Inclusion of Techno-Pedagogical Model in Mathematics Teaching-Learning Process

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Abstract

The aim of this work is to know the perception of the high-school student towards the teaching process of the Financial Mathematics under the modality assisted by the technology. The EAPHMF Scale designed by García-Santillán and Edel (1988) was used, which collects data associated with the student's perception of the variables that make up the techno-pedagogical model. Fifty two students were surveyed in a private institution. The results show that, as a whole the variables of the techno-pedagogical model favor the student's perception toward financial Mathematics being the ones that most contribute, the workshop-type class followed by the design of financial simulators. Furthermore, in the intervention process, an improvement in the evaluation of learning was observed.

Keywords: techno-pedagogical model, ICT, perception, financial mathematics, spreadsheet.

1. Introduction

During the 21st century, the use of technology has extended to every human activity around the world. Certainly, the automatization of productive sectors, to name an example, has been favored with the inclusion of information technologies. Another area that has been benefited – related to the object of study – is education. Namely, the teaching-learning processes of different graduate programs and special fields offered in the educational system worldwide, and specifically in Mexico, include ICT in their formation processes, seeking to develop more and better competencies in individuals.

From the 1970s, some researchers like Feierabend (1960) dedicated their study to measure the attitude of students towards Mathematics, but these studies were not done in detail (Garcia, Juárez, 2011). In 1976, Fenemma and Sherman took on the task of measuring anxiety towards

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Mathematics in men and women. Other studies were added to the body of knowledge in such period, trying to measure different factors that students usually present towards Mathematics, such as confidence, mathematical interaction and computers, commitment, motivation or usefulness, among others (Galbraith, Hines, 2000). Nonetheless, from such studies, research about this area has notably increased, especially since students perceive Mathematics as a difficult, boring and abstract subject, as stated by Gil, Guerrero and Blanco (2006).

In recent years, teaching-learning processes have changed due to the presence of information technologies, and Mathematics has not been an exception because ICT have been included in learning formation models assisted by technology, an example of that are the proposals by García-Santillán and Edel (2008) and García-Santillán et al. (2010) about a techno-pedagogical model that has aided students in improving the understanding of subjects in financial Mathematics by visually – with financial simulators – taking the image of concepts they considered abstract to a more technological view, specifically regarding financial simulators.

The later are the resulting elements of the setup of the afore-mentioned model since these financial simulators are designed by the students as product evidence of the techno-pedagogical model. In support of this, some authors have shown the great accomplishments obtained by students in the Mathematics teaching processes aided by ICT, since they can simulate scenarios and present different economic situations to be solved (Domínguez et al., 2008 quoted in Maz et al., 2012).

Along those lines, ICT provide different tools that can be used for the teaching of Mathematics: the use of spreadsheets, simulators and virtual platforms, among others; these tools can change the student’s perception towards this subject and have improved students’ final grades (Benítez et al., 2011).

On this regard, Williamson and Kaput (1999) pointed out that everything related to computational means is linked to the virtual culture, because it is there where the fifth cognition stage is developed, that is where new forms of mathematical representation can be created, which can be manageable and changing. The former causes the development of processing and storing capacity for each step and rules within the system, which derives in the virtual culture.

It is clear that technology does not come to solve the existing problems in the Mathematics teaching-learning process, but it is indeed a fact that technology is an aiding or assistance tool for a transformation in Mathematics education (Gómez-Chacón, 2010). The important role of ICT is precisely the transformation of said education processes for visual representation and creation of scenarios in mathematical problems modeling and solving.

In the field of Mathematics, solving a problem that involves too many variables or elements that constitute theorems is certainly complex and developing them manually has been the traditional teaching school. This argument can sustain the justification about the importance of technology use in the teaching process, since the aim of its use is exactly being able to show the way of simplifying the complexity of a certain situation.

Technology can also assist in the area of environment, as it intervenes with symbolic systems too; technology has an important role that goes from designing to aiding the teaching didactic system (Gómez-Chacón, 2010).

Student considers more attractive to be able to interact with several possible scenarios of problem solving because thought connects to the visual algorithm and hence, makes the student think while interacting with cognition. In such manner, he/she learns by using visual and graphic representations with algorithms through graphic calculators and computers which eases the learning of Mathematics (Hitt, 1998).

Lupiánez and Moreno (2001) state that the main use of computer and calculators is using the calculation system, through computerized formulas. They argue that with the use of computers vectors, matrices, numeric solutions and trigonometric operations, among others, can be appreciated. Even if it is true that technology can benefit the learning process, its regular use can be detrimental since the student can forget what he/she can do with paper and pencil. Technology must be used carefully, to enrich the process of developing cognitive skills and not as a substitution (Lupiánez, Moreno, 2001).

Likewise, Santos (2001) points out that technology is essential for the teaching and learning of Mathematics, as it eases the making of calculations in an efficient and precise manner. In his study, he mentions that students focus more because they pay attention when making decisions,
helps them ponder and reason each scenario to be solved using calculators and computers. It is obvious that Mathematics has a steady presence in our environment, hence examples and situations that can be shown to students are a way of proving its application on different moments of daily-life and the areas it can encompass (Godino et al., 2003).

With the support of these arguments, the main focus of this study is to identify the factorial structure of latent variables that allow measuring the student’s perception towards the Mathematics teaching process supported by a techno-pedagogical model. As a result, it is pertinent to question: how does the undergraduate student perceives the Mathematics teaching-learning process assisted by ICT? And similarly: what is the factorial structure of the techno-pedagogical model that contributes more to the explanation of the study phenomenon?

