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Investigating the Impact of Lesson Study and Pedagogical Content Knowledge on Mathematics Teaching Practices of Minority Pre-Service Teachers

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Abstract

Contextualized in an urban college's teacher education program and its partnership inner-city elementary schools, this qualitative study explores the impact of lesson study on the emergence of a stronger pedagogical content knowledge on mathematics teaching practices of minority preservice teachers. Aiming to raise the mathematics achievement of K-6 students by increasing the effectiveness of preservice teacher candidates, this study focuses on developing and deepening 11 prospective teachers' mathematics pedagogical content knowledge by requiring them to analyze their students' mathematics work samples and study the effectiveness of their own teaching practices during their clinical-practice/student teaching year. Findings reveal that participant preservice teachers' pedagogical content knowledge benefited with lesson study. By using the lesson study approach, they were able to change their mathematical identities and beliefs about teaching mathematics in positive ways and become more effective in teaching mathematics. Implications for urban preservice teacher education programs are discussed.

Keywords: Lesson study, pedagogical content knowledge, mathematics identity, pre-service teachers

Introduction

This study focuses on developing and deepening 11 prospective teachers' mathematics pedagogical content knowledge by requiring them to analyze their students' mathematics work samples and study the effectiveness of their own teaching practices during their clinical-practice/student teaching year. In this study, minority students are defined as first-generation-college-going immigrant or non-immigrant non-White students of lower socioeconomic background, who finished their high school either outside of the U.S. or in inner-city U.S.-schools that followed traditional mathematics curricula and teacher-centered mathematics teaching strategies. Most policy makers and scholars in mathematics education stress that teachers' effectiveness is very much mediated by their beliefs, perceptions, and the way they learn mathematics (Ball, Lubinsky, & Mewborn, 2001; Bray, 2011, Darling-Hammond, & Bransford, 2005; Darling-Hammond, 2006; Frank, Fennema, & Carpenter, 1997, NCTM 2007). In order to increase teachers' effectiveness and students' outcomes, teacher education programs should focus on preparing teachers who have profound knowledge of reform- based mathematics content and "knowledge of students' mathematical thinking and learning" (Hill, Ball & Schilling, 2008, p.373).

To support reform-based mathematics, it is essential that teacher education programs have a deep understanding of how teachers' beliefs, pedagogical and content knowledge, and teaching practices are related (Ball et al. 2001; Bray, 2011, Darling-Hammond & Bransford, 2005; Darling-Hammond 2006; Frank et al. 1997). Battista (1994) asserts that in mathematics education,

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changing teachers' traditional beliefs and practices is crucial to the success of reform efforts. In a study on mathematics' teaching practices, Goldhaber'(2006) concluded that what teachers learn in education method courses is fundamental to their teaching practices, effectiveness in the classroom, and their Pedagogical Content Knowledge (PCK). The National Academy report on higher educational outcomes emphasizes that effective teachers with a higher level of PCK are successful in raising students' achievement (Hill et al. 2008). Aligned with Hill et al.'s (2008) notion on teacher effectiveness, this study argues that in addition to the subject matter knowledge, Pre-Service Teachers (PSTs) need the knowledge of students' mathematical thinking and learning. By utilizing the lesson study approach, pre-service mathematics teachers develop a higher level of PCK. Thus, the questions that guided this study were:

- How does lesson study help minority PSTs teach mathematics lessons and enable them to study the effectiveness of their own teaching practices?
- How lesson study helps PSTs to develop stronger PCK?
- How higher level of PCK, a deeper mathematical understanding, and teaching strategies (learned in the mathematics methods course) enable minority PSTs to develop in-depth knowledge of the mathematical concepts they teach?

Conceptual Framework

This study is grounded in the concept of mathematics identity (Gee, 2001; Gresalfi & Cobb, 2011; Martin, 2007; Yackel & Cobb, 1996; Taylor, Puchner, Scheibel, 2006), and the importance of beliefs and experiences (Fennema et al., 1996; Thompson, 1992). The concept of identity emphasizes that mathematics identity encompasses: (a) individual teacher's self-perception with regard to their knowledge of mathematics; (b) their confidence level to teach mathematics; and (c) a set of practices and expectations that shape individual teacher's beliefs about their mathematics teaching competencies (Gee, 2001; Gresalfi & Cobb 2011). The process of preservice teachers' mathematical identity formation is profoundly influenced by the norms, values, and practices of the specific context of their prior mathematics experiences, mathematics method course(s), and their early field experiences.

