ACTA DIDACTICA NAPOCENSIA
Volume 11, Number 2, 2018

THE KNOWLEDGE QUARTET IN THE LIGHT OF THE LITERATURE ON SUBJECT MATTER AND PEDAGOGICAL CONTENT KNOWLEDGE

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Abstract: The purpose of this study is to introduce the indicators of the Knowledge Quartet based on the literature on subject matter knowledge and pedagogical content knowledge, and to emphasize its importance. Knowledge quartet consists of four units named foundation, transformation, connection and contingency. In this study, indicators of the each unit were identified. The importance of the so-called indicators was highlighted in the literature on subject matter knowledge and pedagogical content knowledge related to mathematics education. The results showed that the knowledge quartet constitutes a detailed framework to investigate how the subject matter and pedagogical content knowledge of in-service and pre-service mathematics teachers were reflected in their teaching.

Key words: knowledge quartet; subject matter knowledge; pedagogical content knowledge, mathematics education, mathematics student teacher.

1. Introduction

It is vital to understand and support student teachers' "subject matter knowledge" (SMK) and "pedagogical content knowledge" (PCK) to improve their mathematics and teaching (Kahan, Cooper & Bethea 2003). In the literature on the SMK and PCK of mathematics teachers, the "Knowledge Quartet" (KQ) has been used since 2003 as a model that helps evaluating and improving SMK and PCK together (Huckstep, Rowland & Thwaites 2006; Petrou 2009; Rowland, 2005, 2007; Rowland et al., 2009; Rowland, Huckstep & Thwaites 2003, 2005; Rowland & Turner, 2007; Turner, 2007). The KQ is a framework for the observation, analysis and development of mathematics teaching, with a focus on teachers' SMK and PCK (Rowland, Huckstep & Thwaites, 2005; Rowland et al., 2009). The KQ also provides a way of organizing the situations in which mathematics teachers' knowledge "plays out" in the practice of teaching (Rowland & Zazkis, 2013). Liston stated that Mathematics teachers' knowledge related to teaching could be improve by using KQ (Liston, 2015). The KQ has four dimensions-called foundation, transformation, connection, and contingency-each of which is associated with several codes see in Figure 1 (www.knowledgequartet.org).

Foundation, the first unit, involves theoretical background about SMK and PCK, as well as beliefs regarding mathematics and mathematics education (Petrou 2009; Thwaites, Huckstep & Rowland, 2005). Transformation comprises the ways in which knowledge can be transmitted clearly by teachers to learners, the use of examples and selection of procedures to form concepts, and the choice of illustrations and representations (Rowland, 2008; Rowland, Huckstep & Thwaites, 2003; Thwaites, Huckstep & Rowland, 2005). Connection involves decisions about sequencing subjects or lessons, associating lessons with previous lessons and with students' knowledge, associating procedures with concepts, and anticipating and carefully sequencing the introduction of complex ideas in the lesson (Rowland, Huckstep & Thwaites 2004). Contingency, the last unit, involves unplanned examples in lessons, students' unexpected ideas, the use of unpredictable opportunities at the time of teaching, and deviation from the lesson agenda in response to an unplanned opportunity (Rowland, Huckstep

Received: 6 November 2017, accepted 8 June 2018.

Cite as: Kula Ünver, S. (2018). The Knowledge Quartet in The Light of The Literature on Subject Matter and Pedagogical Content Knowledge. *Acta Didactica Napocensia,* 11(2), 27-42, DOI: 10.24193/adn.11.2.3.

&Thwaites, 2003). When the literature on KQ is examined, it is seen that only the units of the KQ are defined, the codes of these units are named, and some codes are exemplified by in-class applications.

