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The Importance of Automaticity Development in Mathematics


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The Importance of Automaticity Development in Mathematics

Abstract

This study examined whether students were reaching automaticity with single digit multiplication facts. A fourteen question interview was used to collect data. The first three questions asked the student basic information about themselves and their current math teacher. The next seven questions were math facts. The math facts chosen for the interview were a range of difficulty, starting with a simple problem like 1×9 and increasing in difficulty to 6×9 . The last four questions were open-ended with the intent of gaining insight into whether the students were using strategies to complete the problems or if they were showing automaticity. The study took place in one elementary school and one middle school. The participants were students from 3rd, 5th, and 8th grade. There were nine students from 3rd grade. Four were male and five were female. Ten students were from 5th grade. Four were male and six were female. Five students were from 8th grade. Three were male and two were female. The findings from this study are consistent with research included in this study in that students are continuing to struggle with single digit multiplication problems.

Keywords

automaticity, fluency, mathematics, multiplication, skills, development

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Literature Review

Mathematics skills are essential for every student to learn. They are fundamental to the success of a student through their education and into their professional careers. Unfortunately, students are not meeting the academic targets put in place by the Common Core or National Council of Teachers of Mathematics (Stickney, Sharp, & Kenyon, 2012). This has led to research on the concept of automaticity. The definition of automaticity provided by Stickney et al. (2012) is the ability to deliver a correct answer immediately from memory without conscious thought, as opposed to relying on calculation. Also in the related research is another term called fluency. According to Lin and Kubina, (2005) fluency requires students to be both fast and accurate when solving basic math facts. Automaticity is a piece of fluency. Fluency is the end goal and considered true mastery of the concept when reached. Now connect these two terms to mathematics and we develop the idea that students will develop automaticity first, then fluency, and by doing this, they will develop a pattern of sustained success in their mathematics career (Cumming & Elkins, 1999; Lin & Kubina, 2005; Stickney et al., 2012; Woodward, 2006).

Importance of Automaticity Development

The concept of automaticity is critical for the general success of students in mathematics. There are studies that have found math fact automaticity to be a predictor of performance on general mathematics tests (Stickney et al., 2012). Not only does automaticity help with general math, information-processing theory supports the view that automaticity in math facts is fundamental to success in many areas of higher mathematics (Woodward, 2006). Without this ability to retrieve facts directly, students are likely to experience high cognitive loads and produce work that is inaccurate. As the complexity of math progresses past single digit multiplication, calculations like finding common denominators when adding and subtracting fractions become an extremely difficult task. Lin and Kubina (2005) found that only 13% of the 155 students assessed achieved fluency for basic multiplication facts. Even fewer students, 3%, achieved fluency with the more complex multiplication problems. This data shows the strong correlation between automaticity with the retrieval of basic facts and the ability to achieve fluency on more complex multiplication problems.

According to Woodward (2006), decades of research show that academically low-achieving students as well as those with learning disabilities exhibit considerable difficulty in developing automaticity. Research on elementary aged students indicates that students with learning disabilities are more likely to rely on counting strategies than direct retrieval when working with single-digit fact problems. Failing to reach automaticity results in students relying on different counting strategies. Whether it is a student with a learning disability or a student without, the results are very similar. The students begin to fall behind at the elementary level and continue this pattern into secondary level math. Cumming and Elkins found that elementary aged students who were computing simple addition problems made the most errors in a miscalculation of the fact and not with the addition algorithm (1999). This supports the idea that fluency in basic facts, whether addition or multiplication, affects the student's progression in mathematics. Research suggests there are effective strategies for students with disabilities and students without (Stickney et al., 2012). However, there are varying opinions as to which strategies are effective and when each strategy should be implemented.

Multiplication Interventions

There are numerous strategies that claim effectiveness when dealing with multiplication.

