors Related to Retention of At-Risk College Students. *Learning Assistance Review, 16*(1), 31-43.

Lave, J., & Wenger, E. (1991). *Situated learning: legitimate peripheral participation*. Cambridge University Press.

Leinhardt, G. (2001). Instructional explanations: A commonplace for teaching and location for contrast. In V. Richardson (Ed.), *Handbook for research on teaching* (4th ed.). Washington, DC: American Educational Research Association.

Leseman, P. P. M., & Sijsling, F. F. (1996). Cooperation and instruction in practical problem solving. Differences in interaction styles of mother-child dyads as related to socioeconomic background and cognitive development. *Learning and Instruction, 6*(4), 307-324.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Beverly Hills, CA: Sage

Loui, M. C., & Robbins, B. A. (2008). Work-in-progress: Assessment of peer-led team learning in an engineering course for freshmen. In *Proceedings of the Thirty-Eighth ASEE/IEEE Frontiers in Education Conference, Saratoga Spring*, NY. FIF-7 – FIF-8.

Liou-Mark, J., Dreyfuss, A. E., & Younge, L. (2010). Peer assisted learning workshops in precalculus: An approach to increasing student success. *Mathematics and Computer Education, 44*(3), 249-260.

Malm, J., Bryngfors, L., & Mörner, L. (2010). Supplemental instruction (SI) at the Faculty of Engineering (LTH), Lund University, Sweden: An evaluation of the SI program at five LTH engineering programs autumn 2008. *Australian Journal of Peer Learning, 3*(1), 38-50.

Marshall, C., & Rossman, G. (1999). *Designing Qualitative Research* (3rd ed.). London: Sage.

Martino, L. (1993). When students help students. *The Executive Educator, 15*(1), 31-32.

Maxwell, J. (1992). Understanding and Validity in Qualitative Research. *Harvard Educational Review, 62*(3), 279-301.

Merkel, C. J., & Brania, A. (2015). Assessment of Peer-Led Team Learning in Calculus I: A Five-year Study. *Innovative Higher Education, 40*(5), 415-428.

Merriam, S. B. (1998). *Qualitative research and case study applications in education*. San Francisco, CA: Jossey-Bass.

Merriam, S. B. (2002). *Qualitative research in practice: Examples for discussion and analysis.* San Francisco, CA: Jossey-Bass.

Miller, L. J., Kohler, F.W., Ezell, H., Hoel, K., & Strain, P. S. (1993). Winning with peer tutoring. *Preventing School Failure, 37*(3), 14-19.

Neubert, J., Khavanin, M., Worley, D., & Kaabouch, N. (2014). Minimizing the Institutional Change Required to Augment Calculus With Real-World Engineering Problems. *PRIMUS, 24*(4), 319-334. DOI: 10.1080/10511970.2013.879970

Parkinson, M. (2009). The effect of peer assisted learning support (PALS) on performance in mathematics and chemistry. *Innovations in Education and Teaching International, 46*(4), 381-392.

Pigott, E. H., Fantuzzo, J. W., & Clement, P. W. (1986). The effects of reciprocal peer tutoring and group contingencies on the academic performance of elementary school children. *Journal of Applied Behavior Analysis, 19*, 93-98.

Prince, M. (2004). Does active learning work? A review of the research. *Journal of Engineering Education, 93*(3), 223-231.

Reinholz, D. L. (2015). Peer-Assisted Reflection: A designed-based intervention for improving success in calculus. *International Journal of Research in Undergraduate Mathematics Education, 1*, 234-267.

Reinholz, D. L. (2016a). Improving calculus explanations through peer review. *Journal of Mathematical Behavior, 44*, 34-39.

Reinholz, D. L. (2016b). The Assessment Cycle: A Model for Learning through Peer Assessment. *Assessment & Evaluation in Higher Education, 41*(2), 301-315.