To provide answers for these questions and achieve the main purpose of this research, the theoretical model posed by García-Santillán and Edel (2008) will be replicated, considering the variables of the scale that measures attitude and perception towards Financial Mathematics (EAPHMF) and thus, the construct is presented next:

![Study conceptual model EAPHMF (García-Santillán, Edel, 2008)](URL: http://blogconamat.blogspot.mx/2012/05/objetan-ensenanza-tradiconal-de.html)

Where: VC: Virtual communities; DFS: Design of financial simulators; MHWC: Content of Mathematics history and workshop class; SSP: Spreadsheet programing; ITP: IT platforms.

Source: Images taken from Google.

Therefore, in order to measure perception towards Financial Mathematics (FM) and the inclusion of Information and Communication Technologies in the teaching processes, it is necessary to discuss the theory and empirical evidence of both constructs.

### 2. Literature review

The variables implied are framed in the following theories and empirical evidence: Mathematics History promotes a change in the attitude towards the subject, according to the statement Furinghetti and Somaglia (1998), the students still holds the belief that Mathematics is
abstract and is only found in the mind of professors and that practically everything has been
discovered in this area; the author also mentions that for them, it is a boring subject lacking
imagination. However, Bell (1985: 54) postulated that Mathematics is one of the disciplines that
loses the most when its history is excluded from the teaching process.

In the study by González-Urán eja (2004), the author presents very significant theoretical
arguments. Along the document, it extracts important arguments from great mathematicians,
pedagogues and historians that have provided theoretical basis to this variable of Mathematics
history as didactic resource, mentioning: Poincaré, Klein, Toeplitz, Köthe, Bell, Courant, Puig
Adam, Lakatos, Kline, Santaló..... (sic).

It is important to identify the origin that builds the historic roots of Mathematics, as Courant
has referred in the prologue of “The History of the Calculus and its conceptual development”
(Boyer, 1949). For instance, Fauvel (1991), based on the opinion of Gellert (2000), states that it is
necessary to use Mathematics history in the teaching-learning-evaluation (TLE) process of the
subject, by using comparative studies between: the methodology used in the moment and context
of the student and analogously, in a different cultural context (Pizzamiglio, 1992; Bidwell, 1993;
Murugan 1995).

Likewise, Clinard (1993) and Fauvel (1991) support the argument of including “Mathematics
History” as an important component in T-L-E processes of this subject, but also for the teacher,
since it allows him/her to visualize Mathematics from another perspective. Clearly, the great absent
of Mathematics teaching process has been its own history, that is, its birth and how it has evolved
until today.

It is interesting to know the controversies and conflicts between the great scientists for the
development of this discipline. Also, it must not be forgotten than these debates caused regression
and stagnation too, until the progress known nowadays was achieved (Vidal, Quintanilla, s/f).

The benefit obtained by this methodologic resource is remarkable; actually, in a study carried
out by Chávez and Salazar (2006), the authors assert to have incorporated this technique in an
algebra subject with excellent results, among which the following stand out: development of oral
and written expression skills, attitude change in students (interest, collaboration and disposition).

On the other hand, Salinas (2010) made an experiment in which Mathematics history (MH)
is applied with the purpose of observing geometry students to know their attitude towards MH
topics, besides observing if this didactic resource encourages the student to introduce him/herself
in the deductive sense of geometry.

Nowadays, one of the strongest tendencies in education is the one that presents the
Mathematics teaching process with the use of technologies (Hitt, 1998; Lupiánez, Moreno, 2001;
Goldenberg, 2003). In that area, the use of computer spreadsheets has allowed a noticeable
advancement because it has enabled the development of financial projections automatically, just by
using a set of values. In this respect, Moursund (1999, 2003 and 2007) has stated repeatedly that
for solving problems from a business context, exact and social sciences, as well as other knowledge
disciplines, the spreadsheet offers a beneficial environment for the modeling of said problems.

In the teaching of financial Mathematics, other significant mechanism for learning is the
creation of a virtual community, that is, an space in IT platform where people (who will learn) and
instruments (means to learn) can be integrated; hence, in the words of Pazos et al. (2001), “virtual
communities” are considered as web-based environments that group people related to a specific
subject, who besides the distribution lists (first node of the virtual community) share documents,
resources, among others.

In the same idea, Hunter (2002) declares that virtual communities have been created to
analyze or solve problems and they support the construction of knowledge jointly with its
members, in such manner that students would have a higher involvement, active participation,
autonomy, interdependence and responsibility, all of that regarding the learning process,
culminating in a collaborative and cooperative work. In the same way, Mendoza (2015) carried out
a research which shows the formative potential of students in virtual communities. On said study,
data revealed that students showed higher skills in the learning process, which lead them to
improve their grades, besides leaving them satisfied with the learning experience.

By including the TLE process of financial Mathematics, the variables: simulation and
simulators as tools that are created in the “Workshop class”, it is pertinent that said tools are
shared to other people, institutions or any other interested in obtaining them, that is, creating a virtual community, where they can be shared.

On that matter, the perspective of Salinas (2003) is relevant as he states that there is a higher probability of achieving virtual learning communities when there are individual interests and affinities among the students who are taking the same subject.

