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Exploring the genealogy of the concept of 'innate mathematical ability' and its potential for an egalitarian approach to mathematics education

Noel Colleran

Tipperary Education and Training Board, Tipperary
<ncolleran@tipperaryetb.ie>

Abstract

Recent work by a number of researchers has argued that the capacity for mathematical thinking is innate to human intelligence. Much of the evidence for this conclusion is based on findings in fields as diverse as linguistics, genetics, evolution, archaeology, psychology, and philosophy. This paper argues that the genealogy for this development is sourced in the philosophy of the Enlightenment, particularly the work of Immanuel Kant. Kant's seminal idea suggests that human intelligence had a natural and necessary capacity for mathematical thinking in the forms of space and time. This paper will explore the ideas of Immanuel Kant regarding space and time, particularly his views that the intuition of space provides the source for geometry while the intuition of time provides the source for number. A limited, yet sufficient, evaluation of recent relevant literature will be employed to illustrate that 'new insights' regarding innate mathematics ability can be 'genealogically' traced to the work of Immanuel Kant. Ultimately, this paper argues for the debunking of generally accepted agreement among some that many mathematics students have an innate capacity to do mathematics while others are innately incapable in this regard. With an acknowledgement of this 'initial state' regarding universal mathematics ability among young, as well as adult students, an egalitarian perspective regarding students' expectations and achievements in mathematics is in view.

Key words: Enlightenment, Kant, space and time, innate maths, maths gene, egalitarian education.

1. Introduction

It is my contention that there are many who believe that people in general, and students in particular, are either good or bad at mathematics. On the basis of this belief students are regularly encouraged or discouraged from doing higher level and more challenging mathematics in schools and colleges. There is no doubt that students come to school, and adults return to education, with proficiency in mathematics which is wide-ranging, however, I will argue that this range, spanning from what might be called 'prodigies', on one side, to those who have 'math phobia', on the other, results neither from the learners' giftedness nor from intellectual deficiencies, but from the amount of time they spend thinking mathematically and doing mathematics. I will argue that the element that encourages mathematical practice, on the one hand, and discourages practice, on the other, is accounted for through socialisation processes experienced by each individual. This paper will provide evidence that all of us, young and old, have an innate, natural ability to do mathematics to the highest level: we are all endowed with a 'maths gene'. And, if Peuquet's (2002) is correct when she asserts that adults learn in a similar fashion to younger students when confronted with novel situations, my argument is relevance to both adult and younger learners of mathematics. The basis for this argument is derived from the work of the Enlightenment philosopher Immanuel Kant. His philosophy provided revolutionary insights regarding space and time and many of his foundational ideas are continuing to reverberate nearly 250 years later. In the first instance, I will argue that Kant provides the 'genealogical' source for the concept of innate mathematical ability among humans. This will be followed by more current research, which will support Kant's most important ideas, and provide sufficient evidence to reasonably postulate that the Enlightenment thinker provides the most influential source for the concept of innate mathematical ability. I will then provide evidence that the amount of time spent doing mathematics is directly proportional to the proficiency levels achieved. I believe that the acceptance and employment of these two principles - innate mathematics ability, and practice makes perfect – among teachers of mathematics will, no doubt, lead to a re-evaluation of practice with young and, in particular, adult students of mathematics. Ultimately, I believe that practice based on these principles can provide for a more egalitarian approach to mathematics education for all.

2. Immanuel Kant's contribution to the concept of innate mathematical ability

Most serious philosophers will agree, to a greater or lesser extent, that Immanuel Kant brought about a transformation in western philosophy the likes of which had not been seen since the ancient Greeks: 'and [Kant's] work did indeed change philosophy permanently' (Hatfield, (2004 p. ix); 'within a few years of the publication of his Critique of Pure Reason in I78I, Immanuel Kant was recognised…as one of the great philosophers of all time' Guyer and Wood, (1998 p. vii); 'the Critique of Pure Reason is a philosophical classic that marks a turning-point in the history of philosophy' Kemp Smith (1918 p. viii); 'the most important phenomenon which has appeared in philosophy for two thousand years… the principal works of Kant' Schopenhauer (1818 p. xv). Much of this reputation is based on his most famous publication in 1781 entitled A Critique of Pure Reason. The metaphysical transformation that Kant brought about with his Critique was centred on the question; what is the range of human understanding? Or, from a negative perspective, what are the limits of human understanding? Accordingly, he turned his attention not to the product of human understanding but the producer; the instrument by which human understanding is generated i.e. human rationality. At the outset Kant was satisfied that human understanding and knowledge were constituted by both sensed experiences and reason, and both had a range within which they operated effectively – outside this range human understanding and knowledge was vulnerable to attack and could not be defended. The philosophical clearing in which Kant's position regarding the range and limits of human cognition is a good place to start, in particular the manner in which he distinguishes the noumenal and the phenomenal world.

