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An Investigation of Prospective Mathematics Teachers' Knowledge of Basic Algorithms with Whole Numbers: A Case of Turkey

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Abstract: The aim of this qualitative case study is to investigate prospective mathematics teachers' subject matter knowledge of the underlying concepts of standard and nonstandard algorithms used to solve the problems with whole numbers. Twenty three prospective mathematics teachers enrolled in the Elementary Mathematics Education Program of one of the most successful universities in Turkey were the participants of the study. The data was collected through four tasks containing basic algorithms. More specifically, the Ones Task assessed participants' understanding of the underlying place value concepts of standard algorithms. The Andrew Task and the Doubling Task required participants to conceptualize and interpret nonstandard strategies. In the Division Task, participants were expected to provide in-depth explanation for the difference between multiplication and division and between partitive division and measurement division. The content analysis method was used to analyze the data. The results of the study revealed that more than half of the prospective mathematics teachers had knowledge about the place value of 1 in addition and subtraction, and also multiplication. However, most of the prospective teachers could not explain the underlying principle and the meaning of the nonstandard algorithm in subtraction. Similar to their knowledge on subtraction, prospective teachers' knowledge on division was limited.

Keywords: *Content knowledge, basic algorithms, whole numbers, prospective mathematics teacher.*

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Introduction

In the last few years, there has been an increasing interest in knowledge needed for effective teaching. Many researchers have focused on investigating the answer to one of the constantly evolving questions related to knowledge needed for effective teaching. The central focus of these questions is related to the knowledge needed about a subject which is to be taught and the way of teaching it (Grossman, 1990; Ma, 1999). The questions have been tried to be answered by considering them from different perspectives (Ball, Thames & Phelps, 2008; Gess-Newsome, 1999; Grossman, 1990; Rowland, Huckstep & Thwaites, 2005; Shulman, 1986; 1987). Shulman (1986), one of the pioneers of teachers' knowledge, identified pedagogical content knowledge (PCK), curricular knowledge (CK) and subject matter knowledge (SMK) as knowledge needed for effective teaching. Shulman described PCK as the combination of content knowledge and pedagogical knowledge. According to him, CK is the knowledge of the topics, learning outcomes, activities, books, and instructional materials in a specific curriculum. It is also related to knowledge of the organization of the topics and the relationship of the topics in a specific grade level with other disciplines. Lastly, Shulman defined SMK as knowing the facts, principles, and issues, knowing why these facts, principles, and issues are taught, and presenting justifications for mathematical explanations and procedures. Shulman emphasized the importance of SMK by stating that "The teacher need not only understand that something is so; the teacher must further understand why it is so" (p.9)

Building on Shulman's framework, Ball, Thames and Phelps (2008) proposed an extended and a more detailed framework called Mathematical Knowledge for Teaching (MKT). In their framework, they divided each category (SMK and PCK) into three sub-categories. In order to refer to general mathematical knowledge that is not specific to mathematics teachers, they proposed Common Content Knowledge. The second category, Specialized Content Knowledge (SCK), is mathematical knowledge unique to teachers. As the last category, Horizon Content Knowledge is defined as "an awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (Ball et al., 2008, p.403). On the other hand, Shulman's PCK is specialized into knowledge of content and

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students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC). KCS is described as the integration of knowledge about mathematics and knowledge about students, whereas KCT is the combination of knowledge about teaching and knowledge about mathematics. Lastly, KCC refers to the integration of knowledge about content and knowledge about curriculum.

Furthermore, Rowland, Huckstep and Thwaites (2005) generated the "Knowledge Quartet" framework for mathematical knowledge for teaching. The first category, foundation, is related to pedagogy and usage of mathematical terminology, and the second category, transformation, is the knowledge needed to plan teaching. The third category, connection, on the other hand, concerns the coherence of the planning and teaching and the sequencing of topics of instruction within and between lessons. Lastly, contingency category refers to unexpected classroom events.