Further, it has been proposed that *racialized forms of experiences* (Oppland-Cordell, 2013, p. 91) influence the *mathematics identity* of minority teachers and minority students. Martin (2007) defines mathematics learning as "structured by the relations of race that exist in the larger society" (p.5) and mathematics identity as "the dispositions and deeply held beliefs that individuals develop about their ability to participate and perform effectively in mathematical contexts" (p. 150). Martin's mathematics identity co-construction theory examines Black PSTs' mathematics learning and racial identity construction. Since all of the participants in this study were Black (American or foreign born), it is assumed that their mathematics learning, mathematics identity, and self-perception would be influenced by their racial identity.

The co-construction theory provides explanation for negative outcomes in mathematics by highlighting factors such as cultural differences, knowledge of mathematics, problem solving skills, family's socioeconomic status, and oppositional behavior triggered by school contexts that influence mathematics achievement of African American students (Martin, 2009, 2012). The theory depicts that African American students' mathematics identity is socially constructed in the social climate that labels African American children as mathematically deficit (Ackerman, Heafner, & Bartz, 2006; Stinson, 2007; Gregory, Skiba, & Noguera, 2010; Zilanawala, Martin,

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Noguera, & Mincy, 2017). Research has identified many cognitive, non-cognitive, institutional, and structural factors that influence the nature of participation and socialization and the mathematics performance and achievement of Black children (Berry, 2008; Martin, 2007; Terry, 2010; Jackson,2009; Oppland-Cordell, 2013). For instance, negative social realities and the ideology of Black inferiority (Martin, 2009a, 2009b; Jackson 2009) influence negatively Black children's confidence in mathematics learning. The influence of Black inferiority ideology is often evident in classrooms impacting mathematics achievement and performance. In sum, it can be proposed that mathematics identity construction is not only mediated by performance in mathematics, but rather it is deeply influenced by society's perception and stereotyping of minority students' mathematics competencies.

Conceptualization of Lesson Study

Among mathematics education researchers, the widespread belief is that PCK contributes to effective mathematics teaching and students' mathematics learning in the classroom (Abell, 2007; Akerson et al. 2017; Bausmith & Barry, 2011; Carpenter, Fennema, Peterson, & Carey, 1988; Van Driel & Barry, 2012; Hashweh, 2005; Hill et al. 2008; Tröbst, et al. 2018). PCK is defined as "teachers' combined knowledge of content and their students' learning" (Hill et al. 2008, p. 373). Effective teachers possess mathematics teaching methodology and exclusive knowledge of their students' mathematical thinking, conceptual and procedural understanding, and competencies. The research by Hill et al. (2008) stresses that effective teachers have knowledge of the mathematical concepts they teach and understanding of how students learn and know mathematics. At the same time, PSTs understand how PCK is related to students' mathematics achievement and performance (Hill et al. 2008). The insights into students' understanding of mathematics provide a significant foundation for PSTs' PCK. Teacher education programs that concentrate on the development of PCK by teaching PSTs to investigate students' mathematics learning practices analyze their students' misconceptions and misunderstandings of mathematical concepts, produce effective mathematics teachers, and improved students' mathematics achievements.

Lesson study provides opportunities for preservice teachers to examine their teaching practices and understand their strengths and areas that need improvement (Taylor et al., 2006). The lesson study inquiry involves systematic, intentional study of one's own professional practices (Puchner, & Taylor, 2006; Taylor et al. 2006). It follows a collaborative framework in which cooperating/mentor teachers, PSTs, and the college supervisor work together to plan lessons and units of study. It is a type of teacher action research that allows PSTs to examine their own teaching strategies, their students' understanding, and consequently improve their mathematics teaching practices. Accordingly, teachers identify a problem, design the research project, collect and analyze data, and change their teaching practices based on their findings (Dana & Yendol-Silva, 2003; Taylor et al. 2006). The most salient feature of lesson study is collaboration. The participating teachers collaborate to design and plan the lesson. In this study, the PSTs were involved in the lesson study process that provided them opportunities to understand and examine their teaching skills and practices, and based on the insights gained through this examination, to improve their teaching practices and teaching methodology.