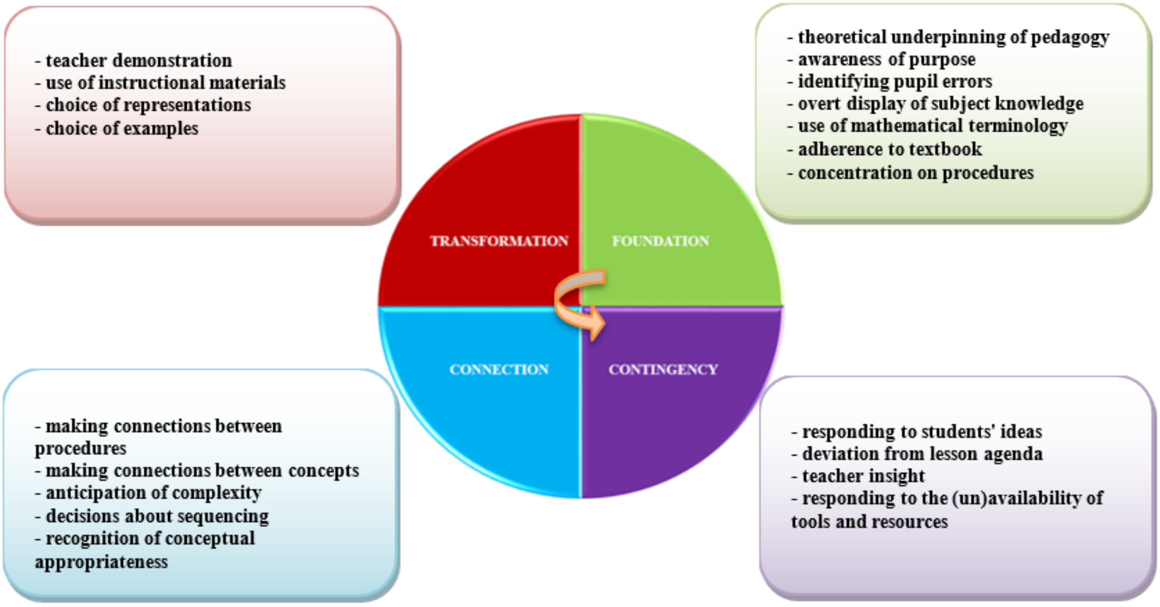

Figure 1. *Knowledge Quartet and its codes (Rowland, 2013 cited in Kula & Bukova Güzel, 2014)*

D*eveloping Primary Mathematics Teaching: Reflecting on Practice with the Knowledge Quartet*, a book published by Rowland et al. (2009), provides guidelines to support and assess pre-service teachers' mathematics teaching (p. 35-37). These guidelines involve units, a brief introduction to these units, and the questions about the reflections of so-called units to the teaching of pre-service teachers. Within the scope of this study, it was decided to transform the questions in these guidelines into indicators. Although each KQ unit has their own code, it was thought that units including indicators would be more comprehensive and thus, would contribute more to examining mathematics student teachers' teaching. Furthermore, it seems that the codes of the KQ are not determined in the context of the studies on the SMK and the PCK. Therefore, after forming the indicators, which are more comprehensive than the codes, we tried to answer the following question: "What is the importance of the indicators relating to the units of KQ in the literature?". In this direction, the study aims to examine the indicators relating to the units of KQ in accordance with the literature on SMK and PCK, as well as to emphasize the importance of KQ.

2. Methodology

While the reviewing the literature on SMK and PCK, I came across with the studies that used KQ. Subsequently, I investigated the occurrence of the KQ and also studies, between 2003-2017, using this framework. Particularly, after reading the book called Developing Primary Mathematics Teaching: Reflecting on Practice with the Knowledge Quartet published by Rowland et al. (2009), it was seen that each question statement in pages 35-37 were introduced the key ideas concerning the four units of KQ. I also decided to transform the question statements into indicators of KQ. It was also seen that the key idea behind each indicator was handled sometimes separately and sometimes in groups in the reviewed literature on SMK and PCK.

Rowland, Huckstep and Thwaites (2005) based the framework of KQ on Shulman's (1987) definitions of SMK and PCK. While this study was being carried out, studies related to SMK and PCK - generally about mathematics education - were examined in detail and it was examined which researchers place importance on the determined indicators. As much as possible, the studies referring to the generated indicators are tabled and the KQ's importance has been tried to be revealed once more in the light of the literature on SMK and PCK.

