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Pre-school children's understanding of Mathematical patterns

Abstract

This study explored the patterning abilities of eight children between the ages of four and five-and-a-half. Task-based interviews were conducted where children were required to produce their own repeat pattern, to copy and extend a given repeat pattern, and complete missing parts of a repeat pattern. The findings show that most children were able to copy and extend a pattern, but few could produce their own repeat pattern. While most children tended to focus on "what comes next", some children paid attention to pattern structure, but were not able to isolate the pattern element.

Keywords: mathematical patterns, repeat patterns, pattern element, early childhood numeracy

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Introduction

> Emma is four-and-a-half and loves to make necklaces. She carefully threads the beads onto a string, starting with orange then pink then green; then orange, another pink, and a green, and so she continues. She does not yet know that the twelfth bead will be green but once she has threaded the eleventh bead, she will know what comes next.

Threading beads is a typical example of young children's experience of pattern. They soon learn to identify a mistake and to correct it, but few are able to predict the colour of a bead in a particular position and few can easily determine how many times the pattern has been repeated because they do not see orange-pink-green as the basic unit of the pattern. They simply see orange, pink, green, orange, pink, green, and so on.

The exploration of pattern is a key element of doing mathematics. Indeed Steen (1990, p. 5) described mathematics as the "science and language of pattern". English (2004) argues that the development of children's mathematical reasoning is dependent on *inter alia* their ability to identify, extend and generalise patterns. Pattern is a phenomenon that young children encounter daily in many different forms and contexts. They may recognise patterns in fabric, in designs of buildings, in clapping to music, and in their own play.

Papic and Mulligan (2005, p. 609) define pattern as "a numerical or spatial regularity" and suggest that pattern structure is constituted by "the relationship between the various components of the pattern". They distinguish between different types of patterns, as follows (Papic & Mulligan, 2005; 2007):

- *Repeating patterns* contain an element that recurs continuously. This pattern element or unit of repeat contains one cluster of the individual components that form the repeat. The following pattern can be coded as an ABCD pattern with three repeats: □◆⌘●□◆⌘●□◆⌘●. The unit of repeat is delimited as □◆⌘●. Repeating patterns range in complexity and may contain more than one variable, for example in a pattern with beads the variables may be shape and colour. Repeat patterns are particularly important because of the links with iteration in multiplication and measurement (Mulligan & Mitchelmore, 2009).
- *Spatial structure* patterns refer to patterns where the individual components are organised in a recognisable physical arrangement, such as a triangle of dots or a rectangle of stars.
- *Growing patterns* increase or decrease in a systematic way. They are frequently associated with spatial structure patterns where the spatial structure increases in size, such as triangles of dots where successive triangles have a larger number of dots and the increase in dots is systematic. However growing patterns could also take the following form: ABAABAAABAAAAB.

It seems fair to say that pre-school children's experience of pattern tends to focus mostly on colour and shape, as well as repeating patterns. Consequently, it may be

more useful to consider pattern at this level as the repetitive regularity of colour, quantity and/or shape.

Literature review

There can be little doubt that mathematical patterning is an essential part of mathematics. Cuoco, Goldenberg and Mark (1997) identify 'pattern sniffing' as a habit of mind that mathematicians develop, and argue that learners of mathematics also need to develop this competence, where they deliberately look for regularities in the mathematics under investigation, and where they see regularities in their daily lives which may then prompt a question that can be investigated through mathematics. Patterning skill is particularly important for the development of algebraic thinking (Warren, 2005) as well as functional thinking (Blanton & Kaput, 2004). For example, typical tasks in introductory algebra require learners to generate algebraic expressions as generalisations of visual (growing) patterns. These algebraic representations express functional relationships in the pattern.

Pattern exploration is also a fundamental component of young children's development, and extends beyond mathematical patterns to include identifying regularity and variance across the pre-school curriculum in diverse areas that include language, art, science, music and physical education (Fox, 2005). Ginsburg, Inoue and Seo (1999) investigated the relative frequency of different kinds of mathematical activity in a pre-school setting and found that children in their study were working on activities involving shape and pattern for 30% of the observed time.

Economopoulos (1998, p. 231) suggests that the key shift for children to make with repeating (red-blue) patterns is to move from seeing that "red comes after blue and blue comes after red" to seeing that "red-blue is a fundamental unit of the pattern". Once children have made this shift, she argues they will then be able to use the known information to predict the unknown. Thus, they will shift from a focus on "what comes *next*" to see the structure of the pattern, and will recognise that it comprises repeating units that are made up of two or more basic elements.

Economopoulos describes tasks used in a professional development programme with kindergarten teachers, where participants engaged in small action research interventions with their own children. One task involved a pattern of four tiles, alternating green and blue, with the next four tiles covered by cups. Someone was required to point to a particular cup and a child was required to say what colour tile was under the cup. While some children read the entire green-blue pattern from left to right to work out the answer, others simply looked at the last visible tile and worked out the alternating pattern, saying something like: "I looked at the last colour, and that was green; so in my mind I just went green-blue-green and knew that the next one had to be blue" (p. 231-232). Economopoulos suggests that those who worked from the start of the pattern were using a "what comes after what" strategy, whereas those who looked at the last visible tile were developing some understanding that green-blue was the smallest unit of repeat in the pattern.