One method is a mnemonic strategy approach. Several studies (Greene, 1999; Mahler, 2011; Nelson, Burns, Kanive, & Ysseldyke, 2013; Zisimopoulos, 2010) provide data for using this type of intervention to achieve math fact automaticity. A mnemonic strategy involves some type of picture or word to associate with a certain number (Greene, 1999; Nelson et al., 2013). The pictures produce a visual image for the student and therefore create a better chance of recall. Mahler's (2011) and Zisimopoulos' (2010) studies each included very small sample sizes so we must use this information with caution. However, in both studies the students did make gains in their math fact achievement levels. According to Greene's (1999) study, the mnemonic training was more effective in stimulating the retention of the facts compared to the traditional method. The results showed a significant difference between groups based on the order they received the instruction. There were two different groups. Group A received the mnemonic training then traditional. The control group received the traditional training first, then the mnemonic training. The group who received the mnemonic strategy first produced higher achievement levels. Again, with this study the sample size was too small to make any generalizations. We see in Nelson et al.'s (2013) research that the mnemonic strategy itself did not have a statistically significant effect. What we did see was that the group receiving the instruction via technology out-performed the other groups. The mnemonic group and technology group improved overall, but only the technology group produced statistically significant differences when it came to recall. This study suggests that technology-based programs are an effective tool for improving multiplication fact retention.

Recent research has indicated that students might benefit from an integrated approach to achieve automaticity with multiplication facts (Woodward, 2006). In this particular study, the first approach was a use of different strategies while the second approach was the use of timed practice drills. The study assessed 58 fourth grade students in the same elementary school. The integrated group was taught fact strategies while the control group was taught using the multiplication algorithm and controlled practice. Both methods were effective in raising the mean performance level on a mix of math facts to mastery or near mastery. The biggest difference between the groups was the performance on the extended facts test. The integrated group performed much better than the group with the timed practice drills only. This study produced an interesting limitation due to the fact that it was difficult to tell which students were actually learning the facts and achieving automaticity and which ones were still using other strategies to come up with the answer.

Kroesbergen and van Luit (2002) examined another approach in a study comparing guided instruction to structured instruction. Guided instruction is student-directed and focuses on the real-life application of math problems. The students are encouraged to "discover" the processes and methods of multiplication. The structured instruction is just the opposite. It is strictly teacher-directed, and students are only given information from the teacher. All communication goes through the teacher. There were three groups involved in Kroesbergen and van Luit's study: the guided instruction, structured instruction, and the control. Of the three groups, the structured group's performance was the weakest. All groups improved, but the guided instruction group outperformed the other two. An interesting finding was there was an interaction effect found that suggested that guided instruction may be better for children in regular education and structured instruction may be better for special education students.

Effect of Students' Level of Prior Knowledge

Prior knowledge is the information a student has previously learned. Research suggests that teachers can use students' level of prior knowledge to determine what intervention might help them reach automaticity with multiplication or other basic mathematics functions (Burns, Coddling, Boice, & Lukito, 2010; Burns, Ysseldyke, Nelson, & Kanive, 2014; Coddling et al., 2007; Kroesbergen & van Luit, 2002). Burns et al. (2014) and Stickney et al. (2012) both used

data from a national database (MathFacts in a Flash) to gather information about students' ability to solve basic multiplication problems. Burns et al. (2014) inquired about three different research questions. The first looked at the effect of math fact type and number of repetitions needed, the second question looked at the effect of math skill on the number of repetitions needed, and the third question looked at the effect of grade level on the number of repetitions needed. They found that the grade level and math skill affected the number of repetitions it took for a student to achieve mastery on a certain math fact. Across the board, more repetitions were needed on digits 4 through 7. With such a large sample, this study provides a unique view of the difficulties elementary aged students are having with single digit multiplication facts.

Stickney et al. (2012) used 89,159 students' data and grouped them as severely deficient, at risk, and low risk. The large majority of students (86%) were able to master at least one level during the school year. The low risk students mastered more levels than the at-risk group, and the at-risk group mastered more than the severely deficient. However, Stickney et al. found that the MathFacts instruction in a Flash program was more beneficial to the at-risk group than the low-risk group or severely deficient group. Even though the majority of students (86%) were able to master at least one math fact level, the number of students meeting the Common Core goal of automaticity by the end of 2nd grade for addition and subtraction facts was low. Based on the information in the study, 36% would have met the goal, and that only includes data from one year. This low percentage suggests students were not on track to display automaticity at the appropriate age, thus putting them at risk of falling increasingly behind their classmates.