For the former, it is necessary to have a special infrastructure and a platform to create virtual classrooms (VC), being Moodle one of the best options, since it has a pleasant environment. Moodle pretends to be a platform to create virtual courses of any subject, as well as being an excellent tool that complements educational community, without space and time limits (Dougiamas, Taylor, 2002, 2003; Costello, 2014).

It is important to highlight that in the learning-teaching process, perception plays an important role because a large part of interest in learning depends on the attributes of each student and therefore, the teacher must take into account the influence of this factor in the teaching-learning process.

From a pedagogic point of view, the teaching-learning process of this subject can be configured around the theories of Bruner and Ausubel (Roca, 1991), these authors affirm that learning is produced by the interaction of the previous mental schemes of the subject, as well as the new information from the environment, where new information in the knowledge and learning process does not substitute previous knowledge of the student, but it is an interaction with the ones already present.

To sum up, the variables involved in the techno-pedagogic model have been fundamental in several studies, such as the case of Mathematics History as methodological resource in the teaching-learning process (Fauvel, 1991; Russ, 1991; Pizzamiglio, 1992; Moreno, Waldegg, 1992; Clinard, 1993; Toumasis, 1995; Barbin, 1997; Nuñez, Servat, 1998; Ernest, 1998; Fauvel, Manen, 1997; Furinghetti, 1997; Chávez, Salazar, 2006 and Salinas, 2010), the construction of simulators as didactic tool for the teaching process and the case of Excel spreadsheet (Barbin, 1997; Goldenberg, 2003; Lewis, 2006; Mousround, 2007; Niess, 1998; García-Santillán et al., 2008).


After the afore-mentioned analysis, the research question is ratified: How does the undergraduate student perceives the Mathematics learning-teaching process that uses ICT? As well as, what is the factorial structure of the techno-pedagogical model that better contributes to explaining the study phenomenon?

Consequently, the hypothesis is: \( Hi = \) There is a factorial structure of latent variables that explain the undergraduate student´s perception towards the Financial Mathematics teaching process that uses technology.

### 3. Method

It is a non-experimental study, descriptive and with factorial analysis of main components. The sample selection is non-probabilistic since it was selected by convenience. The selection criteria consisted on including students of all the bilingual programs at the Business School who were enrolled in the third semester who would take Financial Mathematics as subject. The request for the empiric study and authorization was made by the Business School Dean. The study subjects are shown on Table 1.
Table 1. Studied population

<table>
<thead>
<tr>
<th>Career program</th>
<th>Students per program</th>
<th>M</th>
<th>W</th>
<th>M</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company Management and Direction</td>
<td>10 = 19 %</td>
<td>2</td>
<td>8</td>
<td>3.85 %</td>
<td>15.38 %</td>
</tr>
<tr>
<td>Accounting and Finances</td>
<td>14 = 27 %</td>
<td>6</td>
<td>8</td>
<td>11.54 %</td>
<td>15.38 %</td>
</tr>
<tr>
<td>Economy</td>
<td>9 = 17 %</td>
<td>5</td>
<td>4</td>
<td>9.62 %</td>
<td>7.69 %</td>
</tr>
<tr>
<td>International Markets and Business</td>
<td>19 = 37 %</td>
<td>7</td>
<td>12</td>
<td>13.46 %</td>
<td>23.08 %</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>52</strong></td>
<td><strong>20</strong></td>
<td><strong>32</strong></td>
<td><strong>38.00 %</strong></td>
<td><strong>62.00 %</strong></td>
</tr>
</tbody>
</table>

As observed in Table 1, 62 % are female, which is a larger number of participants in this study against 38 % of male students. All of them are enrolled at the programs of Company Management and Direction, Accounting and Finances, Economy, International Markets and Business.

The distinctive characteristic of this study group is that they belong to the area of Business school (bilingual system) during the first five semesters of their college program and later, they study the specialization during the rest four semesters. There is even the option of double certification with a US college (South Eastern Louisiana University), for that, they are required to study the last two years in the USA.

3.1. Instrument

The scale designed by García-Santillán y Edel (2008) and modified in García-Santillán et al. (2010) was used; it collects information about the student’s perception towards variables related to Mathematics History, spreadsheet programming, simulators’ design, computer platforms and virtual communities. The instrument is designed in a Likert scale that ranges from 1, totally disagree to 5, totally agree (Table 2).

Table 2. Dimensions and indicators of the EAPHMF scale

<table>
<thead>
<tr>
<th>Variable</th>
<th>Code</th>
<th>Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>-Mathematics history and workshop class</td>
<td>MHWC</td>
<td>1,2,5,6,7,9,10,11,12,13,14,15,17</td>
</tr>
<tr>
<td>-Spreadsheet programming</td>
<td>SSP</td>
<td>3,8,16,20,21,22,23,26,</td>
</tr>
<tr>
<td>-Design of financial simulators</td>
<td>DFS</td>
<td>18,24,25,27,28</td>
</tr>
<tr>
<td>-Computer platforms</td>
<td>CP</td>
<td>4,19</td>
</tr>
<tr>
<td>-Virtual communities</td>
<td>VC</td>
<td>29,30,31</td>
</tr>
</tbody>
</table>

The internal consistency reported in the original scale design was $\alpha=.902$ and standardized $\alpha=.905$ however, the consistency obtained for this study was $\alpha=.775$ (31 items), $\alpha=.791$ (typified elements) and $\alpha=.770$ (grouped in five dimensions), which is an average internal consistency in relation to the theoretical statements recommended by some authors (Hair et al., 1979).