2.1 The Noumenal and Phenomenal World

Central to Kant's argument in the Critique is his contention that there are two distinct versions of the world: the noumenal world and the phenomenal world. The noumenal world is the world as-it-is-in-itself; the world of beliefs, spirituality, feelings, etc., which are not accessible to human sense organs. And while we may speculate about the noumenal world, humans cannot know it. Humans can understand and know the phenomenal world because we have mediated access to this world through our senses. Knowledge of the phenomenal world is limited by human senses and our capacity to cognise perceptions mediated through those senses. Therefore, the range and limits of human cognition and knowledge lie within the phenomenal world. However, while the human faculty for knowledge is 'limited' to the world of phenomena, it seems to have 'limitless' capacities to generate knowledge within this context, particularly in the sciences, mathematics, information technology, etc., and we must remain ever vigilant of our own limitations and refrain from stepping outside the bounds of the phenomenal world.

2.2 Empiricism and Rationalism

The philosophical milieu from which Kant’s Critique emerged was dominated by a spectrum of two competing doctrines regarding what constituted genuine human knowledge. This comprised, at one end, a form of radical empiricism, positing that there is an objective out-there-now-real world that we engage with and know, not immediately but mediately, through our five senses i.e. seeing, hearing tasting, touching, and smelling. According to Locke (1690), the human mind enters the world as a ‘tabula rasa’ (a blank slate) and human experiences cover this blank slate with our knowledge of the world. There are obvious difficulties with this approach because all humans see, hear, etc., differently and the development of any knowledge based on subjective experiences could never approach general or universal understanding. However, the empiricists were convinced that the only true source of human knowledge is through human experiences and were satisfied to push the breaks at this point and conclude that humans know through individual perceptions that aggregate and combine into ever more complex ideas and knowledge. While Kant accepted that our senses provide access to the phenomenal world he rejected the idea that knowledge is an aggregation of increasingly complex sensed experiences. Without some non-empirical faculty that forms, unifies, establishes coherence, and makes sense of these impressions, there is no possibility of knowledge. The empiricists were adamant that these forming and binding capacities are not given in experience and anything not so derived should be ‘committed to the flames’ (Hume 1748).

On the opposite side of the knowledge spectrum was a form of radical rationalism and chiefs among the rationalists were Descartes and Leibnitz. Like the empiricists, they too questioned the validity of human sensed experience in providing objective knowledge. However, instead of accepting that human senses experiences provide access to knowledge of the world, the rationalists rejected human experiences as much too vulnerable. The rationalists relied on the capacity of human intelligence alone to provide such knowledge. Rationalists:

> ‘… held that it is possible to determine from pure a priori principles [thinking and speculating without reference to vulnerable human experiences] of the ultimate nature of God, of the soul, and of the material universe’
>
> (KEMP SMITH, 1912, p. 13).

Descartes and Leibnitz contended that human thought, unfettered by subjective sensed experience, can determine objective reality. Again, Kant was satisfied that there was some validity in this view as human intellectual capacities play a fundamental role in forming and synthesising human perceptions to constitute human understanding. He contended, while the empiricists had stopped short, the rationalists had gone too far. Kant agreed that the rationalists provided the cognitive capacities to ‘interpret’ human experiences, and significantly, these capacities are available without reference to sensed experience. Fundamentally, the human knowing process is available innately, or, in Kant’s term, a priori; unadulterated and without reference to human experiences and so ‘pure’. While accepting the innate presence of pure reason, a nod of sorts to the rationalists, Kant equally accepts the empiricists’ view that the context and source of knowledge is the world of human experiences. Kant provides an accommodation of the limiting aspects of both perspectives by accepting that human knowledge is derived only subsequent to human experiences - a posteriori - and these sensations are necessarily categorised and synthesised by an a priori, innate intellectual capacity, awaiting stimulation.

> ‘But, though all our knowledge begins with experience, it by no means follows that all arises out of experience’
>
> (CRITIQUE, B 15).

[5] All references to the *Critique* henceforth will have a prefix ‘B’ to suggest the second edition in 1787. The Politis translation of 1993 is the version used throughout unless otherwise stated.

His conclusion is that humans have a capacity for receptivity through the senses (content), and a capacity for conceptualising through the intellect (concepts). Both are universal and necessary for the possibility of human knowledge.

> Thoughts without content are empty; intuitions without concepts are blind
>
> (B 74).