However, there has been very little research from a mathematical perspective on young children's patterning abilities (Jones & Peters, 2004). Most of the research that has been done has occurred in the last decade and has come from Australia (e.g. Fox, 2005; Mulligan & Mitchelmore, 2009; Papic & Mulligan, 2005; 2007; Waters, 2004). One of the consistent claims of this body of research is that children have the potential to perform more complex patterning tasks than they are typically given, but this potential will only be realised if teachers pay more deliberate attention to pattern structure. For example, in the pre-school years where repeat patterns are dominant, teachers need to emphasise the unit of repeat and to provide children with the language to talk about it.

Waters' (2004) case study of two classes of five and six year olds, involved two teachers who both acknowledged the importance of mathematical patterning in the curriculum. In one class children referred to the 'patterns' on their clothing. One noted an AB pattern around the cuff of her jeans, another referred to the matching pattern (i.e. symmetry) of the butterfly on her shirt, and a third child noted the pattern on his shorts which consisted of a visually random design. The teacher referred to all three instances as patterns without noting the characteristic differences. This lack of attention to carefully defining what a pattern is may result in children considering any kind of picture as being a pattern. In another activity involving a worksheet with a repeating AB pattern, children failed to identify the repetition of shapes and the teacher did not intervene appropriately to define the pattern for them. Waters observed ten patterning activities across the two classes and concluded that the teachers did not work with a consistent definition of patterning, demonstrated limited understanding of patterning language, did not increase the complexity of the patterning tasks, and did not integrate pattern activities across the curriculum. By implication Waters suggests that these are important aspects of mediation to support the development of learners' patterning abilities, and we would agree.

Papic and Mulligan's (2005; 2007) study shows that interventions with pre-school children had a substantial impact on the children's performance on patterning tasks. The study initially focused on children in their final year of pre-school at two day-care centres. An intervention was implemented in one of these centres and they assessed children's progress using three interview-based assessments one of which was done after the first year of formal schooling. They concluded that the children who participated in the intervention were able to develop more sophisticated patterning skills, and their patterning skills continued to be more advanced than the non-intervention children after the first year of formal schooling. In particular they found that the ability to identify the unit of repeat and hence the structure of simple patterns was a key difference between the two groups of children. Whereas the non-intervention children focused mainly on alternating colours in solving patterning problems, the intervention children could identify the unit of repeat and used this to solve tasks. For example, when copying towers of alternating colours with an ABABAB pattern, intervention children identified the AB pattern element and the number of repetitions, while non-intervention children simply remembered the alternating

colours: red, blue, red, blue, red, blue in a similar way to the responses discussed by Economopoulos (1998). This simple, yet profound, difference shows how the intervention children were paying attention to structure, and the research suggests that without teacher intervention, children do not readily do this.

Most of the research we reviewed has focused on the overall performance of groups of learners. By contrast, our research focuses on individual children and their performance on a range of tasks involving repeating patterns. This enabled us to get a more nuanced perspective on some of the complexities of children's growing understanding of repeating patterns. While the children had been exposed to patterning work across the curriculum, they had not yet been explicitly taught the notion of 'pattern repeat', nor to identify the pattern element. Consequently, our findings reveal what these children could do based on general exposure to patterning. In particular, we will focus on the extent to which children have a sense of pattern repeat.

Research design

Setting and participants

The research was conducted in an independent pre-primary school in the northern suburbs of Johannesburg, which draws children from middle to upper socio-economic backgrounds. The first author is the principal of this school, the class teacher of the children who participated in the research, and the primary researcher. Although the original intention was to select a sample from the class, there was so much enthusiasm to "play the games with Erica" that, with parents' consent, all 17 children were given the opportunity to complete the tasks in a task-based interview setting (Goldin, 2000). This provided additional opportunity to pilot the interviews. We report here on the responses of eight children, all of whom had been at the school since the age of three, and were thus in their third year of pre-school education. They ranged in age from four years and five months to five years and three months. All children spoke either English or Afrikaans as their home language. Prior to the data collection, which took place four months into the school year, the children had done pattern work weekly. This work included pegboard activities where they were required to work individually to copy a given 'picture' on a card by placing pegs in the appropriate places on their pegboards. These pictures included various patterns. Children also threaded beads onto sticks and arranged blocks or counters in patterns as a group activity. Children were familiar with some of the apparatus that were used in the interview, but had not worked with the card-based activities that are described below.

The data were collected in a room adjacent to the children's classroom. Each child sat individually with the teacher-researcher, and each interview took 20 – 30 minutes. Careful attention was taken to avoid interrupting the children as they worked on the tasks. Questions were only asked of them after they had completed each task. It was important to give them the opportunity to explain what they were doing immediately after completing the task, as, due to their age, they may not have remembered this at

a later stage. In some cases the children talked aloud as they executed the task, which gave us insight into their thinking as they worked. All interviews were video-recorded, with the camera focusing on the child's hands and the manipulatives available on the table.