In relation to the knowledge types that the teachers should have to teach the subject in an effective way, Hiebert and Lefevre (1986) defined conceptual and procedural knowledge. These knowledge types have served as a widely used framework to examine teachers' knowledge. Conceptual knowledge is described as the "knowledge that is rich in relationships" (p. 3), while procedural knowledge is defined as the "rules or procedures for solving mathematical problems" (p.7). Put differently, conceptual knowledge involves understanding of basic arithmetic facts, mathematical ideas, and procedures by being able to interpret and perform them correctly in different situations. However, procedural knowledge includes solving a problem via procedures, rules, formulas, algorithms and symbols (Engelbrecht, Harding & Potgieter, 2005; Rittle-Johnson, Siegler & Alibali, 2001).

Among the frameworks for teacher knowledge, the present study was grounded on Shulman's SMK. Because of its importance for high quality teaching, many research studies were conducted to investigate teachers' SMK related to various mathematics topics such as functions, division, fractions, and probability (Ball, 1990; Even, 1993; Isiksal, 2006; Lucus, 2006; Nilsson & Lindstrom, 2012; Philipp, Schappelle, Siegfried, Jacobs & Lamb, 2008). Among these studies, Philipp et al. (2008) investigated the effect of a professional development program on K-3 teachers' mathematical content knowledge of basic algorithms with whole numbers. They concluded that the prospective teachers had procedural knowledge related to basic algorithms with whole numbers. In other words, the prospective teachers had a tendency to calculate basic algorithms rather than conceptualize them. As basic algorithms with whole numbers is an important subject to teach and learn mathematics conceptually (Van de Walle, 2003), teachers need to be able to interpret and explain the underlying meaning of the algorithms to empower students mathematically (Thanheiser, 2009). In order to determine whether teachers' mathematical content knowledge of basic algorithms with whole numbers is sufficient to teach the subject effectively, it is significant to explore teachers' knowledge of basic algorithms with whole numbers. As prospective teachers are the future teachers, studies conducted with them have a vital role in mathematics education research. For these reasons, teachers' knowledge of recognizing and explaining the underlying concepts, knowledge of applying nonstandard algorithms, knowledge of reasoning students' solution strategy, and knowledge of recognizing the problem structure are the focus of the current study. Within the scope of this focus, the aim of this study is to explore subject matter knowledge of prospective mathematics teachers. The study addresses the following research questions:

1) What is the nature of the prospective mathematics teachers' subject matter knowledge of the underlying concepts of standard algorithms used to solve the problems with whole numbers?

2) What is the nature of the prospective mathematics teachers' subject matter knowledge of the underlying concepts of nonstandard algorithms used to solve the problems with whole numbers?

3) What is the nature of the prospective mathematics teachers' subject matter knowledge of the measurement-division problem?

Methodology

Design of the Study

As this research study aims to gain a deeper insight into Turkish prospective mathematics teachers' subject matter knowledge, the most appropriate research method was a qualitative case study design. Creswell (2007) stated that the purposes of case studies are to analyze a case or multiple cases within a bounded system and to develop a richer and deeper description about the cases. The cases were prospective mathematics teachers whose boundaries were being in the fourth year of the teacher education program in one of the most successful universities in Turkey.

Participants

In order to achieve the aim of the study, the most appropriate method for selecting participants was purposive sampling. The important criteria for selecting the participants were to select the most knowledgeable and accessible prospective teachers. Based on these criteria, 23 prospective middle school mathematics teachers enrolled in the Middle School Mathematics Education program of a state university located in Ankara, Turkey were selected as the participants of the study. The students attending this program take mathematics courses (e.g. Calculus, Discrete Mathematics, and Analytic Geometry), mathematics education courses (e.g. Instructional Principles and Methods, and Nature of Mathematical Knowledge for Teaching), and education courses (e.g. Educational Psychology, and Turkish

Educational System and School Management). They mostly take mathematics courses in the first 2 years and the mathematics education courses in the subsequent years. At the time of data collection, the participants had already taken Methods of Teaching Mathematics I-II and Nature of Mathematical Knowledge for Teaching. Students taking the Methods of Teaching I-II course can understand the basic concepts related to school mathematics and recognize connections among mathematical ideas in the middle school mathematics curriculum, analyze students' misconceptions related to mathematics topics, and design and perform lesson plans and activities for mathematics instruction using different teaching strategies. Furthermore, students completing the Nature of Mathematical Knowledge for Teaching course can distinguish between the types of mathematical knowledge, describe the misconceptions that students may have and generate strategies to overcome these misconceptions, and evaluate their conceptual and procedural knowledge by explaining their underlying reasoning. The boundary of the cases, who are prospective mathematics teachers, is that they have taken these courses and they are senior students in the Middle School Mathematics Education program of one of the most successful public universities in Turkey.