And so, as a result of Kant's distinction of the noumenal and phenomenal worlds, and his exploration of the competing aspects of the contemporary field of philosophy, he began his critique of the relevant organ: human intelligence. In the section entitled the Transcendental Aesthetic ('aesthetic' here refers to senses), which consists of not more than forty pages, he describes the mediating role played by human senses and, more importantly for this paper, his contention that all human experiences are grasped by the natural, pure, and innate forming capacities of space and time. According to Guyer and Wood, (1998, p. 7):

> … the "Transcendental Aesthetic" [has] been the subject of a very large proportion of the scholarly work devoted to the Critique in the last two centuries.

2.3 Innate Space and Time

The first key contribution of Kant's Critique, mentioned above, was his distinction between the noumenal and phenomenal world and all that flowed from that position. His second contribution, directly relevant to this paper, is the one element of the 'bridge' he constructs to accommodate his version of empiricism and rationalism. This element is provided by the 'forming' intuitions of space and time. According to Kant, the intuitions of space and time, necessary for human understanding, are not derived from experience. This means that space and time are cognitively available prior to any experiences of the world and so must be innately and necessarily available to all human knowers. Space and time provide the necessary and only faculties by which human experiences are formed, shaped, grasped, or, according to Robinson (2011), how we come to 'behold' the external world. This provides the basis for my hypothesis that the intuitions of space and time are there in the 'initial state' (Chomsky, 2000 p. 7), unlearned, innately accessible to all irrespective of one's experiences in the world. Later I will refer to Kant's contention that space and time provide the foundations for the sciences of geometry and number respectively, and this extra layer provides the source of my argument that the genealogy of the concept of innate mathematical ability originates with Kant's assertion that all human understanding requires the forming intuitions of space and time and, because of their relationship to geometry and number, all humans have an innate capacity to understand and do mathematics.

2.4 Exploring Space

The empiricists argue our understanding is derived from human senses and if it is not so derived then it is baseless. Kant, however, suggests that while all our knowledge arises out of human senses it is not the source of all our understanding. The concept of space, which he argues is not sourced through the senses, is non-empirical and therefore a priori. Without this a priori forming intuition, Kant argues, human understanding is impossible.

> By means of the external sense, we represent to ourselves objects as outside us, and these all in space. Therein alone are their shape, dimensions, and relations to each other determined or determinable
>
> (B 35).

Kant postulated that space is not derived from the relations of external objects but that external experiences are possible only with the a priori intuition of space. He builds his argument for the innate faculty of space by suggesting a number of thought experiments such as: we can never contemplate the non-existence of space, while we can imagine empty space. We can only think of one space, and when we talk of different spaces they remain parts of the one same space: space cannot be built out of parts of space (B 37). Space is intuitively infinite in quantity;

it can be bigger or smaller by an extra measure no matter how large or small (B 39). Space provides the intuition by which we put a shape on the world external to us and ‘through space alone is it possible for things to be outer objects for us’ (Guyer and Wood, 1998, B 44).

One of Kant’s most innovative arguments regarding the innate intuition of space with regard to the appearance of objects was his employment of chiral objects: an object is said to be chiral if it cannot be superimposed onto or does not coincide with its mirror image. Kant employs left-handed and right-handed objects like human hands and gloves, spiral shells, etc., which ‘obviously’ appear to be chiral or incongruent counterparts when we look at them.

> …for the left hand cannot, after all, be enclosed within the same boundaries as the right (they cannot be made congruent), despite all reciprocal equality and similarity; one hand’s glove cannot be used on the other
>
> (KANT, 1783 pp. 37-38).

If one describes a left hand in detail each detail is similar for a left hand as it is for a right hand, yet they appear different. And while they appear incongruent, the incongruence is not amenable to rational explanation. The apparent, yet ‘obvious’, difference can only be accounted for by the innate spatial intuition. (For an in-depth discussion on incongruent counterparts and chiral objects see Severo, R. (2005) and Bennett, J. (1970).

2.5 Moving on to Time

As for time Kant argues that ideas such as co-existence, succession or change could not be perceived were it not for the foundational and a priori intuition of time (B 45). All appearances are connected to time and cannot be contemplated outside the substratum of time i.e. past, present, and future. Principles of time that cannot be derived from sense organs, i.e. experience, such as: ‘‘Time has only one dimension’, ‘Different times are not co-existent but successive’’ demonstrate the a priori nature of time (B 46). And like space, different times are part of the one and the same time; time progresses infinitely into the future and regresses infinitely into the past; and so, is unlimited (ibid). Kant emphasised:

> …the concept of change, and with it the concept of motion, as change of place, is possible only through … and in time
>
> (IBID).