The tasks

Each child was asked to complete a series of tasks. The first task was to produce their own pattern with 'touch and count' (unifix) cubes. The second task was to fill in or complete the missing part of a repeat pattern, and the third task was to copy and repeat patterns using beads on dowel sticks. Our discussion views the task from our adult perspective. We are well aware that the children were not necessarily conscious of all the aspects we mention. Conversely, it may be that the children saw features of the task, which we are not aware of. One obvious issue is that the children in the study had not as yet learned to read, and so they were not yet conditioned by the Western convention of reading from left to right.

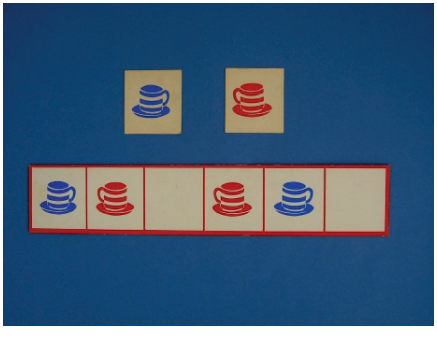
Task one – Building a pattern

Each child was given the same instruction: to create a pattern using blocks and then to repeat the pattern. The instruction stipulated that they could make any pattern using any amount of blocks. Our intention was that this task would give insight into the children's ability to produce a pattern. Also, being an open-ended task, we believed that this would help the children to settle down in the interview.

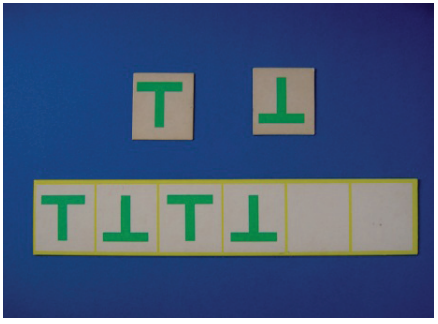
Task two – Fill in and complete patterns

This task investigated repeat patterns where children were asked to complete a given pattern. The materials used for this task came from an 'educational game' where each card has six divisions, four with pictures, while two are blank. We shall refer to this as the 'strip'. Two loose picture cards are provided to place on the strip, in order to complete the missing parts of the pattern. Four tasks were chosen from this game and were presented one at a time. We refer to these as Tasks 2a, 2b, 2c and 2d.

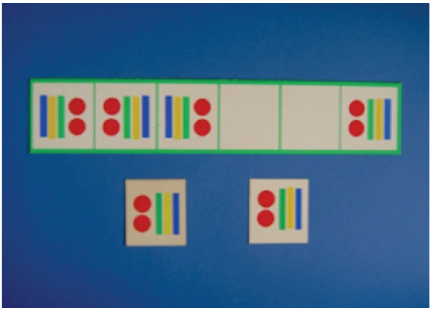
Task 2a contains an AB pattern of red and blue cups and saucers. Blue cups have the cup handle on the right and red cups have the cup handle on the left. Thus the orientation of the cups produces the same AB pattern as the colour. The structure of the strip is AB_BA_. Where _ indicates a blank place for the child to place a loose card.



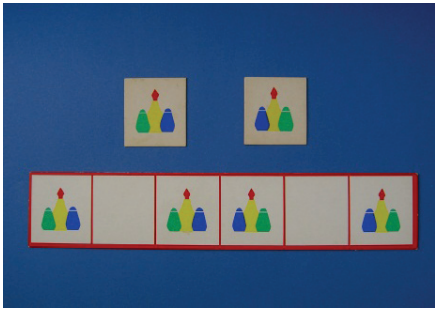
Task 2b contains an AB pattern of green T-shapes. Here the same colour has been used throughout but the orientation of the T alternates. Thus all the loose cards are identical and the pattern emerges by changing the spatial orientation of the loose cards. The structure of the strip is ABAB_ _.



Task 2c is the most visually complex task with an ABA_ _ B structure on the strip. As with Task 2b, the loose cards are identical and the task requires a spatial orientation of the loose cards.



Task 2d consists of pictures of three skittles. The colours of two skittles are laterally reversed on some positions in the strip so, as with task 2a, the loose cards are different. The structure of the strip is A_AB_B.