Codding et al. (2007) also found that a student's level of prior knowledge helped predict what intervention might help that student. This study used three groups: explicit timing, cover-copy-compare, and a control group. Explicit timing is a procedure that requires students to mark their progress in one-minute intervals when completing an academic assignment, which provides students with feedback about how many problems are completed at each interval. Cover-copy-compare is a strategy that provides a series of learning segments within a short period of time through 5 steps: Look at the mathematics problem with the answer, cover the mathematics problem with the answer, record the answer, uncover the mathematics problem with the answer, and compare the answer. They found that for students whose initial fluency level fell within the frustration range, the cover-copy-compare and control conditions resulted in the best performance. For students whose initial fluency range was in the instructional range, the explicit timing method was most effective.

Summary of Findings

According to the research presented, automaticity in basic math facts is important for mathematics students. It is not just important at the elementary age, but it is of most importance at that age. When automaticity is achieved at the desired developmental stage in a child's life, they are more likely to have continued success in mathematics. Automaticity is an achievable goal for students. The research presented not only provides strategies to achieve this, but also information as to which strategy would work best for students achieving at different academic levels.

Research Questions

Multiplication facts are one of the most important building blocks when it comes to mathematics development. There are also many students who struggle with the task of memorizing or learning these facts. Mathematics is a content area that builds on previous concepts, multiplication being one of those building blocks. Not being able to recall this information not only affects the student in the classroom but can also have a negative impact on

the students' view of math in general. This research is important to the overall and long-term success of students, ranging from students who are learning these facts for the very first time to the students who have seen these facts for several years.

Automaticity is the ability to automatically process information without occupying the mind with the low-level details, allowing it to become an automatic response pattern or habit. Students who have reached automaticity with their math facts tend to produce better overall math results throughout their education than those students who never reach that point. This study will use interviews of students at different grade levels to gain insight into the research questions. The questions that will be investigated are the following:

- 1) Are students using hybrid strategies to solve single digit math facts, and if so, are they solving them within the time frame that could be considered automatic?
- 2) Have any of the students reached automaticity?
- 3) Is there a relationship between grade level and the point at which automaticity is reached?

Method

Participants.

The study took place in one elementary school and one middle school. The elementary and middle schools are in the same county located northeast of Atlanta, Georgia. The area is mostly rural with some schools located in suburban areas. Six out of the eight middle schools are considered to be in low-income areas and nineteen out of twenty of the elementary schools are considered to be in low-income areas (The Governor's Office of Student Achievement, 2015). The elementary and middle school used in the study are considered low-income. The vast majority of students come from working class and lower middle class socioeconomic backgrounds with 59% of the students qualifying for free or reduced meals. For the 2015-2016 school year the racial demographics were 51% White, 41% Hispanic, 5% Black, 2% Multiracial, and 1% Asian.

The participants were students from 3rd, 5th, and 8th grade. There were nine students from 3rd grade. Four were male and five were female. Ten students were from 5th grade. Four were male and six were female. Five students were from 8th grade. Three were male and two were female. These students were randomly selected from each class to take part in the interview. A total of twenty four students were interviewed. The students chosen for the interview were chosen from classes that were considered grade-level. The classes that were chosen were from teachers that volunteered to take part in the research.

Materials/measures.

A fourteen-question interview was used to collect data. The first three questions asked the student basic information about themselves and their current math teacher. The next seven questions were math facts. The math facts chosen for the interview were a range of difficulty, starting with a simple problem like 1×9 and increasing in difficulty to 6×9 . The last four questions were open-ended with the intent of gaining insight into whether the students were using strategies to complete the problems or if they were showing automaticity. An example of the open-ended format was question number eleven, "Explain to me how you thought of the answer for 2×6 ." This same format was used for question numbers eleven, twelve, and thirteen. Each question asked about a different math fact as they increased with difficulty. The interview protocol can be found in Appendix A.

Procedures.