Table 2.1. Case processing summary

<table>
<thead>
<tr>
<th>Cases</th>
<th>N</th>
<th>%</th>
<th>Cronbach´s Alpha</th>
<th>Cronbach´s Alpha based in typified elements</th>
<th>Number of elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valid</td>
<td>52</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Excluded</td>
<td>0</td>
<td>.0</td>
<td>.775</td>
<td>.791</td>
<td>31</td>
</tr>
<tr>
<td>Total</td>
<td>52</td>
<td>100.0</td>
<td>.770</td>
<td></td>
<td>5</td>
</tr>
</tbody>
</table>

a. Elimination by list based on all the procedure variables

The descriptive statistics from the five dimensions are shown on Table 3 (mean, deviation variation standard and coefficient), where it can be seen that the CP dimension presents a higher
variation between its mean and standard deviation (15.54 %) against SSP, which presents the lower variation (8.14 %).

Table 3. Descriptive Statistics

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Mean</th>
<th>Typical deviation</th>
<th>N of analysis</th>
<th>VC= St Dev./µ</th>
</tr>
</thead>
<tbody>
<tr>
<td>MHWC</td>
<td>49.9615</td>
<td>4.31578</td>
<td>52</td>
<td>8.64%</td>
</tr>
<tr>
<td>SSP</td>
<td>31.0385</td>
<td>2.52797</td>
<td>52</td>
<td>8.14%</td>
</tr>
<tr>
<td>DFS</td>
<td>19.6923</td>
<td>2.20106</td>
<td>52</td>
<td>11.18%</td>
</tr>
<tr>
<td>CP</td>
<td>7.7885</td>
<td>1.21003</td>
<td>52</td>
<td>15.54%</td>
</tr>
<tr>
<td>VC</td>
<td>12.2500</td>
<td>1.15258</td>
<td>52</td>
<td>9.41%</td>
</tr>
</tbody>
</table>

3.2. Procedure for data analysis

For the statistical analysis the program Statistical Package for the Social Sciences (SPSS) version 23 was used and the multivariate statistical procedure of exploratory factor analysis (EFA) with main component extraction was carried out, since the information will be summarized in a minimal quantity of factors with prediction purposes in order to answer the main question of the study and achieve the set goal, after the hypotheses have been contrasted.

4. Data analysis and discussion

In order to corroborate the model factor structure of the Scale EAPHMF by García-Santillán and Edel (1988), a EFA is done using the main components method (with Varimax rotation). The results from the measures shown on table 4 are statistically optimal. The sample adequacy KMO (.684), Bartlett’s test of Sphericity (Chi² 71.924, df 10 p <0.01), the MSA values (which range from .626 to .851) are theoretically acceptable (.>5) and the positive correlations with a low determinant (.227). These EFA measures allow surpassing significantly the validity and pertinence of the data matrix and continue with the analysis.

Table 4. Bartlett´s test of sphericity KMO, MSA, Correlation and significance

<table>
<thead>
<tr>
<th></th>
<th>MHWC</th>
<th>SSP</th>
<th>DFS</th>
<th>PI</th>
<th>VC</th>
<th>KMO</th>
<th>MSA</th>
<th>Bartlett’s test (Chi² df10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MHWC</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.675(α)</td>
<td>71.924 (α = .000)</td>
</tr>
<tr>
<td>SSP</td>
<td>.509</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td>.626(α)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DFS</td>
<td>.356</td>
<td>.725</td>
<td>1.000</td>
<td></td>
<td></td>
<td>.684</td>
<td>.710(α)</td>
<td></td>
</tr>
<tr>
<td>CP</td>
<td>.111</td>
<td>.445</td>
<td>.321</td>
<td>1.000</td>
<td>.179</td>
<td>.705(α)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VC</td>
<td>.144</td>
<td>.394</td>
<td>.379</td>
<td>.179</td>
<td>1.000</td>
<td>.851(α)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a Determinant = .227

Afterwards, two important steps in the development of the factorial technique take place, the first of them is the analysis of main components with latent root criterion > than 1 with the 31 items grouped in the five dimensions. The second is using the same main components method, but now under the factor criterion (31 items of the scale) with Varimax rotation and Kaizer normalization, as to try to minimize the number of variables that present high saturations and being able to simplify the interpretation of each of the obtained factors.

In table 5, there is the calculation of main components using the Eigenvalues criterion > 1, where it can be seen that the analysis shows a single component with an Eigenvalue of 2.515, which represents 50.30 % of the total variance of the five dimensions in the conceptual study model from Figure 1 (MHWC, SSP, DFS, CP and VC). A second and third component attain a value > than 1 and remain under this criterion (.901 and .824), which combined would provide 84.80 % of the variance.
Table 5. Total Variance

<table>
<thead>
<tr>
<th>Component</th>
<th>Extraction sums of squared loadings</th>
<th>Rotation sums of squared loadings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total % of Variance</td>
<td>Cumulative %</td>
</tr>
<tr>
<td>1</td>
<td>2.515</td>
<td>50.299</td>
</tr>
<tr>
<td>2</td>
<td>.901</td>
<td>18.025</td>
</tr>
<tr>
<td>3</td>
<td>.824</td>
<td>16.470</td>
</tr>
<tr>
<td>4</td>
<td>.531</td>
<td>10.629</td>
</tr>
<tr>
<td>5</td>
<td>.229</td>
<td>4.576</td>
</tr>
</tbody>
</table>

Extraction method: principal component analysis.
In the second procedure the factor matrix was rotated, for that, the 5 grouped dimensions were analyzed (Table 6) and afterwards, the 31 items of the scale (Table 6.1), both with the criterion of factorial loadings > .5 and ranked by factor type.