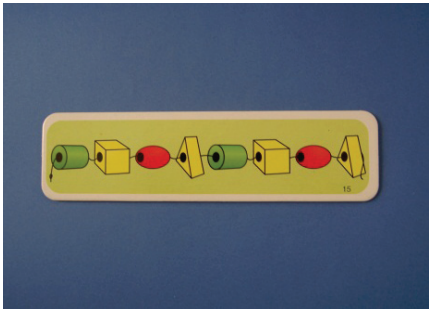
The strip in Task 2b is the one that provides clear indication of the pattern element with two repeats, and since the open spaces are at the end of the sequence, this task required pattern completion rather than filling in the missing parts of the pattern. The given information made this the easiest task for the children and with hindsight; perhaps, we should have given this task first. Neither Task 2a nor Task 2d make the alternating repetition explicit. We anticipated that some children might place the loose cards so as to group like colours together, thus producing an ABBBAA pattern for Task 2a, and an AAABBB pattern for Task 2d. The symmetry of this arrangement on Task 2c could provide a strong indicator to children that this is a correct response. However, in both cases we are treating alternating AB patterns as the correct response. The repeating AB structure on strip 2c was less obvious than on the other cards, because of the more complex image on each card. We wanted to see what criteria children

would use in their decision-making to complete the pattern, for example, would they draw on the spatial structure of the strips and/or the given pattern to place the loose cards, and if so, what aspects of the design would they focus on? The loose cards for each task were placed randomly on the table. We do not know what impact this might have had on each child's responses. For example in the diagram for Task 2b above, the cards are placed in the correct order and orientation. A child could take this as a cue to move them both into position without making any changes to the order and spatial orientation.

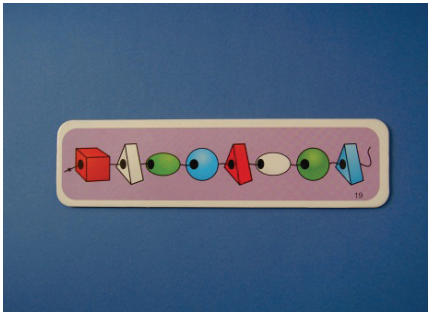
Task three – Copy and extend patterns

This task also focused on repeat patterns. Children were given different picture cards, one at a time, containing a pattern of beads on a string. The beads differed in colour and shape. The children were then required to select the appropriate beads to copy the pattern, and then to repeat the pattern several times. This required them to distinguish shape and colour. Children were given trays with all the necessary beads in each colour and each shape as shown alongside. We gave the children dowel sticks for threading the beads (rather than string) to capture their actions more clearly on camera. We report here on only two of these tasks, which we refer to as Task 3a and 3b.

Task 3a contains an ABCD pattern that is repeated twice, with both colour and shape changing. However, if one focuses only on colour, then the pattern is ABCB. Children had to recognise the pattern element, choose the correct shape and colour, and notice the repetition.



Task 3b contains an ABCDEFGH pattern with no repeats. However, if one focuses only on colour, then the pattern element is ABCD and there are two repeats of the pattern. If one focuses on shape only, there is an ABCABCA structure with a cube on the left end. If children focused on both colour and shape, they had to recognise that there were no repeats on the card and then had to repeat the entire ABCDEFGH pattern.



After the interviews had been conducted, we selected eight children for the analysis. In making this selection, we wanted to include responses that covered a wide range of performance on the tasks, as well as instances of inconsistency where children were able to complete some tasks correctly, but not others. We anticipated that potential inconsistencies and instability in children's patterning ability might give insights that have not as yet been reported in the literature. We excluded interviews where children appeared to lose concentration too soon and where the technical quality of the video footage made analysis difficult. Interviews were transcribed, and analysis began with repeated observations and descriptions of the video footage together with the transcription.

In analysing the video footage, we paid attention to children's actions and their talk. For example, we noted what children did when they started each task; whether they worked from left to right or right to left, whether they turned their dowels sticks around during the task; and whether they pointed to particular parts of the card. Since the children were not easily able to express their thinking, we had to match carefully their talk with their actions, such as pointing to particular shapes or parts of the pattern. There were several instances where what the children said, did not match with what they were actually doing. This meant the transcriptions could not be used without the footage.

The initial focus of the analysis was on the extent to which each child was able to complete a task correctly. We then focused on each task, comparing the children's productions in order to identify similarities and differences. Finally we focused on each child in order to explore consistencies and inconsistencies in their performance across tasks.

Findings and discussion

We begin with an overall summary of the children's performance on each task. Thereafter we discuss the strategies of three children in more detail, in order to highlight some of the key findings of this research.

Task one – Building a pattern

Three children produced repeat patterns. They worked with ABCD structures and their number of repetitions ranged from two to seven. Despite producing the repeat patterns, the children's talk about their patterns indicates that some were not attending to the pattern element. They were asked questions such as: "How many times have you done your pattern?" or "Explain to me how your pattern has gone". In response, Grant and Janet both counted all the blocks they had used. Jack counted the numbers of each different coloured block. Only Tessa was able to identify the pattern element:

> Researcher: Do you know how many times you have done the pattern?
>
> Tessa: (Pointing to each pattern element of four counters connected green, blue, red, yellow)
>
> Ok, that's one, that's two, that's three, that's four, that's five, that's six, that's seven.