3. Results

The findings obtained as a result of the literature review will be presented in separate subheadings for all units of KQ. In addition, the definition of each unit will be given in support of the literature.

Examination of Foundation

The first unit of KQ focuses on the beliefs of in-service and pre-service mathematics teachers about mathematics and mathematics teaching and deals with their theoretical background on SMK and PCK which they bring to their teaching environments (Petrou, 2009; Rowland, Huckstep & Thwaites, 2003; Rowland, Thwaites & Huckstep, 2003a; Rowland et al., 2009; Thwaites, Huckstep & Rowland, 2005; Turner, 2007). It is rooted in the foundation of the teacher's beliefs (Rowland, 2013). The main components of this theoretical background are: mathematical knowledge and understanding per se; follow-up of the literature on mathematical teaching and learning; thinking about them and reflect their achievements in their teaching; and espoused beliefs about mathematics, including beliefs about why and how it is learnt (Rowland, Huckstep & Thwaites, 2005; Rowland et al., 2009). Rowland et al. (2009) explain the relationship between the theoretical structure of foundation and SMK-PCK as follows:

> We saw that the first category in Lee Shulman's typology of knowledge for teaching was subject-matter knowledge. This may be described as knowledge of the facts, concepts, processes and connections within the subject (substantive knowledge) as well as the way in which knowledge within that subject is investigated and developed (syntactic knowledge). Both of these aspects of subject-matter knowledge are important facets of foundation knowledge, but they do not make up the whole picture. Theoretical pedagogical content knowledge is also seen as a key component within foundation knowledge. Teachers need to understand the ways in which pedagogical strategies relate to the mathematics they are trying to teach in order to make decisions about which strategies to use. These actual decisions would be considered to be part of the transformation dimension; however, the theoretical understanding that underpins them is part of the teacher's foundation knowledge (Rowland et al., 2009, p. 153).

Rowland, Huckstep and Thwaites, (2005) stated that the foundation coincides to a significantly with Shulman's (1987) 'comprehension', which is the first stage of his six-point cycle of pedagogical reasoning. This unit includes the knowledge, insights and beliefs that pre-service teachers acquire through their own individual efforts, both in college and university education (Petrou, 2009; Rowland, 2013; Rowland et al., 2009; Thwaites, Huckstep & Rowland, 2005). Foundation differs from the other three units of the KQ in the sense that it is about knowledge possessed, irrespective of whether it is being put to purposeful use (Rowland, 2012). Also, the other three units based on foundation (Turner, 2007). The foundation is fundamental because it underpins all the decisions about which examples or representations to use, connections to make, or how to respond to students' ideas (Rowland, Thwaites & Huckstep, 2003b; Rowland et al., 2009). Evidence of foundation can be found in both planning and teaching process (Rowland et al., 2009). Table 1 was formed by author to reveal the importance and place of the indicators of foundation (Rowland et al., 2009, p. 35) in the literature on SMK and PCK.

Table 1. *The place of foundation's indicators in the literature on SMK and PCK.*

The indicators of foundation	The key idea of the indicator and related studies
Have a clear and coherent belief about the purposes of mathematics education and why his/her pupils are compelled to learn it	Beliefs and purposes: Ball, 1988; Ball & McDiarmid, 1989; BCooneyorko & Putnam, 1996; Boulton-Lewis et al., 2001; Cooney, 1994; Davis, 2003; Fernández- Balboa & Stiehl, 1995; Graeber, 1999; Grossman, 1990; Kahan, Cooper & Bethea, 2003; Leinhardt & Smith, 1985; Ma, 1999; McDiarmid, Ball & Anderson, 1989; NCTM, 1989; Nespor, 1987; Ponte, 1999; Schoenfeld, 2006; Shulman, 1987; Simon & Blume, 1994; Szydlik, 2000; Thompson, 1984, 1992.