Data Collection Tools

In this study, to explore prospective mathematics teachers' subject matter knowledge related to basic algorithms with whole numbers, four tasks given in the Appendix were used. These tasks, prepared by Philipp, Schappelle, Siegfried, Jacobs, and Lamb (2008), were applied after receiving permission from the authors. Three tasks were related to algorithms and one task was designed to evaluate whether participants recognize problem structure. More specifically, the *Ones Task* assessed participants' understanding of the underlying place value concepts of the standard algorithms. Particularly, the aim of the task was to define 1 in addition as regrouping 10 ones into one group of ten, and in subtraction as regrouping one hundred into 10 groups of ten. In contrast to the *Ones Task*, the *Andrew Task* and the *Doubling Task* required participants to conceptualize and interpret nonstandard strategies. In the *Division Task*, participants were expected to provide an in-depth explanation for the difference between multiplication and division and between partitive and measurement division. The participants were asked to explain their answer to the tasks in detail to gain an in-depth understanding of their knowledge of basic operations with whole numbers.

Data Analysis

In the data analysis phase, content analysis defined by Strauss and Corbin (1990) was applied. In the first stage of the data analysis, participants' written responses for each task were examined and coded by two researchers. The data gathered from four tasks were coded separately. This first analysis resulted in combining some codes together, removing some of them, or adding some others to the list. At the end of this stage, categories were generated for each task. In the second stage of the analysis, the frequency tables were formed to reveal the number of participants in each category. The final lists of categories and the frequencies are presented in Table 1, Table 2, Table 3 and Table 4.

To establish inter-rater reliability, each task was coded by two independent researchers who are expert in mathematics education. The inter-rater reliability was calculated and a 90 percent correlation was found between ratings.

Findings

The findings were presented in four parts. In each part, prospective teachers' knowledge of basic algorithms with whole numbers was analyzed based on each task.

The Ones Task

The Ones Task involved standard algorithm in addition (Problem A) and in subtraction (Problem B) and the participants were required to explain the value of 1 in both problems. Also, they clarified the reason for adding 1 to 5 in Problem A (Appendix) and the reason for adding 10 to 2 in Problem B (Appendix). This content item was evaluated on a 4-point rubric (0-3 scale). The score of 0 reflects no explanation regarding place value, while the score of 1 represents explanation for place value with addition, not subtraction. Moreover, the score of 2 reflects making an explanation for place value for both problems, but not explaining the reasons for adding 1 in the addition problem and adding 10 in the subtraction problem. Finally, the score of 3 represents explaining both the place value for both problems and the reasons for adding 1 and 10 in the addition and subtraction problems, respectively. The scores, the explanation of the place value of 1, the explanation of the reasons for adding 1 and 10 and the number of participants for each score are presented in Table 1.

Table 1. The 4-point rubric for the Ones Task

Score	The place value of 1	The reasons for adding 1 and 10	n (%)	Participants who received this score
0	No explanation	No, incorrect or missing explanation	3 (13 %)	14, 15, 16
1	Correct explanation for addition, not subtraction	No, incorrect or missing explanation	1 (4.4 %)	11
2	Correct explanation for both problems	No, incorrect or missing explanation	3 (13 %)	1, 7, 12
3	Correct explanation	Correct explanation	16 (69.6 %)	2, 3, 4, 5, 6, 8, 9, 10, 13, 17, 18, 19, 20, 21, 22, 23

Among the 23 prospective mathematics teachers, 3 prospective teachers (13%) (P14, P15 and P16) did not mention the place value of the 1 in each problem. For this reason, they could not explain why in the addition problem, 1 is added to 5, but in the subtraction problem 10 is added to 2. The examples from their explanations are given below:

Participant 14:

"The value of 1 is different since 297 and 395 are not the same."