Table 6. Rotated component matrix (a)

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>DFS</td>
<td>.919</td>
</tr>
<tr>
<td>MHWC</td>
<td>.971</td>
</tr>
<tr>
<td>CP</td>
<td>.977</td>
</tr>
<tr>
<td>VC</td>
<td>.975</td>
</tr>
<tr>
<td>SSP</td>
<td>.795</td>
</tr>
</tbody>
</table>

Extraction method: Principal component analysis. Rotation method: Varimax with Kaiser Normalization.
a The rotation has converged in 5 iterations.

On Table 6, it can be seen that the five factors corresponding to each of the model dimensions (with the 31 grouped items), present high saturations in the following order: Computer Platforms (.977), Virtual Communities (.975), Mathematics History and Workshop Class (.971) and Design of Financial Simulators (.919), although with a lower saturation in comparison to the others, but still with an acceptable factorial loading we find the dimension of Spreadsheet Programming (.795).

Table 6.1. Rotated component matrix (a)

<table>
<thead>
<tr>
<th>Items</th>
<th>Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSP-22</td>
<td>.814</td>
</tr>
<tr>
<td>SSP-23</td>
<td>.766</td>
</tr>
<tr>
<td>DFS-27</td>
<td>.723</td>
</tr>
<tr>
<td>SSP-26</td>
<td>.683</td>
</tr>
<tr>
<td>DFS-24</td>
<td>.599</td>
</tr>
<tr>
<td>DFS-28</td>
<td>.528</td>
</tr>
<tr>
<td>DFS-25</td>
<td>.691</td>
</tr>
<tr>
<td>CP-19</td>
<td>.633</td>
</tr>
<tr>
<td>MHWC-17</td>
<td>.593</td>
</tr>
<tr>
<td>SSP-8</td>
<td>.833</td>
</tr>
<tr>
<td>MHWC-7</td>
<td>.817</td>
</tr>
<tr>
<td>DFS-18</td>
<td>.529</td>
</tr>
<tr>
<td>MHWC-10</td>
<td>.581</td>
</tr>
</tbody>
</table>
Extraction method: Principal component analysis. Rotation method: Varimax with Kaiser Normalization. a The rotation has converged in 10 iterations.

The saturations presented by the indicators that have been grouped in each of the five components obtained provide very significant information, given that the integration of each component provides routes or indications of how is the student’s perception regarding the technopedagogic model proposed for the teaching of the Financial Mathematics subject.

Therefore, next there is the interpretation of the rotated matrix, with which the extracted latent indicators are identified according to the procedure of rotated component extraction by factors criterion, where the ones with higher saturation (> .5) are extracted.

Table 7. Rotated component matrix (a) Factor 1

<table>
<thead>
<tr>
<th>Code</th>
<th>Factor 1 ((Y_1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSP-22</td>
<td>Program the formulas in Excel and work with them in the workshop class aids my learning.</td>
</tr>
<tr>
<td>SSP-23</td>
<td>Designing financial tools in an Excel spreadsheet complements my learning.</td>
</tr>
<tr>
<td>DFS-27</td>
<td>I really like to learn FM if I can transform the formulas seen during class in financial simulators.</td>
</tr>
<tr>
<td>SSP-26</td>
<td>I feel that Excel programming strengthens my learning in FM.</td>
</tr>
<tr>
<td>DFS-24</td>
<td>The design of simulators provides added value to my teaching-learning of FM.</td>
</tr>
<tr>
<td>DFS-28</td>
<td>It is an incentive when the teacher promotes competition for the best design of simulators.</td>
</tr>
</tbody>
</table>

In factor 1, the six items with saturations > .5 are related to spreadsheet programming and design of simulators, which leads us to think that the student perceives that Excel programming to design financial tools strengthens his/her learning together with the teacher’s incentive of promoting competition for the best simulator.

Table 7.a. Rotated component matrix (a) Factor 2

<table>
<thead>
<tr>
<th>Code</th>
<th>Factor 2 ((Y_2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>DFS-25</td>
<td>Excel programming and design of simulators help me not to reject the teaching-learning of FM.</td>
</tr>
<tr>
<td>CP-19</td>
<td>Use of ICT in the learning process of FM generates more interest in me.</td>
</tr>
<tr>
<td>MHWC-17</td>
<td>FM generates more interest and expectation when I relate it to real cases and present them in class.</td>
</tr>
</tbody>
</table>

For factor 2, the importance of ICT is noted again, since programming of mathematical formulas that are analyzed in each class session are turned into financial tools in a workshop class, that is, the student designs financial simulators that will also help him/her in professional life, which generates more interest to take the subject under a modality aided by ICT. Also, the use of previously designed financial simulators allows the creation of diverse scenarios to simulate decisions in several possible contexts.
The terms and symbols used in Mathematics are never difficult for me to understand and handle because the teacher encourages me to create new ways to code them.
The FM course helps to teach how to think and I can propose some alternatives of solution.
I learn better when the FM subject is taught using other didactic techniques.

The result for factor 3 highlights once again the relevance of ICT, given that the student is motivated by the implementation of alternative ways of learning, different from traditional sessions in the Mathematics teaching process. In the same manner, students are incentivized by being able to code to computer language, the symbols and terms used in the formulas, which helps them propose solution alternatives to the cases been solved in class.