Use appropriate teaching strategies to promote the required mathematical understanding in pupils	Teaching strategies: Ball & Sleep, 2007; Bukova Güzel, 2010; Carlsen, 1999; Chick et al., 2006; Fernández-Balboa & Stiehl, 1995; Graeber, 1999; Grossman, 1990; Leavit, 2008; Magnusson, Krajcik & Borko, 1999; Marks, 1990; Schoenfeld, 1998; Shulman, 1986, 1987; Tamir, 1988; You, 2006.
Demonstrate knowledge of factors which have been shown to be significant in the teaching of mathematics	-
Concentrate on developing understanding rather than excessively on procedures	Procedural and conceptual knowledge: Ball, 1988; Baker, Czarnocha & Prabhu, 2004; Ball, Thames & Phelps, 2008; Bossé & Bahr, 2008; Chick et al., 2006; Fennema & Franke, 1992; Hiebert & Lefevre, 1986; Leinhardt & Smith, 1985; Ma, 1999; NCTM, 1991; Thompson, 1984.
Make use of his/her own resources and teaching strategies rather than adhering to textbook or National Numeracy Strategy unit plans	Adapting textbooks to teaching Ball, Thames & Phelps, 2008; Gess-Newsome, 1999a; Hill et al., 2008; Nicol & Crespo, 2006; Chick et al., 2006.
Show, in his/her planning, knowledge of common errors and misconceptions and take steps to avoid them	Errors and misconceptions: Ball & McDiarmid, 1989; Ball, Thames & Phelps, 2008; Chick et al., 2006; Cornu, 1991; Davis & Vinner, 1986; Graeber, Tirosh & Glover, 1989; Grossman, 1990; Grouws & Schultz, 1996; Hart, 1981; Kovarik, 2008; Marks, 1990; NCTM, 2000; Özaltun, 2014; Schoenfeld, 1998, 2000; Shulman, 1986, 1987; Sleep & Ball, 2009; Szydlik, 2000; Thompson, 1984; Williams, 2001; Williams & Ryan, 2000; You, 2006.
Show care in writing mathematical expressions correctly	Mathematical expressions: Ball, 2003; Sleep & Ball, 2009; NCTM, 1989.
Show a good understanding of the process	Leinhardt & Smith, 1985.
Demonstrate a knowledge of quick mental methods	-
Use mathematical language correctly	Mathematical language: Ball, 2003; Ball & Sleep, 2007; Ball, Thames & Phelps, 2008; NCTM, 1989, 2000; Owens, 2006; Sleep & Ball, 2009.
Demonstrate an accurate understanding of mathematical ideas or concepts	Conceptual understanding: Ball, Thames & Phelps, 2008; Bransford, Brown & Cocking, 2000; Hiebert & Lefevre, 1986; Ma, 1999; Sleep & Ball, 2009; Simon & Blume, 1994; Szydlik, 2000; Thompson, 1984.

The literature review on SMK and PCK revealed that the indicators of foundation were addressed in different studies, which underlined the importance of the relevant indicators. Table 1 presents the key idea of each indicator and related studies. For example, the key idea behind the indicator “Concentrate on developing understanding rather than excessively on procedures” was determined as constructing conceptual understanding along with procedural knowledge instead of focusing on procedural knowledge and the so-called indicator was briefly named as “procedural and conceptual knowledge”. Some of the indicators could not be directly associated with any study as they are topic-specific. In their research, Rowland et al. (2009) focused on the situations in which elementary mathematics student teachers’ teaching the four operations. In this context, some indicators such as "demonstrate a knowledge of quick mental methods" are meaningful about number concept. However, with a more detailed research it is possible to find places of indicators in the literature.

Examination of Transformation

As different from the foundation, the remaining three units focus on the ways and contexts in which possessed knowledge is brought to bear on both processes of planning and teaching (Rowland, Huckstep & Thwaites 2005; Rowland et al., 2009; Rowland, 2013). One of these three units, transformation, includes the presentation of the ways in which the teacher’s own knowledge is transformed to make it accessible to the students (Turner 2007). This unit also includes the selection of examples and procedures to assist concept formation, the uses of multiple representations and presentations, and focuses on planning and conducting to teaching process (Rowland et al., 2009; Rowland, Huckstep & Thwaites, 2003, 2005; Thwaites, Huckstep & Rowland 2005). Petrou (2009) defines transformation as the knowledge-in-action and indicates that this unit includes the representations and examples used by teachers, as well as, teachers' explanations and questions asked to students in the teaching process.