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Methods

The study was conducted in a large Northeastern urban city. The Brown College (pseudonym) is a part of a large urban university. The college educates and prepares preservice teachers to teach racially, ethnically and linguistically diverse students in urban communities. The study specifics included:

- *Research schedule and timeline:* The timeline for this research was two semesters of clinical practice in two elementary school classrooms. The grades ranged from first to fifth. The grade level of the PSTs was determined by the college's clinical practice supervisor.
- *Research sites:* Brown College's partner schools in urban neighborhoods. All partner schools were within the vicinity of five miles of the college. These schools were situated in lower socioeconomic neighborhoods with high unemployment rates. On the average, 80-90 percent of the students in these schools qualified for free lunch.
- *Participants:* Out of eleven participants, there were nine female and two male participants. All participants were of Black heritage. Ten participants had emigrated to the U.S. from the Caribbean Islands of Jamaica, Trinidad, and Guyana. Only one participant was from South Carolina, United States. The participants were enrolled at the Elementary Education Teacher Preparation Program at Brown and completed their student teaching practices requirement at the college's partner schools.
- *Curriculum Focus:* PSTs were expected to use reform-based mathematics curriculum (NCTM, 2007), the Common Core mathematics standards, and quality mathematical activities.
- *Instructional Methodology*: PSTs were supposed to follow the Concrete- Pictorial-Abstract (CPA) methodology (Bruner, 1966). According to Bruner (1966) there are three stages of cognitive development: the concrete, the pictorial, and the abstract stages. It is proposed that children learn best by constructing their own knowledge through concrete-pictorial-abstract activities that include three stages of learning: *enactive*, *iconic*, and *symbolic*. The *enactive* representation stresses that children learn associating the concrete experiences with past experiences. The *iconic* representation is comprised of mental and visual images of an activity. Finally, the *symbolic* representation is the final stage of mathematics learning. At this stage, individuals connect the mathematical concepts with symbols or language (Bruner, 1966).

Data Sources and Data Collection

The qualitative data for this study comprised of lesson study, reflective journals, field notes of informal interviews, pre and post surveys of PSTs perception/beliefs of their mathematics teaching, and re-teaching lessons. PSTs worked with cooperating teachers to plan a unit of study, taught the lesson while collecting data on student learning (the research lesson), discussed and revised the lesson (debriefing), and re-taught the lesson with the modifications that were made based on the findings from the prior research lesson. The researcher examined PSTs perceptions about their classroom environment and experiences and its impact on their self-efficacy toward mathematics, and development of PCK through lesson study.

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PSTs constructed two mathematics units consisting of six lessons each were designed and enacted each semester. They developed a rubric for each lesson. More specifically, the pre-service teacher participants:

- Developed a formative assessment to assess students' conceptual understanding, computational and procedural fluency, mathematical reasoning, and problem-solving skills.
- Defined the evaluation criteria they would use to analyze student learning related to the mathematical understanding described above.
- Collected student work from the selected assessment and analyzed students' mathematics learning practices, mathematical errors, confusions, and partial understandings learning in graphic (chart or table) or narrative form to identify patterns of learning.
- Selected and submited 5 work samples that demonstrated an area of struggle identified in their analysis and analyze the errors or misconceptions related to the struggle.
- Designed and taught a re-teaching lesson based on the targeted learning objective/goal and to teach 5 focus students during one-on-one, and small group implementation.
- Analyzed the effectiveness of the strategies they used during the re-teaching lesson to develop students' mathematical understanding misconception(s) in relation to the identified areas of struggle.
- Wrote reflective essays evaluating and reflecting on the effectiveness of their teaching practices and its impact on students' learning. The reflective essays were an essential part of this study. The PSTs reflected on their teaching practices and wrote their reflective thoughts on their teaching methods and children's mathematics learning. To write realistic reflections and correct conclusions, PSTs observed their students' mathematics learning, responses, and class work. In their reflective essays, PSTs were required to analyze their students' work. Often in the classroom settings, teachers tend to treat their lesson through teachers' perspective and interpretations (Fernandez, Cannon, & Chokshi, 2003) not from students' perspective. In order to honor students' perspectives, PSTs reflected on their lessons and methods of instruction through their students' perspective (Fernandez et al. 2003). The discussions of reflective thoughts, reflections, and students work analysis provided PSTs opportunities to examine strength and areas of improvement of their mathematics teaching.

Data Analysis

Following the qualitative research design (Guba & Lincoln, 1989), the analysis of qualitative data began as soon as it was collected. Data was analyzed utilizing Grounded Theory (Strauss, 1987) where theorizing grows from the data rather than from a pre-existing framework used to confirm or disconfirm a theory. Through document analysis of reflective narratives, surveys, transcripts of informal interviews, re-teaching lessons, lesson study, pre and post surveys of PSTs perceptions and beliefs, and field notes, codes were developed based on categories which emerged within mathematics identities, conceptions of the nature of mathematics, and best practices in mathematics teaching and learning. The video analysis enabled PSTs to compare the

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salient features of the teaching practices from the beginning of the semester to the end of the semester.