2.6 Space and Time as the basis for geometry and number

Kant makes the plausible connection between human intuition of space with the more formal science of Euclidian geometry. He also connects human intuition of time with number and motion.

> Geometry bases itself on the pure intuition of space. Even arithmetic forms its concepts of numbers through successive addition of units in time, but above all pure mechanics can form its concepts of motion only by means of the representation of time
>
> (KANT, 1783 p. 35).

2.7 Space and Time as Causality

The concept of causality, or cause-and-effect, is, by its nature, structured according to the sequence of time, i.e. succession of events, and provides the answer to ‘why’ questions with such answers beginning with ‘be-cause’.

> The… causality of a thing is the real which, when posited, is always followed by something else. It consists in the succession of the manifold insofar as that succession is subject to a rule
>
> (B 183).

Schopenhauer’s *The world as Will and Representation* (1819) acknowledged the importance of a priori space and time, however, he collapsed the remaining categories, identified by Kant in the Critique, into the single notion of 'causality'. Schopenhauer went further by arguing that causality is constituted by space and time.

> What is determined by the law of causality is therefore not the succession of states in mere time, but that succession in respect of a particular space, and not merely the existence of states at a particular place, but in this place at a particular point in time. Thus change, i.e., variation occurring according to the causal law always concerns a particular part of space and a particular part of time, simultaneously and in union. Consequently, causality unites space and time
> (SCHOPENHAUER, 1818 p. 10).

It entails the logical succession of events of time in space and provides the ground for the, sometimes unconscious yet necessary, 'if-then' structuring in mathematics.

2.8 Summary

Implications regarding the growth of the science of geometry from the roots of a priori space, and the science of number, sequences, series, motion, etc., deriving from a priori time; and furthermore supported by the view that causality, a concept indispensable to mathematics, is constitutive of space and time, cumulatively provide the basis for my hypothesis that the genealogy of the concept of innate mathematical ability begins with Kant. Taken together, these innate knowledge-constituting intuitions provide an empowering worldview regarding human mathematical ability.

After concluding my argument that Kant provides the genealogical source for the concept of innate mathematical ability, I will now turn to more recent research, which argues, in a more focused manner, that human intelligence is innately mathematical. I will begin with the work of Donna Peuquet's (2002) Representations of Space and Time: her research articulates with Kant's foundational insights i.e. that space and time provide necessary intellectual concepts for the creation of human understanding and knowledge.

3. The necessity of space and time for human understanding

Donna Peuquet's (2002) Representations of Space and Time is a book about geographic space and the dynamics that occur in that space. While the context is computer-based geographic data processing, there is substantial content on theories of how humans acquire, store, and use spatial knowledge. She believes that things change in space over time; both space and time provide an integrated representation of our experiential world. This sounds a lot like the spatial and temporal intuitions proffered by Kant (1781), and indeed Peuquet does make much reference to the work of Kant. Peuquet suggests that space and time, being the most fundamental of notions … 'provide that basis for ordering all modes of thought and belief … Kant's space and time are concepts that we possess at birth (pp. 11 and 21). And while both concepts are innate it does not follow that all will make optimal use of these intellectual resources.

> The various ways that space, time, and their properties may appear to individuals are due to differences in attention to detail … access to technology, education … [etc.] (p. 25)

And while it is evident that people experience the world subjectively there is growing evidence:

> ... that the processes used to organise information are innate and either largely independent of the environmental input or dependant on kinds of environmental input that no human can avoid encountering (p. 28).

In her section on Schema: The Link between Percepts and Concepts, she again refers to Kant's view that schemata allow what we gain through our senses (perceptions) to be interpreted (concepts) and thus to be given meaning: schemata provide the bridge between experiences and meaning (p. 85). Research in cognitive linguistics has identified over twenty-four different image schemata and many of these intellectual bridging concepts are fundamentally spatial and temporal in nature. The schemata include:

> … container, balance, … path, cycle, centre-periphery, and link. Although these are called schemata, they are fundamentally spatial in nature. Our schemata for spatial and temporal orientation are so fundamental and pervasive in our experience that they are usually taken for granted (p. 87).

She refers to studies in visual cognition by Johansson (1973) to illustrate the importance of time in understanding. Given this understanding of schemata she concludes that even the youngest children employ space-time schemata to enable learning about the complex world they experience. Furthermore, learning is a similar process for adults and children in contexts where pre-existing knowledge is unavailable; however, what is significant of an increased capability among adults in a novel context is a larger store of knowledge:

> …Our basic notions of space [and time] are fundamental to learning and understanding [for young and old] in all domains (p. 88).