It is motivating to carry out a workshop class.
I think that using the Web to share knowledge is a good alternative for our education.
Knowing the history of FM helps me become more interested in the course.

The history of Mathematics and the workshop class is seen again in the fourth factor, which points to the student’s interest to take the subject of Mathematics while integrating the historical aspects of the evolution of said discipline.
In addition, it is possible to see the student’s interest for sharing experiences with other students through virtual communities, which undoubtedly will add to his/her learning.

I enjoy FM when the teacher explains how a problem can be solved in different manners.
FM is enjoyable and stimulating for me when the teacher explains its history.
I think I could study harder FM with the use of the spreadsheet.

In the fifth and last factor, the history of Mathematics and spreadsheet programming are present once more, which leads us to think that the student favorably perceives the role of the teacher, when he/she explains the way a particular exercise has been solved historically, meaning how some Mathematics calculations have evolved with time regarding its resolution path. Even the complexity level can increase as long as spreadsheet is used, since this tool would help solving very complex cases.

The five factors that make up the structure of variables that represents the perception towards Mathematics are indicators of a practical and statistical significance, this means that subject teachers can take them into consideration when making teaching-learning strategies in a way that student is more interested in Mathematics and at the same time, help to reduce the desertion rate of the subject.

5. Discussion and Conclusion
Once the results were analyzed, it can be concluded that there is a structure of variables showing the perception that students from the Business School of Cristóbal Colón University have towards the subject of Financial Mathematics. The results allows seeing that teaching is more
dynamic thanks to Information Technologies, which have opened the way to a different teaching manner, this leads to a change in the student´s perception towards certain topics.

In the case of Mathematics, each of the proposed variables contributes to increase the student´s perception towards this discipline and thus, there is a change of perception when classes are done as workshops, including History of Mathematics.

Regarding the History of Mathematics in the teaching process, it is possible to mention a quote by Bell (1985).... No subject loses more than when it is divorced from its history, as it happens in Mathematics. The former leads us to think about the importance of this element in relation to the teaching didactic of this discipline.

Furthermore, the use of simulators has a positive effect in student’s learning and it can be inferred that by using such tools, classes become more attractive.

The same happens for spreadsheet programming, virtual platforms and communities, meaning that when using tools offered by Information Technologies there is a change of the students’ attitude towards this discipline. Besides, if Mathematics History is integrated to the program, the student better understands the importance of the subject in daily life (Bell, 1985; González-Urbaneja, 2004).

These results match the ones stated by Pazos et al. (2001) and Salinas (2003), who claim that virtual communities are spaces of ideas interchange that favors learning; Goldenberg (2003) mentions that CIT can be used in Mathematics to avoid giving a poor impression of this subject. Moursund (2003) maintains that spreadsheet programming offers a fundamental environment for problem solving and the representation of numeric data. Also, the results match the ones by Fauvel (1991) and Clinard (1993) and Furinghetti (1997), who support the argument of including Mathematics History as an important component in the T-L-E processes, since it helps increase the appreciation of the importance, usefulness and value of Mathematics.

In relation to the afore-mentioned, it is necessary for the teacher of this subject to include in its curricula, topics and activities using the CIT and including the variables from the techno-pedagogic model, in such a way that learning is improved but above all that it is oriented to change the student’s perception towards Mathematics in order to attain a favorable perception towards Financial Mathematics.

6. Proposal of the techno-pedagogic model

Next there is a step by step description of the techno-pedagogic model proposal for the teaching process of Financial Mathematics, applicable in college education scenarios. It is relevant to highlight that this subject is taken for the study plan of college careers from the economic-administrative area because of its connotation and application field.

6.1. Development of traditional class

In a traditional Mathematics teaching session, each of the topics that are part of the subject’s curricula are approached. To explain a class taught in the traditional way, we will take as example the topic of arithmetic gradient (\(G_a\)) and geometric gradient (\(G_g\)) to solve a hypothetical case. First, the teacher presents the formulas regularly used for the development of the intended case that will be solved, trying to explain its composition and the meaning of the elements that make up each theorem.

**Arithmetic gradient**

To know current value

\[
VA = \left[ \left( R_p + \frac{G_a}{1/m} \right) \left( \frac{1 + \frac{i}{m}}{\frac{i}{m}} \right)^{n} \right] \frac{n^*G_a}{1/m} \left( \frac{1 + \frac{i}{m}}{\frac{i}{m}} \right)^n
\]

To know future value

\[
M_{ga} = (R_p + \frac{G_a}{1/m}) \left[ \left( \frac{1 + \frac{i}{m}}{\frac{i}{m}} \right)^{n} \right] \frac{n^*G_a}{1/m}
\]

**Geometric gradient**
To know the current and future value, the formulas to be used are different, that is, they depend on the progression reason \((G_g)\) matching the factor \(\left(1 + \frac{i}{m}\right)^m\) or \(\left(1 + \frac{i}{m}\right)^n\) for arithmetic or geometric gradient (of the sum of payments).