Rowland, Huckstep, and, Thwaites (2005) indicate that while naming transformation, they were influenced by the requirement of the teacher's competence to transform his or her possessed content knowledge into forms that are pedagogically powerful (Shulman, 1987) and to distinguish between knowing mathematics for yourself and teaching it in order to be able to help someone else learn it (Ball, 1988). Table 2 was formed by author to reveal the importance and place of the indicators of transformation (Rowland et al., 2009, p. 36) in the literature on SMK and PCK.

Table 2. *The place of transformation's indicators in the literature on SMK and PCK.*

The indicators of transformation	The key idea of the indicator and related studies
Use equipment correctly to explain process in number where appropriate	Use equipment: Higgins, 2005.
Select appropriate forms of representation	Representations: Ball, 1988; Bagni, 2005; Ball, 2003; Ball, Thames & Phelps, 2008; Bukova Güzel, 2010; Chick et al., 2006; Cobb, Yackel & Wood 1992; Duval, 2002; Elia et al., 2007; Even, 1998; Fennema & Franke, 1992; Goldin, 1998, 2000; Grossman, 1990; Hitt, 1999; Izsák & Sherin, 2003; Janvier, 1987; Keller & Hirsch, 1998; Lloyd & Wilson, 1998; McDiarmid, Ball & Anderson, 1989; NCTM, 1989, 1991, 2000; Özaltun Çelik & Bukova Güzel, 2017a; Schoenfeld, 1998; Shulman, 1986, 1987; Sleep & Ball, 2009; Stein et al., 2008; Stylianou, 2010; Turner, 2008; Van de Walle, 2004; You, 2006.
Chose appropriate examples when demonstrating or eliciting an idea	Use of examples: Bills et al., 2006; Bills & Watson 2008; Chick, 2007; Rowland, 2008; Thompson, 1984; Tsamir, Tirosh & Levenson, 2008; Watson & Shipman, 2008; Zaslavsky, 2010; Zodik & Zaslavsky, 2008, 2009.
Give clear explanations of ideas or concepts, possibly making use of analogy	Give clear explanations of concepts: Ball & Sleep, 2007; You, 2006.
Demonstrate clearly and accurately how to carry out procedures	Chick et al., 2006; Thompson, 1984.
Make use of interactive teaching techniques to develop and assess understanding	-
Use questioning effectively to assess and develop children's knowledge and understanding	Use of questioning effectively: Ong, Lim & Ghazali, 2010; Franke et al., 2009; Martino & Maher, 1999; Özaltun Çelik & Bukova Güzel, 2016, Özaltun Çelik & Bukova Güzel, 2017b.

Gess-Newsome (1999b) defines PCK as a transformative model and Shulman (1987) states that the important component of PCK is transformation of SMK, in other words presenting subject in forms that makes it understandable to students. Transformation requires usage of models, analogies, examples, illustrations, representations, and demonstrations that can build a bridge between teachers' understanding about the subject and the understanding that students are expected to achieve (Uşak, 2005). Apparently, the indicators of transformation were also handled in different studies about SMK and PCK (see Table 2). In particular, numerous studies refer to the importance of representations and use of examples. The places of some of the indicators of transformation were not exactly determined in the literature. It is thought that the reason why some indicators could not been achieved in the studies about SMK and PCK is that the KQ is a highly detailed framework to observe, develop and assess the reflection of SMK and PCK to teaching.

Examination of Connection

Connection includes the selection of mathematical topics, the connections between the decisions taken, the sequencing of topics of instruction within and between lessons, and the ordering of tasks and exercises (Rowland, Thwaites & Jared, 2015; Rowland et al., 2009). Turner (2007) indicates that anticipation of complexity, recognition of conceptual appropriateness for students and making connections are significant components of this unit. With this aspect, connection binds together choices and decisions related to mathematical content (Rowland, Huckstep & Thwaites, 2003, 2005; Thwaites, Huckstep & Rowland, 2005). Liston (2015) expressed that connection also includes making connections between concepts and procedures. Petrou (2009) defines connection as creating links between different lessons, different mathematical ideas and the different parts of a lesson and states that it includes being informed about sequencing of activities for instruction and awareness of students' possible difficulties and obstacles. Furthermore, the connection also points out to the importance of the sequencing and selection of required materials appropriately for teaching process (Rowland & Turner, 2009).