To strengthen the trustworthiness of the findings, a framework of prolonged engagement, member checking, and triangulation of data from multiple sources (Guba & Lincoln, 1989) was followed. The member-checking process provides opportunities to verify data with participants who provided them (Guba & Lincoln, 1989). Member checking guaranteed that the PSTs opinions and experiences were truthfully recorded at all times and especially at the close of the research study (Stringer, 2004). During the collection and analysis period, participants were asked to verify their responses for accuracy and consistency between what was recorded and what was intended to communicate, correct errors, and confirm data and judge the adequacy of their responses (Guba & Lincoln, 1989). The researcher was with the participants for one academic year, which provided opportunities to build rapport and establish trust with participants (Guba & Lincoln, 1989).

Results and Discussion

The pre-surveys from the beginning of the study revealed that PSTs' negative belief and perception of their abilities was a manifestation of prior socializing experiences in mathematics contexts. Out of eleven PSTs, ten responded that they did not like mathematics, lacked confidence in teaching mathematics, and possessed very low motivations and rationales for teaching mathematics. All PSTs, except one, voiced the lower expectations for their performance in mathematics teaching. In the beginning of the study, PSTs started with a shared assumption about their mathematical literacy, negative identity, and lower agency. These ideas surfaced in reflective thoughts and informal interviews where PSTs expressed views on the relationship between mathematics performance and Blackness, which were appropriated from their prior experiences in mathematics context. The results support Martin's (2012) assertions that Black individuals' negative self-perceptions and mathematics identities are shaped by deficit ideologies and racial stereotypes.

However, findings revealed that most of the minority preservice teachers who participated in this study changed their mathematical identities and beliefs about teaching mathematics in positive ways and developed motivation to teach mathematics. The main ways in which change occurred are discussed below.

Impact of Lesson Study on PSTs' PCK, Ability to Analyze Students' Mathematics Work and Their Effectiveness in Teaching Mathematics Lessons

The qualitative data of this study such as reflective narratives, surveys, re-teaching lessons, lesson study, and field notes, demonstrated that lesson study helped and provided support to the 11 minority PSTs to teach and re-teach the same lesson. After collecting data and assessing their students' learning and gaining knowledge of their students' mathematical thinking, PSTs developed a higher level of PCK. After debriefing with cooperating and mentor teachers, a college supervisor, and analyzing personal reflections, PSTs found the lesson study method valuable. They mentioned that the input and examples of teaching practices from cooperating teachers and the college supervisor served as a major resource for them and contributed to the development of PCK and mathematics identity.

The lesson study provided them with insights into their students' misconceptions and misunderstandings of mathematical concepts, which caused their students' mathematics errors.

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Most of the PSTs did not want their lessons to be "bombed" (failed) and tried to take preventive measures to avoid "bomb" (Taylor et al. 2006, p. 193; Van de Walle, 2007, p. 57) situations. At the same time, they discovered the benefits of collaboration for the success of their lessons. These findings are exemplified as PSTs shared their reflections:

> Back home (Jamaica) we learned fractions, but I did not get it and I hated fractions. The first time I taught the lesson they did not get it and the lesson was "bombed" [Failed]. I collected their work to learn from their mistakes what I needed to work on. When I analyzed their work many things [mathematical concepts] became clearer to me. The re-teaching lesson was a success … they got it. I think a good lesson plan has less chance to be "bombed." (Sharifa, female PST, second semester of Clinical Practice).

The collaborative experiences boosted moral and motivated her to teach mathematics with ease and confidence. Sharifa acknowledged that the collaborative nature of the lesson study was a source of emotional support for her. She shared her teaching experiences and revealed that the lesson study process helped her prepare for teaching future lessons. With the lesson study process, she became aware of the benefits of collaboration, in-depth lesson preparation, discussion of unforeseen classroom situations, and many other elements that can save a lesson from being "bombed."

What follows is another example in which a participant, Lucinda, emphasized on the importance of good planning and the benefit of prior knowledge:

> Before teaching my first lesson I thought that making a re-teaching lesson and rubric would be very hard. But I realized that we did students' work analysis in our math course and learned to make rubrics… [the] problem was making a "good lesson. I had to work with my cooperating teacher, other student teachers and my professor. The first time …I did not do good planning. The book I used to introduce the concept was not a very good book (the ebook did not work) and I needed to make more charts… I did better next time. (Lucinda, female PST, first semester of Clinical Practice).