Participant 16:

"The value is the same since both represent 10. "

One prospective teacher (P11) explained the place value of 1 in the addition problem; however, she did not clarify its value in the subtraction problem. Similar to P14, P 15 and P16, she could not explain the reason for adding 1 and 10 in the addition and subtraction problems, respectively. The explanation of P11 is presented below:

"In Problem A, we get 10 ones when we add 9 and 8. The 10 ones equal to 1 tens. For this reason, we add 1 to the tens digit."

On the other hand, three prospective teachers (13%) explained that the place value of 1 in each problem represents a different amount. However, they could not clarify correctly, why their amount is different in each problem. As an example, P12's explanation is given below:

"1 doesn't represent the same amount. In here, we decomposed the numbers. In Problem A, 1 represents 10 and we add 1 tens to tens place. However, in Problem B, 1 represents 100 and we add 10 tens to tens digit."

As it can be seen, his explanation related to the value of 1 is correct for both the addition and subtraction problem. The following excerpt gives his explanation regarding the reason for adding 1 in the addition problem and 10 in the subtraction problem.

"*In addition, the operations start from the ones digit and 10 ones equals to 1 tens. This is added to the tens digit. In subtraction, if we don't have enough number, we should decompose the numbers. Thus, 10 is added to 2."*

Regarding the addition problem, P12 explained the reason for adding 1; however, he could not clarify why 10 is added to 2 in subtraction. Although he knows that he should decompose the numbers, he does not know how he will decompose it. Therefore, his explanation was missing.

Most of the prospective mathematics teachers (69.6%) explained the place value of 1 in the addition and subtraction problem, and why 1 is added to 5 in the addition problem, but 10 is added to 2 in the subtraction problem. Among 16 prospective teachers, the explanation of participant 23 is given below as an example.

P23:

They are not in the same amount. Terry solved Problem A by considering that 10 ones is 1 ten. However, in Problem B, he thought that 1 hundred equals to 10 tens. In other words, in Problem A, the addition of 9 and 8 equals to 17 and 17 ones equals to 1 ten and 7 ones. Now, Terry got 1 ten and added it in tens. In problem B, 400 indicates 4 hundreds. He took 1 hundred from 4 hundreds. This means that he took 10 tens. Then he added 10 tens to the tens.

Her explanation regarding the reasons for adding 1 and 10 in the addition and subtraction problems respectively is as follows:

In Problem A, the student added 1 tens to 5 in the tens digit because 10 ones equals to 1 tens. But, in Problem B, Terry added 1 hundred to the tens digit because 10 tens equals to 1 hundred. Since she has already had 2 tens, she got 12 tens.

The remaining 15 prospective teachers agreed with P23, and their explanations were similar. To summarize, more than half of the prospective teachers adequately explained the place value of 1 and the reasons for adding 1 and 10 in the addition and subtraction problems, respectively.

The Andrew Task

Contrary to the Ones Task, the Andrew Task presented in Appendix is related to nonstandard algorithm in subtraction. Prospective teachers were asked to explain the reasoning behind Andrew's strategy and to solve 432 - 162= □ using Andrew's strategy. This content item was assessed on a 5-point rubric (0-4 scale) with scores of 0 and 1 reflecting no explanation related to place value, scores of 2 and 3 reflecting missing or incorrect explanation of place value and lastly score 4 reflecting correct explanation of the place value. The scores 0 and 1 differ from each other in application. Score 0 represents incorrect application, while score 1 represents correct application of the Andrew's strategy to solve 432-162. Similarly, score 2 differs from score 3 in terms of the correctness of the application. The scores, the correctness of both explanation and application, and the number of participants for each score are presented in Table 2.