Before the study could take place, the IRB had to grant approval of the study. Once The IRB approved the study, the superintendent of the school system had to be contacted to receive permission to interview students in that system. Once approved by the superintendent, the principals and prospective teachers were contacted to be sent information about the study. After approval from the principals and one teacher from each grade level consented to participate in the study, the interviews started. Interviews took place at the school where the student regularly attended. The students were chosen at random from a convenient sample of on-level classes. I took recommendations from each principal as to which teacher they felt would be able to best accommodate my research. Every student in the class was given permission forms and sent home. After most of the permission forms were returned, those students' names were placed in a container and pulled at random by the teacher. I interviewed nine students from 3rd grade, ten students from 5th grade, and five students from 8th grade. Each student was asked the same questions at each grade level. The interview duration was no more than seven minutes per interview. The first three questions of the interview provided basic information about the students and their current math teacher. The next seven questions required the student to give an answer to a single-digit math fact problem. Each of those seven problems were timed and documented. The remaining questions were open-ended and the answers were written down as accurately as possible. The students answered all questions verbally and all answers were written down on paper. Notes were made between each interview about each student and how they responded to the survey.

Results

This study focused on three questions around the idea of automaticity in basic math facts. The three questions were: 1) Are students using hybrid strategies to solve single digit math facts, and if so, are they solving them within the time frame that could be considered automatic; 2) have any of the students reached automaticity; and 3) is there a relationship between grade level and the point at which automaticity is reached? A fourteen question interview was used as the data gathering tool. A total of twenty-four students were interviewed from one elementary school and one middle school. Nine 3rd graders, ten 5th graders, and five 8th graders participated.

The interview was created by the researcher and contained fourteen questions. Half the questions were timed and required the students to produce answers to varying single-digit multiplication facts and the other half required the students to provide answers based on their opinion or interpretation of the question. The multiplication problems were timed by the primary researcher using a stopwatch. The other questions were recorded as accurately as possible on printed copies of the interview protocol by the primary researcher. The recorded responses were then used to provide insight into the research questions.

The students were interviewed in their respective schools and to provide a similar atmosphere to their normal classroom setting. A desk and chair were pulled into the hallway just outside each classroom to avoid other students hearing the responses of other students being interviewed. By students knowing the questions that would be asked, this could have produced data that was skewed and not accurate of the general knowledge of each student. After each interview, notes were recorded about each participant if something odd or interesting was noticed. Each interview lasted no longer than seven minutes and there were not any noticeable differences between each grade level and the length of the interview.

The first research question focused on whether students were using hybrid strategies to produce answers to the single-digit multiplication problems. For the purposes of this study, it

was determined that a strategy would be some form of skip-counting, using fingers to count up, or using the hand method with 9's multiplication problems. These are strategies used for students learning their multiplication facts. Skip counting is when someone counts by a certain number, like 2, 4, 6, 8, and so on or 5, 10, 15, 20. It becomes more difficult to use this strategy as the numbers get larger. Another basic strategy is using fingers to count. This is something we learn at a very early age because we can actually see what we are counting. There is another strategy that only works with the 9's multiplication facts up to 9×10 . Let us use 9×3 as an example. You hold both hands up and starting from the left, count three fingers over and put that finger down. On your left hand you will have two fingers still up to the left of the finger that is down and seven fingers up on the right side of the finger that is down. That gives you the answer of 27, which is the correct answer to 9×3 . The first strategy of skip-counting is not as visible unless the person is counting out loud, however, the other two strategies are very easy to observe.

The data collected was displayed in a spreadsheet to have all data together and visible in order to see trends and categorize the findings. After the data was entered, the times for each multiplication fact were color coded using five different colors. Yellow was used to show an incorrect answer. For this study it was determined that an incorrect answer was produced after three failed attempts. After the third wrong answer, the time was stopped. This was chosen because after 3 attempts, the correct answer could have been guessed. The color blue was used when a correct answer was given in under three seconds. Orange was used when the participant used a strategy but did not produce an answer in three seconds or less. Green was used to show the participant used a strategy and produced an incorrect answer. Grey was used to show the participant used a strategy and produced an answer in three seconds or less. This data can be found in Table 1 below.