This insight has clear implications for teachers of mathematics to adults: while the learning process is similar for young and old, adults regularly employ a larger store of knowledge, including mathematical knowledge, not available to younger students. (This sophisticated, and often undervalued 'commonsense mathematics' resource available to adults is explored in some detail in Colleran and O'Donoghue (2007).

In her analysis of our perceptual field she reminds us that all our sense organs operate in a temporarily sequential manner. While most attention in the psychological literature has focused on visual perception, which provides information about size, distance, shape, and texture, she points out that hearing provides information about size and distance, and all senses provide information regarding pattern. All our senses are temporally extended because no single event affords the sequence of perceptions that provide the basis for the emergence of a pattern.

> All our senses are temporally extended…. [With regard to listening, which] is perhaps a more temporally extended activity than other senses... there is typically no single moment in which one hears anything, because sound waves themselves are a space-time phenomenon (105).

The fundamental requirement of pattern in the creation of understanding demands, at a conscious or unconscious level, attending to sequences of sounds, tastes, touches, etc., so that we can relate a particular perception to familiar categories. It is on this basis only that one can identify familiar sounds, images, tastes etc. Obviously, if there is no pattern recognition a new pattern category is developed. All our senses operate within a temporally sequential series of perceptions.

Over time attention has been paid to the contribution of individual senses however Peuquet suggests that an holistic analysis of the contribution of all senses to human knowledge creation can provide a more fruitful approach.

> The current body of evidence supports the view that our senses provide a unified and interrelated suite of sensations and that we understand how these sensations are related very early in life. (p. 108)

Research in psychology provides evidence supporting this unified and interrelated process in gathering spatial information. Furthermore, because of this process, people with deficiencies in one sense area, for example vision, compensate with other senses, such as touch and hearing. Current thinking suggests that… 'encoding our spatial knowledge is innate and not keyed to any particular sensory modality' (p. 110).

In her analysis of language as a symbolic system Peuquet finds that while there are many cultural variations when it comes to languages there is an structural invariance regarding spatial expressions. She suggests that this invariant structure indicates a 'common cognitive structure of spatial knowledge at some deep fundamental level' (p. 166). All languages are constituted predominantly by nouns, verbs, and adjectives, and these can be augmented as the evolving situation demands, for example new technologies. The grammatical elements of a language include prepositions, conjunctions, etc., and these are limited in number. With regard to spatial relationships there are between 80 and 100 relevant prepositions. Spatial and temporal relationships are invariably included within the grammatical structure of a given language (p. 168). The English language provides the verb-ending –ed to indicate past tense; and prepositions ‘above’ and ‘below’; ‘near’ and ‘far’; to refer to space. Temporal relations are referred to with the words ‘before’; ‘during’; and ‘after’. There are also space-time prepositions referring to motion including ‘across’; ‘through’; ‘into’. Peuquet concludes that ‘it does seem to be the case that spatial language encodes the world’ (p. 175). Furthermore, the fact that the number of spatial and temporal terms is very limited and difficult to increment, plausibly implies an invariant, and fundamental structure essential to the manner in which we perceive and understand the world. Further supporting evidence that mathematical concepts, and therefore, mathematical thinking, is integral to language is provided by Devlin (2000) below.

Having created connections between Kant’s contention of the innateness of the mathematical concepts of space and time with more contemporary work, I will now explore recent research providing substantial evidence that our capacity for mathematics is innate and universally available to all humans. This will include Devlin’s *The Maths Gene* (2001) and Butterworth’s The Mathematical Brain (2000). I will first turn to Devlin’s work.

4. Mathematics ability available to all humans

Devlin (2001) provides a human-evolution approach to his argument that mathematical thinking is innately available to all human knowers. His argument is based on the view that the human language faculty is there in the ‘initial state’ not unlike our capacity to walk, or become men and women through puberty: the language faculty just happens. Devlin’s source for this hypothesis is derived from the work of Bickerton (1995), however, many would argue that the seminal work on linguistics was done by Chomsky, (a summary is provided in Berwick and Chomsky, 2016). Devlin argues that the language faculty and the human ability to think mathematically are derived from a single mental human ability: the ability to think off-line.

> The two faculties [mathematics and language] are not separate: both are made possible by the same feature of the human brain … [our] genetic predisposition for language is precisely what you require to do mathematics … thinking mathematically is just a specialised form of using our language faculty
>
> (DEVLIN, 2001, pp. 3-4).