Mathematics is notable for its coherence as a body of knowledge and as a field of enquiry (Thwaites, Huckstep & Rowland, 2005). Connection also deals with this coherence and draws attention to the importance of mathematical discourse in the teaching as well as the integrity of mathematical content (Rowland, Huckstep & Thwaites 2003, 2005; Thwaites, Huckstep & Rowland, 2005). Table 3 was formed by author to reveal the importance and place of the indicators of connection (Rowland et al., 2009, p. 36-37) in the literature on SMK and PCK.

Table 3. *The place of connection's indicators in the literature on SMK and PCK.*

The indicators of connection	The key idea of the indicator and related studies
Make links to previous lessons	Make links: Askew et al., 1997; Ball, Thames & Phelps, 2008; Hiebert & Carpenter, 1992; Ma, 1999; NCTM, 1991; Özaltun Çelik & Bukova Güzel, 2017c; Simon & Blume, 1994; Sleep & Ball, 2009.
Make links between the mental and oral starter and the main part of the lesson	-
Make appropriate conceptual connections within the subject matter	Conceptual connections: Bransford, Brown & Cocking, 2000; Fennema & Franke 1992; Hiebert & Lefevre, 1986; Hill et al., 2008; NCTM, 1991; Simon & Blume, 1994.
Recognise the conceptual appropriateness of mathematical ideas for the children they are teaching	Recognise the conceptual appropriateness: Chick et al., 2006.
Ask questions to elicit children's understanding of connections between mathematical ideas	Ask questions: Fennema et al., 1996; Franke et al., 2009; Martino & Maher, 1999; Livy, 2010; Özaltun Çelik & Bukova Güzel, 2015.
Appear to be aware of the different levels of difficulty in a topic	Aware of the difficulties Bransford, Brown, and Cocking, 2000; Shulman, 1986, 1987.
Anticipate the complexity of an idea and break it down into steps that can be understood by the children	Anticipate complexity: Arnesen et al., 2017.
Introduce ideas and strategies in an appropriately progressive order	Ball, Thames & Phelps, 2008.
Make assessment of children's understanding and amend their lessons accordingly	Make assessment: Leavit, 2008; Marks, 1990; Tamir, 1988; You 2006.

According to Ma (1999), connectedness is one of the four characteristics of teaching performed by the teacher who possesses profound understanding of fundamental mathematics. Table 3 shows the researchers who mentioned the importance of the indicators of connection. Askew et al. (1997) state that the students of the teachers who carry out their courses by making connection develop better in learning mathematics.. Additionally, according to Table 3, "make appropriate conceptual connections within the subject matter" was the indicator that has been most frequently handled by the researchers.

Examination of Contingency

The fourth unit of the KQ differs from possessing a theoretical background, and deliberation, judgment and planning involved in making learning meaningful and connected for students (Rowland et al., 2009). This unit deals with the situations that could not be presupposed and planned before lessons, in other words, the situations that are almost impossible to be planned (Rowland, Huckstep & Thwaites, 2005; Thwaites, Huckstep & Rowland, 2005). Contingency includes deviation from curriculum or agenda, responding to students' unexpected ideas, use of opportunities that could not be presupposed before the teaching but appear in the process of teaching and assumptions of teacher (Petrou, 2009; Rowland, Huckstep & Thwaites, 2003; Thwaites, Huckstep & Rowland, 2005; Turner, 2007). Teaching involves dealing with unpredictable, contingent events in the classroom (Rowland & Zazkis, 2013). In this sense, the idea that almost most of the situations in the class environment could be planned but some of them could not be planned prompted the researchers to create this unit (Rowland et al., 2009).