Reisel, R. R., Jablonski, M. R., Munson, E., & Hosseini, H. (2014). Peer-led team learning in mathematics courses for freshmen engineering and computer science students. *Journal of STEM Education, 15*(2), 7-15.

Rohrbeck, C. A., Ginsburg-Block, M. D., Fantuzzo, J. W., & Miller, T. R. (2003). Peer-assisted learning interventions with elementary school students: A meta-analytic review. *Journal of Educational Psychology, 94*(20), 240-257.

Saunders, D. (1992). Peer tutoring in higher education. *Studies in Higher Education, 17*(2), 211-219.

Shenton, A. K. (2004). Strategies for ensuring trustworthiness in qualitative research projects. *Education for Information, 22*, 63-75.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Slavin, R. E. (1996). Never streaming: Preventing learning disabilities. *Educational Leadership, 53*(5), 4-7.

Solomon, Y., Croft, T., & Lawson, D. A. (2010). Safety in numbers: Mathematics support centers and their derivatives as social learning spaces. *Studies in Higher Education, 35*(4), 421-431.
Springer, L., Stanne, M. E., & Donovan, S. (1999). Effects of small-group learning on undergraduates in science, mathematics, engineering and technology: a meta-analysis. *Review of Educational Research, 69*(1), 21-51.
Tinto, V. (1999). Taking student retention seriously: Rethinking the first year of college. *National Academic Advising Association Journal, 19*(2), 5-9.
Topping, K. J. (1996). Effective peer tutoring in further and higher education: a typology and review of the literature. *Higher Education, 32*(3), 321-345.
Topping, K. J. (2005). Trends in Peer Learning. *Educational Psychology, 25*(6), 631-645.
Topping, K. J., Campbell. J., Douglas, W., & Smith, A. (2003). Cross-age peer tutoring in mathematics with seven- and 11-year-olds: Influence on mathematical vocabulary, strategic dialogue and self-concept. *Educational Research, 45*(3), 287-308.
Topping, K. J., & Ehly, S. (Eds.). (1998). *Peer-assisted learning*. Mahwah, NJ: Lawrence Erlbaum.
Topping, K. J., Watson, G. A., Jarvis, R. J., & Hill, S. (1996). Same- year paired peer tutoring with first year undergraduates. *Teaching in Higher Education, 1*(3), 341-356.
Tosey, P., & Gregory, J. (1998). The Peer Learning Community in Higher Education: reflections on practice. *Innovations in Education and Training International, 35*(1), 74-81.
Treisman, U. (1992). Studying students studying calculus: A look at the lives of minority mathematics students in college. *The College Mathematics Journal, 23*(5), 362-372.
Valkenburg, J., & Dzubak, C. (2009). *The engaged mind: Cognitive skills and learning.* Markton, SC: CreateSpace.
Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes* (M. Cole, V. John-Steiner, S. Scribner, & E. Souberman, Eds.). Cambridge, MA: MIT Press.
Webb, N. M. (1991). Task-related verbal interaction and mathematics learning in small groups. *Journal for Research in Mathematics Education, 22*(5), 366-389.
Wenger, E. (1998). *Communities of practice: learning, meaning, and identity*. Cambridge University Press.
Wertsch, J. V. (1991). *Voices of the mind: A sociocultural approach to mediated action*. Cambridge, MA: Harvard University Press.
Wlodkowski, R. J. (1999). *Enhancing adult motivation to learn* (Rev. ed.). San Francisco, CA: Jossey-Bass Inc.
Wolfe, J. A., Fantuzzo, J. W., & Wolfe, P. K. (1986). The effects of reciprocal peer management and group contingencies on the arithmetic proficiency of underachieving students. *Behavior Therapy, 17*(3), 253-265.
Wood, D., Bruner, J., & Ross, G. (1976). The role of tutoring in problem solving. *Journal of Child Psychology and Psychiatry, 17*(2), 89-100.
Yin, R. K. (2002). *Case study research: Design and methods*. Thousand Oaks, CA: SAGE Publications.

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