Lucinda reflected on the collaborative aspect of the lesson study and the importance of detailed lesson planning in enactment of the lesson. She mentioned that better planning and collaboration helped her create a sound and detailed lesson plan, improved her instructional practices, and increased her teaching effectiveness.

Overall, the participants reported that the lesson study was very helpful in the preparation and enactment of their lessons and that the strategies learned in the mathematics method course enabled them to teach mathematics successfully. The application of mathematics teaching strategies, such as, problem-analysis skills, selecting various problem-solving strategies appropriately, assessing and justifying the validity of answers, and helping students learn by generalizing the problem, provided them with in-depth knowledge of how the subject matter they teach is connected to children's mathematical thinking. They developed higher level of PCK by analyzing children's mathematics understanding and errors, and developing and re-teaching lessons. Most of the PSTs examined their students' work by using analysis skills, such as, integrating writing in math, conferencing, initiating dialogue, and using a rubric, among other methods that they had learned in their math method class. Although in the initial stage of their clinical practice, they had problems in providing explanation for their students' mathematical errors and did not have a sense of why those errors occurred, this changed as they gained experience by doing many sample work analyses through lesson study.

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Impact of PCK, Mathematical Understanding, and Reflective Thoughts on Minority PSTs' Teaching Practices

Initial learning of pedagogical content knowledge: First semester of clinical practice (student teaching). The data suggested that during the first semester of Clinical Practice PSTs' PCK was at an early developmental stage. Although they learned methodology of how to teach mathematics, their PCK was challenged by the real-life situations of the classroom, such as, classroom management issues, the cultural and linguistic dynamics of the classroom, the curriculum, testing challenges, and their own limited PCK. While they were learning to navigate day-to-day classroom discourses and teaching dilemmas, their PCK was developing. Rabina, a first semester Clinical Practice PST possessed stronger subject matter but her knowledge of "how students learn the content" (Hill et al. 2008) was weaker. She had to learn more about her students' mathematics learning practices, teaching mathematics through problem-solving, and teaching reform-oriented mathematics. Rabina shared her frustration about her own teaching ineffectiveness: "Math is my concentration area. I can do Calculus. I get good grades in mathematics but I am not sure why I had a hard time teaching base ten to third graders? What is the problem?" Rabina had strong mathematics content knowledge but her pedagogical content knowledge needed improvement; she had to acquire knowledge of her students and how students learn mathematics. She added, "In my math methods course I was very confident that I will have no problem in student teaching because I know my math, I was wrong. I was not serious in that class." Rabina realized that her content knowledge alone was not enough to become an effective math teacher; she had to learn how to teach math.

Sabrina admitted that her lower level of mathematics content knowledge, and PCK affected the success of her place-value lesson. However, her statement "My re-teaching lesson went well" is indicative of the development of PCK from which to build in the second semester of Clinical Practice. She continued:

> I thought strategies like counting by ones, counting by groups with sticks and plastic cubes in class was too easy activity but now I know they are important strategies to introduce place-value. I always thought that place value is a very easy topic to teach and learn but this is not easy …this is a hard topic to teach. (Sabrina, First Year PST)

The second semester of clinical practice: Strengthening of pedagogical content knowledge: Sharifa's experiences of teaching fractions demonstrate the impact of PCK on teaching practices. Although she came to the methods class with fear of fractions, gradually she gained the conceptual and procedural knowledge of fractions. However, the first time she taught fractions she failed to apply the knowledge gained in her methods course. This changed in the second semester, when she was able to strengthen the content knowledge of fractions, and the knowledge of how students learn fractions, and what strategies are useful to teach the content. She shared:

> Comparing last semester this time (second semester) I did good on fractions. I can explain better why we need to get common denominator or doing problems with mixed numbers. Teaching adding or subtracting fractions are easy for me to teach following the way we learned to teach in college. When we learned fractions in my country it was only rules of doing fractions the concept was not explained … (Sharifa, second semester of Clinical Practice)

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The PSTs shared their ease in teaching the specialized topics and their higher confidence level in teaching mathematics. They became better in interpreting their students' work, detecting reasons for errors and conceptual misunderstanding, and identifying the root causes of the errors while understanding the students' mathematical competency according to their age and grade. Edwin, a second semester PST wrote in his reflective journal:

> The second semester is easier because I know how my students learn. I find it easier to examine their work for mistakes and I can discuss their problems or mistakes better and I find easier to write about their problems. When Martha added 35+16 and her answer was 411, I knew how she got that answer. I have many examples like this….