Devlin, like Chomsky, has a difficulty with the proposition that language is an evolutionary development derived from the need to communicate more effectively. While there is no doubt that language is the most effective means we have to communicate it certainly is not the only medium. We can communicate by the way we dress, the way we do our hair and make-up, our body language, our facial expressions, and so on. Devlin plausibly argues that language is the externalisation of human thoughts, speculations, plans, understandings, etc. In this view language is the most useful means to communicate complex, and not so complex ideas, among humans, however that is not the original evolutionary purpose for language; language was developed because we needed it to think off-line and so, language is primarily the process by which we think. So, in one of those quirky, yet fortunate evolutionary accidents, human thinking, externalised in the form of speech, provided an extraordinary advantage regarding human development in the last 70,000 years. Integral to the development of language, Devlin argues, was the development of our ability to think mathematically: ‘…mathematical ability is nothing other than linguistic ability used in a slightly different way’ (ibid, p. 22).

Devlin refers to substantial research suggesting an innate mathematical capacity among young children. He concludes that it is not just a correlation but, in fact, a symbiotic relationship between language development and our ability to think mathematically.

> I do not believe that a basic mathematical ability is any more unusual than an ability to talk (p. 126)

Devlin supports his position by referring to evidence that all human languages (known presently) have the same universal grammar. Chomsky’s observation that children cannot learn complex syntactic structures because they are not given or taught particular examples by parents, or anyone else that has those structures, leads to the inescapable conclusion that we must be born with the capacity for language.

> ‘… [G]rammatical structure is innate in much the same way that spinning webs is hard-wired into the spider’s brain’ (p. 157).

The synthetic structures inherent in language provided the essential resource for off-line thinking i.e. the capacity to reason in an abstract fashion. This in turn provides the capacity for mathematical thought (p. 162). Furthermore, he argues that while humans have been using language for nearly 200,000 years, with no apparent mathematical uses or developments, it is the mathematical structures inherent in human language that provided the natural source for the development of ‘formal’ mathematics over the past 3000 years.

Devlin’s research points to a two-stage development in the evolution of the human brain: the size of the brain increased over 3,000,000 years to allow for the development of more patterns and capacity to respond in a survival manner to new and various patterns. The second stage - 200,000 to 70,000 years ago – the brain didn’t increase in size but it changed structure.

> Those structural changes… gave us symbolic (i.e. off-line) thought … language, a sense of time, the ability to formulate and follow complex plans of action, and…. to design a … growing array of artefacts (p. 178).

However, even as far back as homo habilis (the size-change phase) there was evidence of capacities around number sense, spatial reasoning, cause and effect, and relational reasoning. It was the brain’s structural change that provided for abstract thinking, and this was the game changer: not a change in degree but a change in kind. An so our basic number sense, developed over 3,000.000 years, and now with the capacity for language from 70,000 years ago, the conditions were ripe for mathematical thinking in the form of numerical ability, algorithmic ability, and logical reasoning ability.

In an exploration of the necessary features of language to represent real-world situations Devlin asks: ‘which features of the world are absolutely necessary … and hence will be incorporated into the syntax and which can be optional? He concludes that ‘subjects’, ‘verbs’, ‘objects’, ‘tense’, gender’, ‘singular-plural’ are elementary to a thinking process capable of representing the world, i.e. off-line thinking.

> Off-line thinking provided the ability to think about past, present and future events, create tools [future orientated]… formulate and follow … plans of future action … logical reasoning (p. 236).

From this description of the necessary elements of syntax coupled with the ability provided by off-line thinking it is clear that many are related to space, time, and causality: verbs and tense are always related to time; singular-plural is related to differentiating space, while logical reasoning is essential in thinking mathematically. Devlin concludes... ‘the maths gene and the language gene are one and the same [and] mathematics is an automatic consequence of off-line thinking’ (237).

And so how is it that language has been used widely for more than 70,000 years while the development of mathematics stretches back less than 4,000 years? While keeping in mind that mathematical thinking is integral to language and language evolved primarily as a thinking process and not as a communication process, Devlin suggests language was hijacked by gossipers, and gossip was used to understand and care more about each other as humans, members of families, groups, tribes etc. Caring more for each other was the result of finding out more about each other. And caring for each other was a definite evolutionary advantage. In this understanding, the use of language developed a caring attitude among humans leading to group cohesion and the obvious advantages arising – language provided a major evolutionary advantage.

And so, for thousands of years this mathematical ability employed in gossip remained active yet invisible and undetected until a few thousand years ago when social and cultural developments, as well as the emergence of unique and exceptional thinkers, developed formal and abstract models to achieve the relevant mathematical outcomes. Gossipers, then and now, remained unaware and unburdened by the mathematical thinking integral to the language used to gossip. Gossip addresses similar questions to those of the mathematician – what is the relationship between? How many are there? What type? Are they the same? Are they equal? What is the property of... what characteristics does he/she have? and so on. Building an understanding of the relationships between people and the characteristics of each person/group is the material of both gossip and mathematics.