Jack repeated his yellow, red, green, blue cluster twice and then added another yellow. These nine blocks were added to the bottom of his tower each time. Then his tower got too tall so he turned it around and started adding cubes to the top. He proceeded to add red then green following his earlier pattern. Then he noticed that there were "not so much blues" and so he added a third blue block. So Jack built his tower systematically using the same order for all 12 blocks. However, when he started adding to the top of the tower, he broke his ABCD pattern. Thus his final tower looked as follows, where the numbers indicate the order in which the block was added to the tower:

D	C	B	A	B	C	D	A	B	C	D	A
12	11	10	1	2	3	4	5	6	7	8	9

Jack's case highlights the fact that the production of a correct repeat pattern was dependent on the way in which he used the manipulatives. He selected the colours in the same order each time, but did not connect them to the tower in a consistent manner. This resulted in an error in his pattern. If he had placed each block separately on the table as some other children did, it is possible that he would have produced the intended repeat pattern.

The other four children who did not produce the expected repeat patterns still worked in deliberate and systematic ways, with three of them using four different colours. Pam systematically took blocks out of the containers (yellow, red, green, blue, yellow, red, green, blue, red, blue) and arranged them in two rows of five blocks, but the arrangement of the cubes did not produce a repeating pattern. Despite this, she appeared to be reflecting carefully on her arrangement because she changed the positions of the last two (red and blue) blocks at one stage. John arranged eleven blocks into a 'pattern snake', but with no repeating pattern element. In both cases the children did not repeat a colour in their 4-block groupings. This suggests that both

children had some sense of the form of pattern – in terms of arranging the blocks – but could not produce recognisable repeating patterns. Janet arranged seven yellow blocks in a circle while Ann arranged four blocks in a cross-shape with a gap in the middle.[1] We would argue that neither Janet nor Ann attempted to produce repeat patterns and it may be that, as in Waters' (2004) study, they interpreted the word 'pattern' as 'picture' and so they produced pictures that did not contain the structure of spatial and/or repeat patterns.

The findings from this task suggest that an ability to produce a repeat pattern does not necessarily imply the ability to identify and talk about the pattern element. In addition, children's production of a repeat pattern is also dependent on their definition of pattern and their facility with the artefacts they are using.

Task two – Fill in and complete patterns

2(a) *Cup and saucer pattern* (Task 2a) – Five children identified the alternating AB pattern correctly. The other three grouped the reds and blues together and so ended up with BRRRBB. It was not immediately evident to these three children that they were required to alternate colours and/or orientations of the cups. All children focused on colour – there was no reference to the different orientations of the red and blue cups.

2(b) *T-shape pattern* (Task 2b) – This was the easiest of the strips since children had to complete the pattern, rather than place cards in the gaps. Seven children completed it correctly. Initially Grant placed the T upside down in position 5 and right way up in position 6. But after some interaction with Erica, he changed the orientations of both these cards.[2]

2(c) *Red dot and stripe pattern* (Task 2c) – Four children completed this task correctly – with red dots facing each other. Some justified their arrangement clearly. For example, Tessa talked about the red dots "facing each other". In other cases we could not tell what criteria were being used to justify the correct arrangement. For example, John who placed the cards very quickly simply said, "it was the right one". Both Janet and Bev placed the loose cards with the red dots on the right hand side. This produced ABAAAB. Bev justified her arrangement by saying that the red dots had to be next to the blue stripes. This is one of several examples where the children appear to focus only on part of the pattern and produce an arrangement that contains a partial pattern. Their reasoning may thus be valid for a section of the pattern, but is invalid if one considers the entire pattern. In some instances this inconsistency was pointed out to the child, but seldom resulted in a change to their arrangement.

2(d) *Skittles* (Task 2d) – The skittle task has the AB pattern element in the centre of the strip, but this is not obvious. Only Ann produced an alternating pattern. The other children grouped all the green-yellow-blue cards together and the blue-yellow-

green cards together, producing an AAABBB pattern. One child placed matching cards over the pictures on the strip.

All these tasks required the children to think about what loose card should be placed in the open spaces. With the exception of the T-task, this forced them to ask the question "what goes here" rather than the simpler question of "what comes next". In order to place the correct card in a particular position, they need to pay attention to all the filled spaces, which requires them to look to the left and the right of any open space. This places high demands on children of this age and, as discussed above, some children were not able to do this. In a later section we will discuss how Ann made use of "what comes here" thinking to complete Tasks 2b and 2c correctly.

Task three – Copy and extend patterns

Task 3a had an ABCDABCD pattern, therefore the repetition was given. Seven of the children were able to copy the pattern, but only five were able to extend it to several pattern repeats. Pam did not reproduce the given pattern. Instead she made up her own EFGH pattern, which she repeated five times. It contained the same colours as the card, but only one of the elements was identical in shape and colour to the card. She had struggled with all the tasks and did not produce the expected pattern for Task 3b either, but her attempt at this task showed that she could produce her own repeat pattern.

Task 3b had an ABCDEFGH pattern; no repetition was given – which meant that the children had to work out how to repeat the pattern themselves. Five children were able to copy and repeat the pattern. Janet completed Task 3 correctly, working slowly and systematically. She identified the colour and shape of each element that she threaded onto the dowel stick, yet there was no evidence in her talk of recognising repetition in the pattern. She arranged the bead trays in the same order as the beads on both cards. This meant that if she took a bead from each tray in order, she would produce the correct pattern. Since she set up this arrangement of trays for *both* cards, we assume that the order of the trays corresponding to the cards was not coincidental. Thus it may be that she recognised the value of working systematically from the card, and this contributed to her success. Once she had completed each pattern, she was asked how many times she had done the pattern. For Task 3a, she hesitantly pointed to the green and red beads and seemed to skip over the yellow beads, but she did not appear to be counting all green or all red, or both. For Task 3b she appeared to be choosing beads randomly as she counted. She seemed to focus mainly on rounded beads irrespective of shape or colour, but did not count all of them.