Impact of Positive Mathematics Identity on Mathematics Teaching Practices

By the end of their Clinical Practice, the majority of the PSTs confirmed a positive change in their mathematics identity. They shared that their lower mathematics teaching confidence was the manifestation of inadequate elementary and high school mathematics preparations. They believed that they were much more confident in teaching mathematics because they had not only refreshed their knowledge of elementary mathematics but had also experienced how much it could be relatable to and useful in their real lives. The PSTs also felt more comfortable and confident because they now had a deeper understanding of the conceptual processes rather than mere procedural knowledge through lesson study and sustained reflection. In conversations with the College Supervisor, two PSTs noted:

> Now teaching math is easy to me… Discussing math lessons before teaching is helpful. It clears problem spots in lessons…I know what to do…no problems (Sabrina, female PST, first semester of Clinical Practice).

> I am teaching them about "time" so I have to give examples to make them understand time… In Guyana our math classes were different. You do not want me to teach math like that… I never liked math but I think I can teach math. They understand it when I teach math (Ustfa, pseudonym, Male PST, second semester of Clinical Practice).

While teaching through lesson study, PSTs were able to see improvement in their students' mathematics understanding. At the same time, their negative perception of their mathematics ability was gradually fading. They were afforded with opportunities to analyze their students' mathematics work samples and study the effectiveness of their own teaching practices of mathematics literacy. Consequently, the PSTs felt more comfortable and confident because they could analyze and modify their own teaching practices. The feelings echoed by the majority of PSTs were that, during their clinical practice, they realized how valuable using manipulatives and concrete-to-pictorial methods (in which individuals learn by working with physical materials) are to teach mathematics.

Edwin, one of the PSTs shared his experiences:

> In my class all Asians were very good in math and we were (non-Asians) slow in math. I thought math was [an] Asian thing. Now I like to teach hands-on math. I think I am good at teaching math to young children (Edwin, male PST, first semester of Clinical Practice).

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Edwin's early mathematics experiences speak of racialized mathematics experiences and how he constructed his mathematical identity. However, his mathematics methods course and Clinical Practice experiences resulted to a change in his preconceived dispositions and beliefs, as he recognized his evolving mathematics competencies.

Implications and Conclusion

The pedagogical content knowledge comprises of the knowledge of how students learn, what misunderstandings and misconceptions negate or facilitate students' mathematics learning, and the insights of a specific subject matter or topic. The development of PCK is an integral part of PSTs awareness of their teaching practices. The lesson study approach provided teachers with specific input, opportunities to enact certain instructional strategies, chances to reteach a lesson, and time to reflect, individually and collectively, on their mathematics teaching experiences. All participants reflected upon their pedagogical experiences and instructional strategies and were able to identify why their lessons were unsuccessful. By teaching one topic and one lesson over and over, PSTs acquired a myriad of teaching practices that contributed to their PCK. Many participant PSTs shared that by analyzing their students' mathematical errors they became familiar with the logic and thinking behind students' errors. In doing, they developed a better ability to identify, recognize, and interpret students' mathematical errors. With the insight into students' mathematics learning and errors, PSTs developed abilities to design better lesson plans. At the same time, they gained confidence in teaching mathematics. The PSTs benefited with lesson study and developed stronger PCK, which shifted their negative mathematics teaching beliefs into positive ones. The positive beliefs can improve their effectiveness in teaching mathematics and set them on more positive teaching journeys when they enter their own classrooms in predominantly urban areas. These positive shifts in beliefs and competence can likely translate into more positive experiences for Black students as they will not perpetuate a negative cycle of mathematics affect. With this shift in beliefs, they are also better able to develop confident mathematics identities and can become autonomous mathematics teachers.

In urban schools, the influence of Black inferiority ideology (Martin, 2012) is often evident in classroom mathematics learning and mathematics performance. Mathematics identity construction is not only mediated by performance in mathematics, it is also deeply influenced by society's perception and stereotyping of students' mathematics competencies. This study can inform practices in urban teacher education programs by its emphasis on the importance of self-analysis and reflection. With these practices, preservice teachers are more likely to develop PCK and positive identity, which can potentially transform into increasing student mathematics achievement in urban settings. With these practices, minority PSTs are more likely to look beyond their racial experiences in mathematics, which could translate into them becoming effective teachers, meeting the expectations, and increasing student mathematics outcomes in urban settings. Finally, the study could inform future research that focuses on the use of lesson study for enhancing PSTs' PCK and their effectiveness in teaching mathematics.