> 'The mental abilities required for gossip – even the most socially denigrated variety – are highly sophisticated, and already structurally adequate to support mathematical thinking… Mathematicians are not born with an ability that no one else possesses. Practically everyone has 'the maths gene' (pp. 249-250).

If mathematical thinking is as natural as learning a language or walking upright, why then do so many people find mathematics so difficult? The first part of the answer, according to Devlin, is that mathematical thinking is highly conceptual and abstract and what distinguishes a great mathematician from a high school student struggling in a geometry class 'is the degree to which the mathematician can cope with abstraction' (p. 253).

The second part of the answer is that we can become proficient at anything in life only by repeated practice. We become good musicians and writers by playing and writing... we become good mathematicians by repeating and practicing, seeing new angles and approaches for doing. Repeated practice is driven by, sometimes obsessive, interest and passion and it is this passion that differentiates those who can do mathematics well and those who claim to find it impossible.

> But for all its difficulty, doing mathematics does not require any special ability not possessed by every one of us (p. 258).

5. Humans have a 'Number Module' located in our brain

The Mathematical Brain (2000) by Brian Butterworth approaches the thesis that all humans have, what he terms, a Number Module, from an evolutionary, historical, neurological, and psychological perspective. Butterworth is a neuropsychologist and his interest in mathematics resulted from tests he carried out with people who had severe inabilities when it came to using numbers. Some of his patients had suffered stoke and other suffered brain injuries, while others, without injuries, appeared to be succeeding quite well but suffered a severe dysfunction with numbers. His hypothesis is that the Number Module is genetically provided and provides the basis for our ability to use numbers to interpret and operate in the world. And while some cultures are more advanced mathematically, Butterworth argues that the sophistication of number use is consistent with the technological levels achieved by that culture.

> Our mathematical brain … contains two elements: a Number Module and our ability to use the mathematical tools supplied by our culture' (p. 7)

However, people without access to the Number Module through injury or dysfunction cannot develop number skills to any level of sophistication and are grossly incapable when dealing with numbers. He proceeds to compare number deficiency i.e. dyscalculia, with dyslexia and colour blindness, as these too, result from a similar type of dysfunction in the brain. Consequently, dyscalculia is derived from the lack of a Number Module.

Butterworth sets the standard for the scientific veracity of his hypothesis - that all who function effectively with numbers have an innate Number Module - by presenting plausible evidence regarding a number of premises including the following:

1. Everybody should show evidence of ability to use nemerositie (categorising the world in terms of numbers of things)

2. Evidence must be shown among infants
3. Brain imaging should be able to locate the Number 'hot spots'
4. The Module must be encoded in our genes and must ten have been passed on by our ancestors
5. This may lead to an understanding why some people a very good while others are hopeless (pp. 9-10).

In his exploration of the history of the use of numbers he concludes that the variety of techniques and sophistication levels used across many cultures provide two conclusions:

1. Number techniques were not invented in one location and then disseminated to other cultures across the globe
2. This localised, cultural variety of number techniques provides plausible evidence that, like language, humans have an innate capacity to employ numbers and to appreciate how numbers can improve the way we live in the world (pp. 23 – 103)

Evidence related to experiments with babies, often as young as three months old, illustrate a capacity to differentiate groups of numbers, to recognise changes by adding and subtracting, and ordering numbers by size. It is these three elements that constitute the basic numerical capacities embedded in our Number Module.

In his study of the anatomy of the brain he discovers that the left side of the brain provides the capacity for mathematics, specifically in the left parietal lobe. He goes on to report on a number of case studies of individuals who had very serious difficulties with numbers while simultaneously being capable of operating very effectively where numbers were not concerned. He describes Charles who had A Levels and a university degree in Psychology. He concluded that Charles was deficient when it came to the innate Number Module and this led to his, and other case study subjects', inability with numbers. Consequently, if the Number Module is working effectively, it would seem that all humans can reach a proficiency equal in sophistication and expertise. However, we all know that this is untrue.

Some of us with a perfect genetically endowed Number Module find mathematics very difficult while others see no limits to what they can accomplish with numbers (while Butterworth did not include geometry skills, he did not directly exclude it either). As mentioned above the Number Module has to have something to work with i.e. culturally provided conceptual tools. And while this creates the limits to which all can reach, the overwhelming evidence is that most of us, in a culture that has developed, and continues to develop very sophisticated mathematics, do not reach those standards. There is something other than the Number Module and the cultural affordances required to ensure that all can become proficient mathematicians.