Turner (2007) indicates that contingency covers the ways in which teachers respond to unplanned instances in a lesson. In this context, Turner (2009) suggests that it is possible to ensure teaching that is more meaningful by responding to students' ideas. Additionally, teachers acquire more information about the nature of students' knowledge construction when a student articulates an unexpected idea (Thwaites, Huckstep & Rowland, 2005). This provides teachers with a forward-looking contribution to recognize their students. Because this unit concerns classroom events that are almost impossible to plan, teachers gain the ability to 'think on one's feet' (Rowland, Huckstep & Thwaites, 2003, 2005; Rowland & Turner, 2009; Thwaites, Huckstep & Rowland, 2005). Table 4 was formed by author to reveal the importance and place of the indicators of contingency (Rowland et al., 2009, p. 37) in the literature on SMK and PCK.

Table 4. *The place of contingency's indicators in the literature on SMK and PCK.*

The indicators of contingency	The key idea of the indicator and related studies
Respond appropriately to children's comments, questions and answers	Children's comments, questions and answers: Ball, 2003; Ball & Sleep, 2007; Ball, Thames & Phelps, 2008; Empson & Jacobs, 2008; Even & Tirosh, 1995; Marks, 1990; Sleep & Ball, 2009; Stein et al., 2008; Thompson, 1984.
Cope adequately with the questions from all children in the group	Ball, Thames & Phelps, 2008; Thompson, 1984.
Deal appropriately with children's responses to activities	Children's responses to activities: Ball, 2003; Ball & Sleep, 2007; Thompson, 1984.
Respond appropriately when children give incorrect answers to questions or make incorrect statements during the course of a discussion	Respond appropriately: Ball, 2003; Ball & Sleep, 2007; Ball, Thames & Phelps, 2008; Leinhardt & Smith, 1985; Sleep & Ball, 2009.
Deviate from their agenda when appropriate	Özaltun Çelik & Bukova Güzel, 2017d; Schoenfeld, 2006.
Make ongoing assessments of children's understanding during the lesson and amend their teaching accordingly	Assessment: Ball, 2003; Ball, Thames & Phelps, 2008; Empson & Jacobs, 2008; Kovarik, 2008; You, 2006.

Sleep and Ball (2009) draw attention to the importance of responding to students' questions and Ball (2003) also acknowledges the importance of organizing class discussions and evaluating of the students' verbal and written responses. Schoenfeld (2006) emphasizes the necessity of reviewing the teaching purposes and making spontaneous decisions at the moment when unexpected situations occur in teaching process. Table 4 shows that the researchers focusing on the different indicators of contingency. As seen in the table, the researchers have most often dealt with the indicator named "respond appropriately to children's comments, questions and answers".

4. Conclusion

This study was carried out to once again verify the importance of KQ in the literature on SMK and PCK. The most important result obtained was that KQ is a detailed and comprehensive framework for examining how mathematics teachers' SMK and PCK are reflected in their teaching process. Different indicators were proposed by different researchers to observe mathematics teachers' SMK and PCK. Even some of these indicators were not taken into account in all these studies. For example; Kovarik (2008) and Ball and Sleep (2007) tried to make the components of PCK in detail. Ball and Sleep (2007) focused on the importance of using mathematical language and notation, while Kovarik (2008) did not take into consideration the importance of this component in the PCK subcategories compiled from the work of different researchers (Ball & Bass, 2000; Schoenfeld, 2000; Shulman, 1986; Wagner, 2003). In a different manner than Ball and Sleep (2007), Kovarik's (2008) paper handles the assessment of students' learning in a detailed way and also, knowing students' misconceptions is a component of this PCK framework. In addition, KQ covers all of the components in the two studies and besides, KQ was made more detailed with some of the components.

Based on the results of this study, it is suggested for mathematics educators and for both in-service and pre-service mathematics teachers to use KQ as a guideline to examine anyone's SMK and PCK. Another suggestion concerns the adaptation of the framework as a comprehensive tool to examine teachers' SMK and PCK in fields like physics, chemistry, biology, and to discuss what changes can take place in this process.

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