Using the chart to interpret the data, out of a possible 168 problems, only 22 answers were produced using one of the strategies mentioned before (skip-counting, using fingers to count up, or the hand method with 9's multiplication). That is only 13.1% of all answers given. It was evident that a strategy was used in the 22 answers because the strategies were visible and audible. If we look at each individual grade level, the percentages of strategies used are very similar. 3rd graders used a strategy eight out of sixty three chances, which is 12.7%. 5th graders used a strategy ten out of seventy chances, which is 14.3%. 8th graders used a strategy four out of thirty five chances, which is 11.4%. An interesting find within the data was that out of the twenty two answers that resulted from the use of a strategy, a female participant provided fifteen of them, which is 68.2%. That is a little more than twice the percentage of male participants who used a strategy. This study gives us an indication that students are using strategies to solve single digit multiplication problems, but not frequently enough to conclude that they are not reaching automaticity because of the use of strategies.

The data does show that, overall, 13.1% of the participants used a strategy; however, a strategy was only used on four of the seven problems. There were three problems that no participant used a strategy on. The four problems that a strategy was used for were the more difficult problems (4×8 , 6×7 , 6×9 , and 9×8). If we look at the percentage of strategies used just on those four problems, the percentage increases to 22 out of a possible 96, which is 22.9%. That is almost double the overall percentage (13.1%), however it is still not a high enough percentage to conclude that students are not reaching automaticity because of the use of strategies, even on the more difficult problems.

alone, it appears that basic multiplication facts are still a struggle for students. If we look at each grade level we see something a little different. 3rd grade produced a percentage of automaticity of 33 out of 63, which is 52.4%. This is lower than the overall percentage (64.9%). This is also the grade in which all students should have mastered their multiplication facts through 10x10 by the end of the school year. 5th grade produced a percentage of 53 out of 70, which is 75.7%. This percentage is higher than the overall percentage (64.9%). This would make sense because we would think in general a student would progress with just basic practice by using the skills for the two school years that have passed by the end of 5th grade. 8th grade produced a percentage of 23 out of 35, which is 65.7%. This is very close to the average of all three grade levels. We would also expect this percentage to at least be similar to that of the 5th graders or even higher because students do not stop using multiplication after 5th grade.

The data also provided some information about the difficulty of the problem and the percentage of participants who produced answers within the automaticity range. There were three problems that produced percentages of correct answers in three seconds or less over 90%. Those problems were 1x9, 2x6, and 5x4. However, the other four problems produced percentages of 50% or less. These problems were 4x8, 6x7, 6x9, and 9x8. The problems 9x8 and 6x7 proved to be the most difficult with only 37.5% of answers considered to be automatic. The next hardest was 4x8, which produced a percentage of 45.8. The problem 6x9 produced a percentage of 50.

The third and final research question addressed whether there was a relationship between grade level and when automaticity is reached. Based on the data collected, 5th grade was the most likely grade level connected with achieved automaticity. However, if we look at the multiplication problems and who achieved automaticity, then each grade level had high percentages of automaticity for the easier problems and low percentages for the more difficult problems. This way of data interpretation seems to represent a more accurate picture of whether automaticity was achieved or not. It is evident with this sample size that no particular grade level did well on the difficult problems. It is interesting to note the differences in percentages of automaticity and grade level because of the drop-off seen in the 8th grade sample. There are five school years between 3rd grade and 8th grade, and multiplication is used constantly throughout the content in those grade levels. The data shows a similar performance between the 3rd graders and 8th graders, and if 8th graders are performing the same as 3rd graders on basic multiplication facts, this could be a problem for the future.

Discussion

The findings from this study are consistent with other research provided by Burns et al. (2014) and Stickney et al. (2012). Students are continuing to struggle with single digit multiplication problems and this is not only affecting them now, it is also putting their success in the future in jeopardy. Developing automaticity is a building block for the success of students in the math classroom. Just as students cannot read with understanding without first learning the correct process for sounding out words and memorizing their sight words, math students cannot “read” math without learning their basic math facts. Those basic facts do not just include multiplication facts, it also includes single digit addition, subtraction, and division problems.