One of the stops put on the 'natural' development of number skills to the highest levels is maths phobia: a learned fear, specific to a situation and accompanied by physiological signs such as increased heart rate, sweating, etc." (p. 333) Students with this infliction do much worse at mathematics and avoid taking mathematics courses. Whether doing badly causes anxiety, or anxiety causes students to do badly is difficult to establish, however, the result is that students are drawn into a vicious cycle of poor performance, external discouragement, internal discouragement, anxiety, avoidance, no improvement, and so on. While the Number Module is available, phobia makes it inaccessible with the result that we avoid spending time with numbers.

On the other hand, Butterworth, like Devlin (2001), argues that differentiation is the result of training. He refers to Ericsson et al (1993) who suggest that the variation in ability to do well at any endeavour, be it music, sport, or mathematics, is 'drive' within the person. And this is manifest in 'deliberate practice, which is usually solitary' (p. 290). He goes so far as to say that 'obsession, however, does seem to be a necessary ingredient' (p. 294). As a result of developments in neuroscience and brain mapping technology, the concept of the 'plastic brain' suggests:

> '…long term, repeated practice at a skill will increase the number of neurons that the brain assigns to that skill on a more or less permanent basis … Long-lasting structural changes in the brain are dependent on practice. Use it or lose it!' … Most of us are born to count, but beyond that the only established limits to mathematical achievement are … zeal and very laborious work (pp. 313 - 314).

In summary, Butterworth is convinced that all of us, excepting those with brain injuries and brain dysfunctions, have a genetic predisposition to use numbers to a level of proficiency limited only by the sophistication of the cultural development one is born into and – agreeing specifically with Devlin (2001) - the amount of time and practice an individual invests.

6. Summarising the argument for innate mathematical ability

The basis for my argument that humans have an innate ability to be proficient with mathematics is sourced in the ideas of the Enlightenment philosopher Immanuel Kant. Kant argued that we are endowed with innate temporal and spatial intuitions. These intuitions provide the basis for the development of all scientific understanding and knowledge while simultaneously providing that basis for the science of numbers and geometry. Racing forward by nearly 250 years Peuquet, in the context of geographical research, provided evidence of the innate nature of space and time. She builds on this conclusion derived from Kant and argues that the structure of human language is also spatial and temporal in nature. Devlin's mathematical research, derived primarily from developments in linguistics, provides persuasive evidence that all of us have a 'maths gene'. He argues that language, currently employed quite effectively as an interpersonal communication medium, was initially used to carry out off-line (conceptual) thinking. Integral to off-line thinking is mathematical thinking employing spatial and temporal concepts among others, and the evidence for this is presented in normal language structures. Both Peuquet and Devlin link innate mathematical abilities to human language which all of us use relatively proficiently.

Butterworth presents evidence regarding a Number Module located in the brain and this Module is innately available to all of us. However, if the Module is damaged, or deficient in any way, the individual will have serious difficulties with numbers. Butterworth points out that while we all have the Number Module, all of us do not reach similar levels of proficiency. He suggests 'maths phobia' will create serious difficulties in developing number potential. Agreeing with Devlin, Butterworth concludes that the level of proficiency is directly proportional to the amount of time given to the practice of mathematics. And so, while we do have the innate capability, it takes drive, even obsession, to become extremely proficient.

7. Innate mathematical ability as the basis for a more egalitarian approach to mathematical education

I have argued that all of us, excepting those with particular intellectual deficiencies, are naturally capable of becoming proficient mathematicians. If this is the case, as adult mathematics educators we need to re-consider the manner in which we approach our profession. There is no doubt that our students come to us with a range of levels of mathematical proficiency. However, there seems to be a prevailing worldview among some teachers of adult mathematics that some people have mathematics abilities while other do not. The preceding argument calls this assumption into question. The evidence provided suggests that the range of proficiency is the result of the amount of time spent doing and practicing mathematics. We can speculate as to why some adult students do very little practice: math phobia, home environments, educational experiences, etc. We can also speculate why other students spend a lot of time practicing mathematics: a love of mathematics, home environment, educational experiences, etc. The challenge for adult mathematics educators is, firstly, to understand that all our learners are capable mathematicians irrespective of the level at which we meet them; and secondly, to provide a learning environment where students can unlearn the negative emotions, derived from various socialisation processes, that produces reduced expectations and motivation when it comes to mathematical thinking and doing mathematics. This is particularly important when it comes to adults as they have had more time than children and younger students to galvanise the negative and confidence-sapping beliefs of their inability to do mathematics. In this way, we can provide a more egalitarian mathematics education for adult as well as younger students.

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