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References

Ackerman, T., Heafner, T., & Bartz, D. (2006). *Teacher effect model for impacting student achievement.* Paper presented at the American Educational Research Association, San Francisco, CA.

Akerson, V.L.; Pongsanon, K.; Rogers, M. A. P.; Carter, I.; Galindo, E. (2017). Exploring the Use of Lesson Study to Develop Elementary Preservice Teachers' Pedagogical Content Knowledge for Teaching Nature of Science. International Journal of Science and mathematics. 15(2) pp 293-312. https://doi.org/10.1007/s10763-015-9690-x

Abell, S. K. (2007). *Research on science teacher knowledge*. In S. Abell & N. Lederman (Eds.), Handbook of research on science education (pp. 105- 149). Mahwah, NJ: Lawrence Erlbaum.

Ball, D.L., Lubinski, S.T., & Mewborn, D.S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed., pp. 433-456). Washington, DC: American Educational Research Association.

Battista, M. (1994). Teacher beliefs and the reform movement in mathematics education. *Phi Delta Kappan, 75*(6), 462-470.

Bausmith, J. M., & Barry, C. (2011). Revisiting professional learning communities to increase college readiness: The importance of pedagogical content knowledge. *Educational Researcher*, 40, 175-178.

Berry, R. Q. (2008). Access to upper-level mathematics: The stories of African American middle school boys who are successful with school mathematics. *Journal for Research in Mathematics Education, 39*(5), 464-488.

Bray, W.S. (2011). A collective case study of the influence of teachers' beliefs and knowledge on error-handling practices during class discussion of mathematics. *Journal for Research in Mathematics Education, 42*(1), 2-38.

Brown, A. (1992). Design-experiments: Theoretical and methodological challenges in creating complex interventions in classroom settings. *Journal of Learning Science*, *2*(2), 141-178.

Bruner, J.S. (1966). *Towards a theory of instruction.* Cambridge, MA: The Belknap Press of Harvard University Press.

Calderhead, J. (1996). Teachers: Belief and knowledge. In D.C. Berliner & R.C. Calfee (Eds.), *Handbook of educational psychology*, (pp. 709-725). New York, NY: Macmillan.

Carpenter, T.P., Fennema, E., Peterson, P.L., & Carey, D.A. (1988). Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for Research in Mathematics Education, 19*(5), 29-37.

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. *The Journal of Learning Sciences*, *10*(1), 113-164.

Dana, N.F., & Yendol-Silva, D. (2003). *The reflective educator's guide to classroom research: Learning to teach and teaching to learn through practitioner inquiry*. Thousand Oaks, CA: Corwin Press.

Darling-Hammond, L. (2006). *Powerful teacher education: Lessons from exemplary programs*. San Francisco: John Wiley and Sons, Inc. 21.

Darling-Hammond, L. and Bransford, J. (Eds.) (2005). *Preparing Teachers for a Changing*

.

World: What Teachers Should Learn and Be Able to Do. National Academy of Education, Committee on Teacher Education. San Francisco: Jossey Bass, Inc., 12. Retrived from http://www.josseybass.com

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V.R., & Empson, S.B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education, 27*(4), 403-434.

Fernandez, C., Cannon, J., & Chokshi, S. (2003). A U.S.-Japan lesson study collaboration reveals critical lenses for examining practices. *Teaching and Teacher Education, 19* (2), 171-185.

Franke, M.L., Fennema, E., & Carpenter, T. P. (1997). Changing teachers: Interactions between beliefs and classroom practice. In E. Fennema & B.S. Nelson (Eds.), *Mathematics teachers in transition* (pp. 255-282). Mahwah, NJ: Erlbaum.

Gee, J. P. (2001). Identity as analytic lens for research in education. *Review of Research in Education, 25*(1), 99-125.

Goldhaber, D. *Everybody's doing it, but what does teacher testing tell us about teacher effectiveness?* http://www.crpe.org. Center on Reinventing Public Education. Paper presented at the AERA annual meeting April 4, 2006, 31.

Gresalfi, M. S., & Cobb, P. (2011). Negotiating identities for mathematics teaching in the context of professional development. *Journal for Research in Mathematics Education, 42*(3), 270-304.

Guba, E. G., & Lincoln Y.S.(1989). *Fourth generation evaluation*. California: Sage Publication, Inc.