Table 2. The 5-point rubric for Andrew Task

Score	Explanation	Application	n (%)	Participants who received this score
0	No explanation,	Incorrect application	0 (0 %)	-
1	No explanation	Correct application	3 (13 %)	7, 17, 18
2	Missing and incorrect explanation	Incorrect application	3 (13 %)	4, 9, 14
3	Missing and incorrect explanation	Correct application	10 (43.5 %)	1, 3, 6, 8, 11, 13, 15, 16, 20, 21
4	Correct explanation	Correct application	7 (30.5 %)	2, 5, 10, 12, 19, 22, 23

As it can be seen in Table 2, although three prospective teachers could not explain Andrew's strategy, they applied the same strategy to solve different subtraction algorithms. Apart from these three prospective teachers, three prospective teachers' explanations were missing or incorrect and their application of the strategy was also incorrect. As an example, the explanation and the application of P9 was presented below. Other two prospective teachers (P4 and P 14) explained and applied Andrew's strategy in a similar way.

P9:

Andrew stated that minuend and subtrahend have the same units so he subtracted 63 from 23. Thus, he got 40. However, he decreased subtrahend as 2, so he should subtract 2 from 40. Then, he got 40-2=38.

Although P9 did not mention the place value, his explanation of the algorithm was correct. Based on his explanation, it cannot be determined whether he had place value knowledge or not. In addition, his explanation related to the application of Andrew's strategy was not clear. He did not clarify his algorithm with respect to place value knowledge. On the contrary, he reported that he subtracted 132 from 432 to make the subtraction easier while applying Andrew's strategy. His explanation was given in the following excerpt.

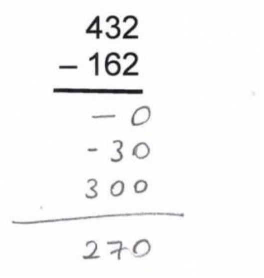
Firstly, we should take subtrahend as 123 to provide easier subtraction. 432-132=300. Then, we should subtract 30 from 300 because the amount of subtrahend decreases by 30. Thus, 300-30=270.

Furthermore, ten prospective teachers' explanations were coded as missing or incorrect explanation; however, they applied the Andrew's strategy correctly. The explanation and application of P13 were given below.

P13:

He generalized the "subtract smaller from larger" expression. It makes sense since they learn to subtract smaller from larger.

His application of Andrew's strategy is as follows:



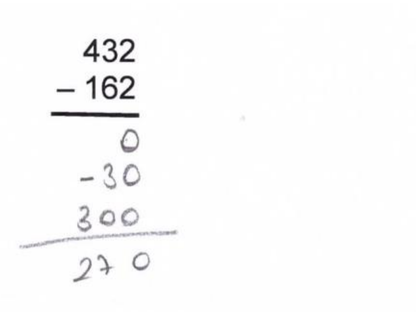
As it can be seen from the explanation of P13, he based subtraction on the "subtract smaller from larger" expression. While explaining the reasoning behind the Andrew's strategy, he did not mention place value. His application was in contrast to his explanation because while he was applying the Andrew's strategy to solve 432-162, he subtracted 30-60 and found -30. Thus, he did not subtract the smaller from the larger. Other nine prospective teachers explained the reasoning behind the Andrew's strategy like P13.

Among 23 prospective teachers, seven of them explained the Andrew's strategy correctly and applied the strategy to solve a subtraction algorithm. An example regarding this type of explanation and application is presented below.

P5:

Firstly, Andrew works on ones, 3-5=-2. Then, Andrew works on tens, 60-20=40. Finally, -2+40= 38

P5's application of the reasoning behind Andrew's strategy is as follows:



As understood from the excerpts of P5, she clarified the Andrew's strategy based on place value. Other six participants explained the strategy in a similar vein. In the light of their explanation, it can be concluded that these prospective teachers have knowledge about place value.

As a result, 13% of the participants have no idea related to the Andrew's strategy, but they could apply it to solve another subtraction problem. Moreover, 56.5% of the prospective teachers tried to explain the reasoning behind the strategy; however, their explanations were missing or incorrect. Among these prospective teachers, 13% of them could not apply the Andrew's strategy to subtract 132 from 432, 43.5% of them applied the strategy successfully. On the other hand, 30.5% of the prospective teachers' explanation and application of the Andrew's strategy were correct.