**Where:**  
- \(M_{ga}\) or \(VF_{ga}\): Future value or amount of a series of fees with arithmetic or geometric gradient (of the sum of payments);  
- \(M_{gg}\) or \(VF_{gg}\): Future value or amount of a series of fees with arithmetic or geometric gradient (of the sum of payments);  
- \(A\) or \(R_p\): Annuity or periodic rent (even fee or annuity);  
- \(VA_{ga}\): Current value of the set of periodic rents;  
- \(i\): nominal interest rate;  
- \(m\): Capitalization (for its type, monthly, bimonthly, etc. the rate is divided: an example of it, if there is a monthly capitalization rate of 12 % = \(\frac{12\%}{12}\);  
- \(n\): Time; \(G_a\)= is the arithmetic gradient; \(G_g\)= is the geometric gradient; \(R_{p1}\)= Annuity or periodic rent number 1.

Once the professor explains the theorem and its application fields, exercises are done in the traditional Mathematics teaching class. As an example of this, let’s assume that it is required to know the amount that has been accumulated in an investment fund established by 10 monthly deposits that grow at a \(G_g\): 5.5 % rate, being the amount of the first deposit \$1,000.00 with a 12 % monthly capitalization rate. From the formula:

\[
Si \left(1 + \frac{i}{m}\right)^m G_g : \quad M_g = R_p \left(1 + \frac{i}{m}\right)^m \frac{(1 + \frac{i}{m})^n - (1 + G_g)^n}{\frac{i}{m} - G_g}
\]

Where: \(R_{p1} = \$1,000.00\); \(G_g = 5.5\%\); \(n = \) number of fees 10; \(i/m = \frac{0.20}{12} = 0.01666667\)

which is the same as 1.6666667% (capitalization rate in \(m\) periods per year).

It is solved as:

\[
M_g = 1,000.00 \left(1 + \frac{0.20}{12}\right)^{10} \frac{(1 + \frac{0.20}{12})^{10} - (1 + 0.055)^{10}}{0.20 - 0.055}
\]

\[
M_g = 1,000.00 \left(1.01666667\right)^{10} \frac{(1.17973879) - 1.07814446}{0.01666667 - 0.055}
\]

\[
M_g = 1,000.00 \left[14.0142386\right] \quad Mg = \$14,014.24
\]

If the fees where past due with \(G_g\), it is derived from the formula:

\[
Si \left(1 + \frac{i}{m}\right)^m G_g : \quad M_g = R_p \left(1 + \frac{i}{m}\right)^m \frac{(1 + \frac{i}{m})^n - (1 + G_g)^n}{\frac{i}{m} - G_g}
\]

Where: \(R_{p1} = \$1,000.00\); \(G_g = 5.5\%\); \(n = \) number of fees 10; \(i/m = \frac{0.20}{12} = 0.01666667\)

It is solved as:

\[
M_g = 1,000.00 \left(1 + \frac{0.20}{12}\right)^{10} \frac{(1 + \frac{0.20}{12})^{10} - (1 + 0.055)^{10}}{0.20 - 0.055}
\]

\[
M_g = 1,000.00 \left(1.01666667\right)^{10} \frac{(1.17973879) - 1.07814446}{0.01666667 - 0.055}
\]

\[
M_g = 1,000.00 \left[13.7844969\right] \quad Mg = \$13,784.50
\]

In the same class dynamic, it is calculated inversely, that is, the exercise about the current value of \(R_p\) is done, so that in order to obtain an amount of \$14,014.24 what must be the amount of
the first of 10 periodic fees (n=10) that increase by 5.5% with an interest rate of 20% monthly capitalization?: Now it is solved in its format of prepaid and past due fees, from the formula:

\[
S_i \left(1 + \frac{i}{m}\right)^i G_g : \quad M_g = R_p \left(1 + \frac{i}{m}\right)^{1 - (1 + G_g)^n} \left(1 + \frac{i}{m}\right)^n - (1 + G_g)^n
\]

Prepaid are calculated

\[
\begin{align*}
\$14,014.24 &= R_p \left(1 + \frac{10 \times 0.055}{12}\right)^{12} \left(1 + \frac{10 \times 0.055}{12}\right)^{12} - (1 + 0.055)^{12} \\
&= \left(1 + 0.01666667\right)^{12} - (1 + 0.055)^{12} \\
&= \left(1 + 0.01666667\right)^{12} - \left(1 + 0.055\right)^{12} \\
&= 13.78449691
\end{align*}
\]

Past due are calculated:

In order to obtain an amount of $13,784.50, what must be the amount of the first of 10 periodic fees (n=10) that increase by 5.5% with an interest rate of 20% monthly capitalization?:

\[
\begin{align*}
\$13,784.50 &= R_p \left(1 + \frac{10 \times 0.055}{12}\right)^{12} \left(1 + \frac{10 \times 0.055}{12}\right)^{12} - (1 + 0.055)^{12} \\
&= \left(1 + 0.01666667\right)^{12} - (1 + 0.055)^{12} \\
&= \left(1 + 0.01666667\right)^{12} - \left(1 + 0.055\right)^{12} \\
&= 13.78449691
\end{align*}
\]

If we now want to know the term, we need to solve for it in the formula from the amount of a series of prepaid fees with geometric gradient:

\[
M_g = R_p \left(1 + \frac{i}{m}\right)^{1 - (1 + G_g)^n} \frac{(1 + i/m)^n - (1 + G_g)^n}{1/m - G_g}
\]

So, we have what

\[
M_g = \frac{(1 + i/m)^n - (1 + G_g)^n}{R_p \left(1 + \frac{i}{m}\right)^{1 - (1 + G_g)^n}}
\]