In order to succeed with Task 3, children had to be able to keep track of where they were on the card in relation to the beads threaded on the stick. Their focus continually shifted between the card, the trays of beads and the dowel stick. If a child could identify the pattern of repeat in Task 3a, this would assist in keeping track of progress. For a child who did not recognise the repetition, the task essentially became a copying task, which could be successfully completed by focusing on what bead

comes next. In Janet's case the deliberate order of the trays was enabling, but other children appeared to place the trays in a random order, which required them to pay more attention to the sequence on the card.

The results for each task have been summarised in the table below. We are well aware that the table does not tell the complete story of the details of each child's performance, but it does provide a useful overview.

Responses to all tasks									
	Task 1	Task 2				Task 3			
Child	Produce own repeat pattern	Cup and saucer	T-shape	Lines and dots	Skittles	Copy pattern with repeat given	Repeat pattern	Copy pattern with no repeat given	Repeat pattern
Tessa	√	√	√	√	X	√	√	√	√
John	X	√	√	√	X	√	√	√	√
Ann	X	√	√	√	√	√	√	√	√
Janet	X	√	√	X	X	√	√	√	√
Bev[3]	√	√	√	X	X	√	√	_	_
Grant	√	X	X	√	X	√	X	√	X
Jack	X	X	√	X	X	√	X	√	X
Pam	X	X	X	X	X	X	X	X	X
Total Correct	4	5	7	4	7	7	5	6	4

Based on overall performance on all tasks, the children could recognise and match attributes of objects, such as size, shape and colour; and they could differentiate and sort objects. Four children could produce their own repeat patterns with several repeats of the pattern element, but all had difficulty in describing their patterns. Seven children could copy a given pattern and six could extend a given pattern. However, only one child could point to the pattern element. The remaining seven showed different levels of awareness of pattern structure. The data in the table suggests that Janet had a better understanding of pattern than Grant, since she completed more tasks correctly. However, when one looks more deeply at her responses to all tasks, there is little evidence to suggest that she is aware of pattern structure. She seems to operate at the level of "what comes next" and her inability to justify her decisions suggests that she is not aware of the properties of patterns. Her inability to count the number of repeats in Task 3 correctly also suggests she has little understanding of pattern element. Janet's case sounds a warning to us that children may produce correct patterns with little understanding. By contrast, a deeper analysis of Grant's

responses suggests that he may have a more developed notion of pattern than Janet, which is illustrated in the next section.

A closer look at pattern repeat

We have chosen to focus on the responses of Grant and Ann in this section. Both children exemplify different and interesting ways of reasoning when working with pattern, thus reflecting different levels of competence. We believe that the texture of these differences may provide insight into how children work spontaneously with mathematical pattern, and thus inform ways in which children might be supported in their learning of mathematical pattern.

In the previous section we noted that only one child was able to identify the pattern element explicitly. In the discussion that follows we will show how Grant and Ann show various degrees of awareness of pattern repeat and the notion of pattern element. We base these claims on what the children did and said. For example, we consider the following as evidence of some level of awareness of pattern repeat and pattern element:

- Use of words like *again* and *another* to indicate more of the same, when referring to beads or pictures; for example, "another red bead", "it goes red, blue again";
- Different speeds of speaking about individual components of a pattern; for example, saying "red, blue, red, blue, red, blue" in the same tone and at the same pace does not necessarily imply recognition of the repeat. However, saying "red, blue" quickly as in "red-blue, red-blue, red-blue" with pauses between each 'word' suggests that the child is recognising the pattern repeat.
- Pointing to key elements of the pattern; for example pointing to alternate elements in an AB pattern suggests awareness of the relationship between alternating positions.

Grant

From Grant's performance on the tasks, we can see evidence that he could produce his own pattern and reproduce a given pattern. However, he did not appear to work explicitly with an idea of pattern repeat. There was no evidence in his talk or in his strategy that he was making use of the structure of the pattern to extend it, or to count the number of repeats. For the most part, he appeared to be operating at the level of "what comes next".

For example in Task 1, he quickly produced an ABCD pattern and repeated it five times. When asked, "how many times have you done the pattern?" he confidently counted all 20 blocks, but did not pay attention to the repeats. He was then asked how many blue, green, red and yellow blocks he had used. Each time he counted all the blocks of that particular colour. He did not recognise that each colour had the same number of blocks.