Hashweh, M. (2005). Teacher pedagogical constructions: A reconfiguration of pedagogical content knowledge. Teachers and Teaching: *Theory and Practice*, 11, 273-292.

Hill, H. C.; Ball D. L.; and Schilling, S.G., (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. J*ournal for Research in Mathematics Education*, 39(4), 372-400. URL: http://www.jstor.org/stable/40539304

Jackson, K. (2009). The social construction of youth and mathematics: The case of a fifth grade classroom. In D. B. Martin (Ed.), *Mathematics teaching, learning, and liberation in the lives of Black children* (pp. 175-199). New York: Routledge.

Martin, D. (2007). Mathematics learning and participation in African American context: The co-construction of identity in two intersecting realms of experience. In N. Nasir & P. Cobb (Eds.), *Diversity, equity, and access to mathematical ideas* (pp.146-158). New York, NY: Teachers College Press.

Martin, D. (2009a). Researching race in mathematics education. *Teachers College Record, 111*(2), 295-338.

Martin, D. (2009b). Mathematics teaching, learning, and liberation in the lives of Black children. London. UK: Routledge.

Martin, D. (2012). Learning mathematics while black. *The Journal of Educational Foundations*. *26*(1/2) 47-66.

National Council of Teachers of Mathematics. (2007). *Mathematics teaching today.* Reston, VA: NCTM.

Gregory, A., Skiba, R. J., & Noguera, P. A. (2010). The achievement gap and the discipline gap: Two sides of the same coin? *Educational Researcher, 39*(1),59– 68.

.

Oppland-Cordell, S. (2013). Mathematical and racial identity co-construction in multiple sociopolitical contexts: A case study of a Latina undergraduate student from an urban community. *Journal of Urban Learning, Teaching and Research*, *9,* 89-99.

Puchner, L. D., & Taylor, A. R. (2006). Lesson Study, collaboration and teacher efficacy:stories from two school-based math Lesson Study groups. *Teaching & Teacher Education: An International Journal of Research and Studies*, *22*(7) 922- 934.

Stinson, D. W. (2007). African American Male Adolescents, Schooling (and Mathematics): Deficiency,Rejection, and Achievement,*76* (4), 477-506 https://doi.org/10.3102/00346543076004477

Strauss, A. L. (1987). *Qualitative analysis for social scientists*. Cambridge, MA: Cambridge University Press.

Stringer, E. T. (2004). *Action research in education*. Upper Saddle River, NJ: Pearson/Merrill/Prentice Hall.

Taylor, A.R., Puchner, L.D., Scheibel, G. (2006). Gwen's story: researching teaching with others in a lesson study transforms a beginning teacher's understanding. In Langrall, C.W. [Ed]. *Teachers engaged in research: Inquiry into mathematics classrooms in grades 3-5.* Greenwich, Connecticut: Information Age Publishing Inc.

Terry, C.L. (2010). Prisons, pipelines, and the president: Developing critical math literacy through participatory action research. *Journal of African American Males in Education*, 1(2), 73-104.

Thompson, A.G. (1992). Teachers' belief and conceptions: A synthesis of research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York, NY: Macmillan.

Tröbst, S., Kleickmann, T., Heinze, A., Bernholt, A., Rink, R., & Kunter, M. (2018). Teacher knowledge experiment: Testing mechanisms underlying the formation of preservice elementary school teachers' pedagogical content knowledge concerning fractions and fractional arithmetic. *Journal of Educational Psychology*. Advance online publication. http://dx.doi.org/10.1037/edu0000260

U.S. Department of Education. (2008). *Foundations of success: Final report of the national mathematics advisory panel*. Washington, DC: U.S. Government Printing Office.

Van De Walle, John A. (2007). Elementary School Mathematics: Teaching Developmentally, Seventh Edition, New York: Longman Publishers.

Van Driel, J.; & Berry, A. (2012). Teacher Professional Development Focusing on Pedagogical Content Knowledge. Educational Researcher, *41*(1) pp. 26-28. Stable URL: http://www.jstor.org/stable/41413082 Accessed: 05-20- 2018 13:00 UTC

Yackel, E. & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education. 27*(4), 458-477.

Zilanawala, A., Martin, M., Noguera, P. & Mincy, R.B., (2017) Math Achievement Trajectories Among Black Male Students in the Elementary- and Middle-School Years, Educational Studies, *54*(2)143-164, DOI:10.1080/00131946.2017.1369414