In terms of automaticity, the 5th grade participants were the highest performers. Based on the data, they produced the most answers that fit into the automaticity range. As for the research

question, it is clear that work still needs to be done with basic multiplication facts. We should not see students regressing with their math skills as we see here. Almost all students showed automaticity on the easier math facts but struggled when it came to the more difficult problems. With such limited information on this topic, the development of automaticity needs to be examined more and taken more seriously by the public education system.

The state of Georgia has standards that are put in place and must be taught in each school. Those standards are supposed to be “mastered” by the end of that school year. This research, along with others (Cumming & Elkins, 1999; Lin & Kubina, 2005; Stickney et al., 2012; Woodward, 2006), shows a different situation. Our math students are struggling when they are in elementary school, they are struggling when they are in middle school, and by the time they are in high school, they are years behind. This study only dealt with on-level students, and students with disabilities were not included. If the students who are supposed to be on-level are struggling, how can we expect the students with disabilities to be successful?

The relationship between learning to read and learning math is more similar than one might think. As mentioned earlier, there is a process that is used to teach reading. There are certain strategies that are used to stimulate learning and eventually it becomes a natural and fluid activity. This is called fluency. According to Pikulski and Chard (2005), “Reading fluency refers to rapid, efficient, accurate word recognition skills that permit a reader to construct the meaning of text. Fluency is also manifested in accurate, rapid, expressive oral reading and is applied during, and makes possible, silent reading comprehension.” Reading fluency is important because it bridges the gap between word recognition and comprehension. Similarly, basic multiplication facts would be analogous to word recognition. Better word recognition leads to better reading comprehension. Learning the basic multiplication facts would lead to a better understanding of more complex math concepts. Before any comprehension can take place, whether in reading or in math, automaticity of sight words or math facts must be achieved to improve comprehension.

The lack of automaticity with math facts is disturbing, but what is contributing to the number of students who are not reaching automaticity? The popularity and dependence on the calculator could be a major factor. Students who have access to a calculator become dependent on this device. What motivation does a student have then to learn the facts when they can use the device that calculates for them? The use of a calculator could lessen the likelihood of the student retaining the facts and committing the facts to memory. It would be of great interest to research the effect calculators have on the development of automaticity within students. Future research on this topic should include questions about how often students use calculators and at what age students begin to use calculators.

Limitations

This study was not without limitations. The first limitation was the sample size. Only twenty four students were used and that included three different grade levels. The sample population was also a limitation. The sample was chosen based on convenience. The schools that were visited were in close proximity and easily accessible. The populations of the schools did not represent great diversity and therefore generalizations cannot be made with these twenty four students.

The qualitative nature of the study was also a limitation in the sense that none of the findings could be generalized. The findings were only observations made by one researcher. Although the observations were recorded, one observer can only collect so much information and is not capable of taking in all information during the experience.

Future Implications

Automaticity is referred to as the ability to deliver a correct answer immediately from memory without conscious thought, as opposed to relying on calculation (Stickney et al. 2012). This is an important piece to the success of a student because as the student progresses through different math concepts, the brain will have difficulty processing two things at once. The math facts have to be committed to memory in order to learn more complex concepts. There does not seem to be substantial research dealing with the issue of the causes of students falling farther and farther behind in math. There are however, people who ask questions as to why students are falling farther and farther behind. It should be of most importance to the State of Georgia to commit to more research on this topic rather than changing the way math is taught year after year. The state places standards for each teacher to follow and the state expects each student to be intellectually ready to master those standards by the end of each school year. If research is being conducted on when students are intellectually ready, maybe it is time to research why the students are not meeting those standards.

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Appendix A

Interview protocol

1. How does math class make you feel?
2. Describe your math ability as one of the following: needs work, average, good, excellent.
3. Describe your math teacher using one of the following words: positive or negative.

Now I am going to ask you to solve seven multiplication problems. Give me the answer as soon as you think of it.

4. 1×9
5. 2×6
6. 4×8
7. 5×4
8. 6×7
9. 6×9
10. 9×8

I am now going to ask you four questions related to the answers you gave for the previous ten questions.

11. Explain to me how you thought of the answer for 2×6 .
12. Explain to me how you thought of the answer for 4×8 .
13. Explain to me how you thought of the answer for 6×9 .
14. Could you describe to me why you chose to answer positive/negative when describing your math teacher?