A similar situation arose in Task 2 where he referred to all elements individually. For example with Task 2c he quickly placed the loose cards in the correct places on the strip. He justified his arrangement by pointing to all colours on all six cards and saying, "blue, yellow, green, red, red; red, red, green, yellow, blue ..." He hesitated a few times while naming the colours and did not show any indication in his pace or tone that he was recognising repetition in the pattern. We cannot be sure that he recognised the pattern element, but his actions do not suggest that his placements were random. In counting or naming every element of the pattern in both tasks described above, Grant did not show evidence of recognising the repetition in the pattern nor of identifying the pattern repeat. However, his ability to produce his own ABCD pattern with several repeats reflects some understanding of the notion of pattern and pattern element. Also, in Task 3a he referred to each bead he was threading by colour and shape, and used 'another' and 'again' which suggests he was recognising the repetition. But his talk did not show evidence of grouping a series of beads for the repeat.

Grant's reasoning with the T's in Task 2b is interesting. When he began the task, both loose cards had been placed right way up. He rotated one card and placed the first T upside down in position 5 on the strip, thus matching the T in position 4, and then placed the last T right way up in position 6. When asked why he put them that way, he said he did not know and moved the two loose cards off the strip, which suggested that he had interpreted the question to mean that his response was incorrect. The interviewer made explicit that she was not implying his response was incorrect, and that she simply wanted to know why he had done what he did. He then put them back on the strip as they were, and pointed to the cards in positions 4 to 1 (in that order) saying "down, up, down, up". When the interviewer asked him to start from the left and tell her the pattern, he said, "Up, down, up, down, up, down" ignoring that the loose cards had been placed "down, up". So he was describing the pattern as AB with three repeats, but his pattern was actually ABABBA. When the interviewer pointed out that what he had said did not match the actual placement of the cards, he did not change the layout of his cards until he was prompted to do so.

Initially Grant placed the loose cards incorrectly so that the T in position 5 faced upwards. When he explained his reasoning, he moved from right to left, starting with position 4 and ending with position one, saying: "up, down, up down". If we extend this to the piece he completed, then his two cards are also oriented "up, down" and so reflect the pattern he has identified, but the pattern is not continuous from position 1 to 6 (or 6 to 1). When asked to read the pattern from left to right, he said, "down, up, down, up, down, up" thus recognising a pattern, but not describing correctly the orientation of the last two cards.

Thus we see two instances where Grant does not appear to recognise the pattern element, and as a result itemises each colour or counts each block. But with the T-cards, he recognises the AB pattern and uses it even when it does not reflect the layout he has produced. This suggests that he was not always working only at the level of "what comes next", but does consider the pattern structure in at least one instance.

Ann

While Ann was busy with Task 1, she was asked if she knew what a pattern was. She confidently replied that a pattern was "something like squares or a little bit like other patterns". Then she produced the arrangement shown below which she described as having two 'sides', referring to the blocks on the left and right, and "one down and one up" referring to the bottom and top blocks respectively.

Earlier we argued that Ann's idea of a pattern in this task could be defined as 'picture'. She made no attempt to produce a repeat pattern and was confident that her production constituted a pattern, as she understood it.

In Task 2, Ann showed evidence of understanding repeat patterns. For example having correctly placed the loose cards on Task 2a, she described the pattern as "blue-red, blue-red, blue-red, blue-red". She said "blue red" quickly (hence we hyphenated the words) with a pause between each one. Also, she indicated four repeats although there were only three repeats on the strip. This indicates to us that she was seeing the pattern repeat and that she was not simply reading the six positions on the card.

Ann also showed evidence on two occasions that she was focusing on the entire pattern across the cards and could thus identify the pattern structure. When working on Task 2b, she correctly placed the loose card in position 6 *before* placing the card in position 5. When justifying her response, she pointed simultaneously to the upward T's (in positions 1 and 3) and then simultaneously to the downward T's (in positions 2 and 4). This suggests that she was focusing on "what comes here" rather than "what comes next".

With Task 2c she quickly placed the two cards correctly with the red dots facing each other, once again placing the loose card in the right-most gap first. When asked how she knew what to do, she indicated by pointing that she had seen the printed pattern on the strip in positions 1, 2, 3 and 6. Although she did not make explicit reference to the orientation of the images in these positions, she was clearly paying attention to the whole strip and not just to the positions adjacent to the empty spaces. Taken together, these responses show that Ann is paying attention to the relationships between different positions on the various strips. She is able to recognise the pattern element even though she makes no explicit reference to it. We are uncertain as to whether Ann could isolate the pattern repeat in order to count the number of repeats.

Ann was the only child to get the pattern in Task 2d correct. She justified her arrangement by referring to the colours: "lellow, green, blue; lellow, green, blue". She did not point or indicate where she was 'reading' this information from so her justification and placement of the cards contradict each other because the green

and blue should be reversed. We cannot tell if she recognised that the cards were different.

Ann's responses across Tasks 2 and 3 show that she can identify the structure of repeating patterns. She identified the pattern element and used this to place the cards in Task 2. This suggests she is working at a more sophisticated level than "what comes next" and is focusing on what comes in any particular position. In Task 3 she selected the wrong bead on two occasions, but corrected herself before selecting the next bead. This suggests she is also paying attention to what comes next. However, she does not appear to be able to isolate the pattern repeat easily as Tessa does and to refer to it explicitly.