The denominator of the right set goes multiplying to the left, thus obtaining:

\[
\frac{M_g}{R_p \left(1 + \frac{i}{m}\right)^{1 - (1 + G_g)^n}} \times (1/m - G_g) = \left[\left(1 + \frac{i}{m}\right)^n - (1 + G_g)^n\right]
\]

The gradient goes as sum to the left, now we need to fulfill the following equation:

\[
(1 + G_g)^n - (1 + \frac{i}{m})^n = \frac{M_g}{R_p \left(1 + \frac{i}{m}\right)^{1 - (1 + G_g)^n}} \times (1/m - G_g) = 0
\]

In summary, we might think in the traditional system this procedure seems tedious or a nuisance for the student, which creates an apparent rejection towards the teaching-learning process. Also, it is very common for the student not to memorize the steps of each formula and that causes him/her to forget them, hence the proposed techno-pedagogic model, which measured the perception of 52 students, is perceived the students as highly accepted for their college training process, but how is the techno-pedagogic model visualized for the teaching of Financial Mathematics?

We must remember that the variables from the EAPHMF scale, which are: contents of Mathematics history, workshop class, use of spreadsheet, design of financial simulators and virtual learning communities. From these variables, the course is planned contextualizing first the topics of Financial Mathematics, a brief outline of its history and how these topics were gestated. The class is organized in teams of three or four participants, encouraging teamwork in each of them.
It is proposed that in the spreadsheet each of the mathematical theorems are programmed. Its transformation to programming is done at all times by the student aided by the teacher. Programming goes from basic Excel to advanced Excel and/or in Visual Basic, or in any other program that allows programming for the development of calculations and interacting in virtual platforms with other participants from other parts of local and/or global context. Each exercise is contextualized in its current reality, meaning that scenarios are created as they happen in daily life, attempting to adapt each topic to a requirement or specific activity of people or company’s financial activities.

### 6.2. Session with the Techno-pedagogic model for the teaching of Financial Mathematics

Next there is a sequence of the process using the same data: \( R_{PI} = $1,000.00 \); \( G_g = 5.5 \% \); \( n \) = number of fees 10; \( i/m = 0.20/12 = 0.01666667 \) (capitalization rate in \( m \) periods per year). Then, we proceed to programming and for that, the templates must be designed considering all the variables of the formula used for the calculations, then the exit sell needs to be programmed in Excel, an example is: \( M_{gg} \) past due yearly fee \( = (F4) \times ((1+F7)^{F6})-(1+F5)^{F6})/(F7-F5) \) and \( M_{gg} \) prepaid yearly fee \( = (F4) \times (1+F7)^{F6} (((1+F7)^{F6})-(1+F5)^{F6})/(F7-F5) \).

![Excel spreadsheet for gradients](source)

**Fig. 2.** Excel spreadsheet for gradients

Source: own

Other example of Excel programming with a higher complexity level would be as follows:
This sort of templates vary depending on the design that each student gives to the simulator, in the same template a series of icons are imbedded and when clicked an instruction is opened for the filling of data. For instance, in the simulator shown on Figure 3, the icons appear and when they are clicked, a menu is opened to use the tool.

**Guide for the use of Financial Simulator.**
1. Use the Arithmetic Gradient formula for Future Value.
2. Introduce in the box “Rate” the given interest rate.
3. Choose if the rate is yearly, monthly.
4. Choose the type of interest, if it is ordinary or exact (remember that for an exact calculation there are 365 days and for an ordinary calculation, 360 days)
5. If you click on the symbol a help message will be opened to present which data must be introduced in each field.

Other simulator model made in Java programming by the students who have taken the Financial Mathematics subject with the use of the techno-pedagogic model is shown next:
The field of Mathematics is very wide, in fact, it could be said that it is present in every activity of human beings, its use and application in daily life has aided scientific and technological development. Likewise, it has also represented an obstacle for a large part of the population during their academic training, which goes from basic education to graduate or post-graduate levels, since its apparent rejection can be said to be due to being poorly understood by those who are students at the time.

Regarding Mathematics, some reflections can be taken from an interesting interview done by Villafrades (2016) to professor Dúwamg Alexis Prada M., a known figure in the teaching of Mathematics, specifically in the subjects of algebra, differential calculus, integral calculus, multivariate calculus, differential equations, analytic geometry, geometry and trigonometry, Mathematics applied to microbiology, Mathematics for biologists, topology for mathematicians.

For professor Dúwamg, Mathematics is a beautiful language that starts from the abstract to make inferences in several branches of knowledge, thus, allowing the approach of study phenomena by using models and theorems, and therefore, contributing to the explanations required from all society activities.

This reflection is relevant when we speak about the importance of being able to contextualize each of the models used for the teaching of Mathematics, because beyond presenting it in its abstract form, it is advisable to relate it to real and tangible cases that are present in the context, searching for a logical explanation.
The knowledge disciplines of Economy and Finances are, among others, a very demanding field of Mathematics application, hence the importance of understanding that Mathematics are present in daily life, in everything we see around us, it will always be present at a time or activity when we need a calculation, even if we do not use complex formulas or specific algebra equations.

“In the language of mathematics, equations are like poetry: They state truths with a unique precision, convey volumes of information in rather brief terms, and often are difficult for the uninitiated to comprehend. And just as conventional poetry helps us to see deep within ourselves, mathematical poetry helps us to see far beyond ourselves—if not all the way up to heaven, then at least out to the brink of the visible universe.”

Michael Guillen – Five equations that changed the world: The power and poetry of Mathematics

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References