The Doubling Task

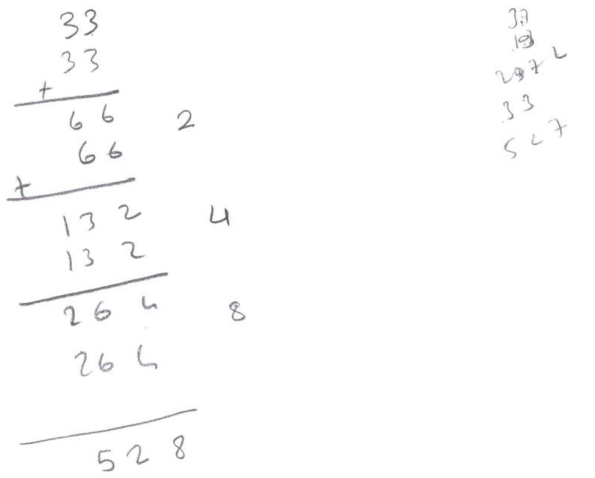
Similar to the Andrew Task, the Doubling Task requires participants to conceptualize and interpret nonstandard algorithm. In this task, Todd, a third grade, solved the multiplication problem using a different strategy. Prospective teachers were asked to solve

33x19= using Todd's strategy. This content item was assessed on a 4-point rubric (0-3 scale). The scores, the explanation of each score, and the number of participants under each score are presented in Table 3.

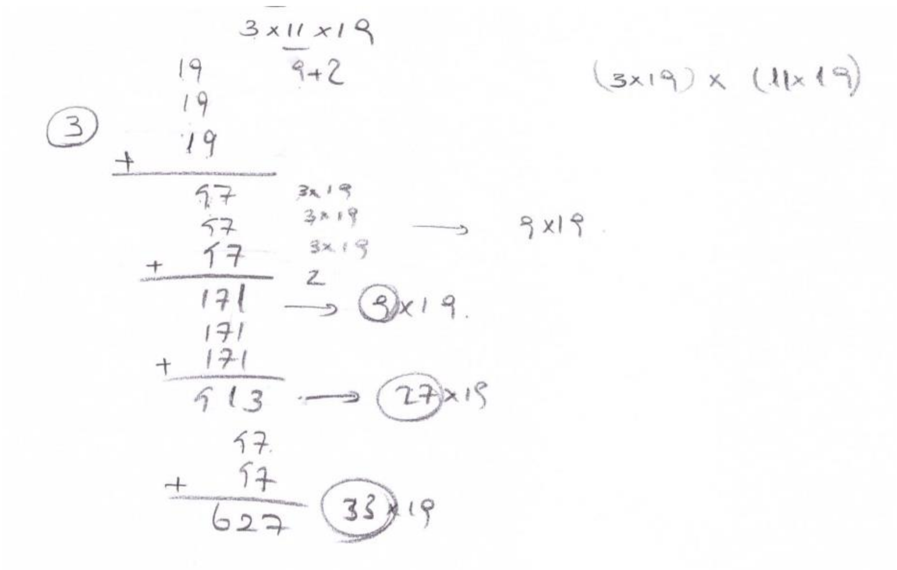
Table 3. The 4-point rubric for Doubling Task

Score	Application	n (%)	Participants who received this score
0	Incorrect or missing calculationally; incorrect conceptually	0 (0 %)	-
1	Incorrect or missing calculationally; correct conceptually	1 (4.3 %)	16
2	Correct calculationally and conceptually but not Todd's way	1 (4.3 %)	7
3	Correct calculationally and conceptually	21 (91.3 %)	1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23

Table 3 shows that of the 23 prospective teachers, one of them (P16) used Todd's strategy to solve 33x19; however, his calculation was missing. He doubled and labeled correctly for 2, 4, 8, but he stopped doubling after 8. His solution is presented below.