Difficulties of doing research with young children

Doing research with young children has several complexities that are less crucial when working with older children and adults, for example, their short attention span and their limited ability to communicate their thinking. This means that all visual and verbal data must be carefully coordinated to establish as accurate an interpretation as possible. When the children were video recorded performing each activity, the camera was deliberately set up not to capture the child's face. This was done to meet ethical concerns about doing research with young children and in an attempt to guarantee anonymity. However, the facial expressions would have been very helpful in the data analysis process and, with hindsight, should have been captured. In the absence of verbal data, the facial expressions may have indicated the extent to which a child was apprehensive or confident in their performance on a task. By capturing the children's faces it may also have been possible to see what they were looking at when working on each task. While we strongly support the need for sensitive and discreet research with young children, we believe that our research would have benefitted from access to the children's facial expressions.

A related difficulty in working with young children is their struggle to communicate their thinking. While some gave responses that gave insight into their thinking, many responses were vague and gave little insight into what the child was thinking. For example the following transcript shows the difficulty of accessing Jack's thinking after working on Task 2c:

Erica:	Why did you choose to put them that way?
Jack:	Because I wanted to.
Erica:	What were you thinking when you put them that way?
Jack:	To put them here.
Erica:	Ok I'm happy with them there. I just want to know why you have decided to put them like that.
Jack:	Cause all the others are like that.
Erica:	What's going this way?
Jack:	The dots.
Erica:	The dots, are you happy with that pattern?
Jack:	Mmmm.

Despite the interviewer's attempts to find out why Jack placed the cards as he had, his answers gave little insight into what he was thinking. He was not being obstinate in his responses. It appeared that he was not able to communicate his decision-making process.

There were many other instances where this happened. In some cases perhaps the children did not actually know what they were thinking. For example, when Ann was asked how many times she had repeated the pattern in Task 3a she answered (incorrectly) six. When asked to explain how she got this answer, she said that she knew how to do it because "sometimes my brain is silly and sometimes my brain is clever."

Another challenge in the research was the multiple roles that Erica played. Erica writes this section in the first person:

> There were three roles that I fulfilled during this research – as the principal of the school, the teacher of the class and the researcher collecting data. The advantage was that I was 'assessing' the children that I teach and was able to give them support and encouragement when they were completing the tasks. This is important when working with young children. The children were also familiar with me, and were comfortable to do the tasks with me. We cannot know how this influenced the results, but we believe it was better for the children that I did the interviews than having a stranger do so. It was also special to work with each child in the class. It was exciting that they each wanted to have a turn to work with me personally and the children tried their best to complete each task, as they had understood it. Their enthusiasm for the project was much appreciated. The disadvantage was that it made it difficult for me to remain objective, as a teacher knows who needs a bit of help and encouragement in order to complete the task, and who can confidently complete a task without assistance. I often wanted to use the opportunity of the data collection session as a lesson and guide each child to work out the correct answer, but I had to restrain myself by letting each child interpret the activity themselves. A teacher also wants the children that she teaches to succeed and I had to be very aware not to use the data collection process as an opportunity to teach patterning skills. It was rewarding to see that some of the children had an understanding of patterning tasks and procedures.

Erica's dilemmas as teacher-researcher are not new, but they may be more acute because the children are young. We believe that the disadvantages of her knowing the children's abilities and the potential for bias in responding to them within the interview and then in analysing the data are outweighed by the advantages of providing a comfortable and familiar environment for the children to participate in the research. If the children had not been at ease, it is likely that their confidence and performance would have been negatively affected.

Conclusion

In this study, which focused on the responses of eight young children to patterning tasks, we have seen clear evidence of a range of abilities in dealing with repeating patterns. While most children could copy and extend a given pattern, fewer were able to produce their own repeat pattern. Based on the data gathered we conjecture that two of the children who did not produce repeat patterns may have been working with a notion of "pattern as picture" on that task. We have shown evidence of some

children operating on the level of "what comes next" when completing a pattern. Some show awareness of pattern structure although they are not necessarily able to identify the pattern repeat, or to count the number of times a pattern has been repeated. Only one child was able to isolate the pattern element and use this to count the number of repeats. It appears that the ability to reason about "what comes here" may be a crucial step in moving towards a greater understanding of pattern structure, and the ability to identify the pattern element. Our conjecture points to the fact that further, and more detailed longitudinal research is required before we can start making universal arguments for the ways in which children respond to patterns more generally. In addition, we join the call for making early childhood practitioners aware of the research on patterning and of the importance of building on children's patterning abilities in the early years. In addition, we concur with Waters (2004) on the provision of support for teachers to do this. Specifically we suggest that more attention should be paid to integrating pattern development across the curriculum, to increasing the complexity of patterns that children encounter, and to develop tasks that enable children to acquire an understanding of the pattern element.

Endnotes

1. See more a detailed discussion below.
2. This is discussed in more detail below.
3. Bev did not want to continue and so stopped before completing Task 3b.

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