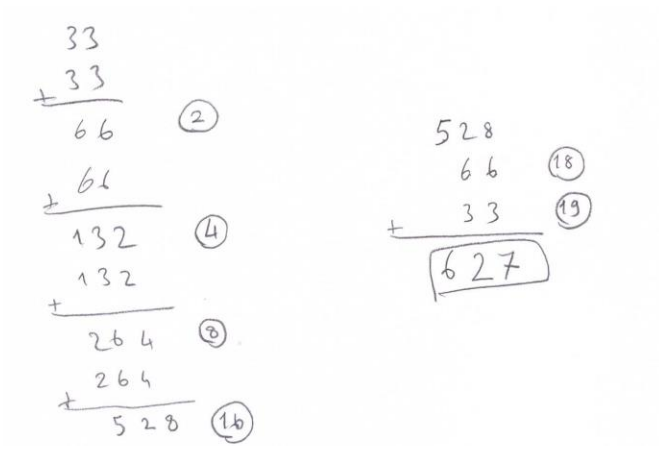


Moreover, one participant (P7) solved the problem not using the doubling strategy, but her strategy was correct both calculationally and conceptually. Instead of using the doubling strategy, she used the triple strategy.

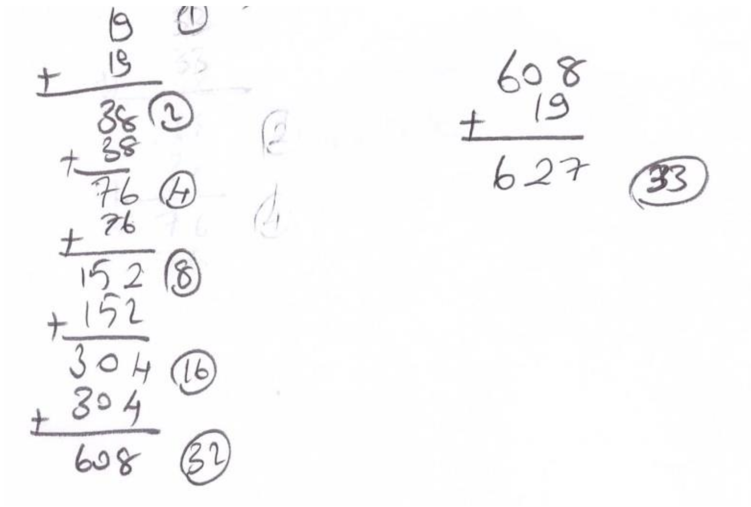


Apart from P7 and P16, the remaining prospective teachers used Todd's strategy to find the result of the 33x19. Some of them doubled based on 33, and the remaining used 19 while doubling. Doubling strategies are presented below.

P3:



P19:



As it can be seen from the examples, both calculations were correct calculationally and conceptually. It can be concluded that for the Doubling Task, nearly all the participants conceptualized the nonstandard algorithm and applied it to another operation.

The Division Task

In the Division Task, participants were expected to provide an in-depth explanation for the difference between multiplication and division and between partitive and measurement division. In order to determine prospective teachers' knowledge of division, a 3-point rubric was prepared. The rubric, presented in Table 4, represents participants' understanding of the distinction between multiplication, division, partitive and measurement division.

Table 4. The 3-point rubric for Division Task

Score	Explanation	n (%)	Participants who received this score
0	No understanding of distinctions between multiplication and partitive and measurement division	16 (69.6 %)	1, 2, 3, 4, 5, 7, 9, 10, 12, 14, 15, 16, 17, 19, 20, 23
1	Understanding of distinctions between multiplication and division, but no distinction between partitive and measurement division	4 (17.4 %)	6, 8, 13, 18
2	Understanding of distinctions between multiplication, and partitive and measurement division	3 (13 %)	10, 21, 22

According to Table 4, most of the prospective teachers (69.6%) did not understand the distinction between multiplication, and partitive and measurement division. All of these prospective teachers solved the problems correctly, but they did not explain their reasoning behind their solutions. However, four prospective teachers (17.4%) provided evidence for their understanding of the distinctions between multiplication and division. They stated that the problems in Part 1 and in Part 3 were division problems, whereas the problem in Part 2 was a multiplication problem. Among the problems in Part 1 and in Part 3, they did not address any distinction between partitive and measurement division. The number of prospective teachers who understood the distinctions between multiplication, partitive and measurement division is three (13%). As an example, the explanation of P10 is provided below.

P10:

Because in the question [in Part 1],the total number of set is given, but the size of the set is not given. This is partitive division. [The question in Part 2 is] multiplication problem since the total number is asked. [The question in Part 3], the size of the set is given and the total number of set is asked. Thus, it is a measurement division.

As it can be seen from Table 4, prospective teachers had difficulty in discriminating between multiplication, and partitive and measurement division. Although all of them solved the problems successfully, they did not know the reasoning behind using multiplication and division.

Discussion and Conclusion

The purpose of this study was to investigate pre-service mathematics teachers' subject matter knowledge of the underlying concepts of standard and non-standard algorithms used to solve the problems with whole numbers. In line with this purpose, the tasks related to each algorithm were presented to the prospective teachers and they were asked to give detail about the answers students gave in the tasks. The results of the study showed that the majority of the prospective students could explain the value of 1 in the addition and subtraction problems and the reasons for adding 1 and 10 in the addition and subtraction problems, respectively. However, a few prospective teachers did not know the place value of 1 and the reasons for adding 1 in the addition and 10 in the subtraction problem. As previously stated, SMK is defined as knowing the facts, principles, and issues, knowing why these facts, principles, and issues are taught, and presenting justifications for mathematical explanations and procedures (Shulman, 1986). As most of the prospective teachers could explain the underlying concepts and meanings of standard algorithms, it could be concluded that the prospective mathematics teachers' subject matter knowledge was adequate to help the students learn and apply the standard algorithms to solve the problems with whole numbers. Furthermore, Hiebert and Lefevre (1986) stated that a teacher who understands the basic arithmetic facts has conceptual knowledge. In this sense, it could be concluded that in the present study the prospective teachers' conceptual knowledge was adequate to interpret the procedures and to perform them correctly in different situations. This result is in contradiction with the results of the earlier studies (Ma, 1999; Philipp et al., 2008) which found that prospective teachers struggle to explain the algorithms. For instance, Philipp et al. (2008) found that prospective teachers had difficulty in understanding standard multi-digit addition and subtraction. Furthermore, they emphasized that teachers' performance on standard multi-digit addition and subtraction was worse than non-standard subtraction and multiplication. The prospective teachers in the present study, on the other hand, had not participated in any professional development program; however, they had taken a number of mathematics courses and mathematics education courses such as Methods of Teaching Mathematics and Nature of Mathematical Knowledge for Teaching. In those mathematics courses, the prospective teachers discussed the underlying meaning of the algorithms in detail and prepared activities related to the algorithms. Their knowledge of the place value of 1 in addition and in subtraction might be higher than the knowledge of prospective teachers who participated in the previous studies due to these discussions and the activities.

The analysis based on the nonstandard algorithm in subtraction indicated that almost half of the prospective teachers' explanation related to the underlying concepts of the nonstandard subtraction algorithm was missing or incorrect. Only one third of the prospective teachers explained why the nonstandard subtraction algorithm made mathematical sense. These teachers based their reasoning of the nonstandard algorithm on the ones and the tens. This finding was in agreement with the study of Siegfried, Gordon and Garcia (2007). In the study of Siegfried et al. (2007), most of the prospective teachers had difficulty in explaining the nonstandard algorithm because of the lack of specific content knowledge for teaching mathematics. This situation could be valid for the present study in which prospective teachers might not have enough knowledge to justify the reasoning behind the nonstandard algorithm, whereas they had knowledge to reason the underlying concepts of standard algorithm. Based on Shulman's definition of SMK, it could be concluded that most of the prospective teachers lacked SMK. In other words, since explaining the underlying concepts of nonstandard subtraction algorithm is a kind of knowledge specific to mathematics teachers (Ball et al., 2008), the prospective teachers' specialized content knowledge was not enough to explain this algorithm conceptually. Although their knowledge is not enough to explain the algorithm, these prospective teachers could apply the nonstandard algorithm to the similar subtraction operation. While applying the nonstandard subtraction algorithm to another algorithm, they most probably tried the same operations by changing the numbers. Interestingly, but perhaps not surprisingly, almost all prospective teachers applied the nonstandard algorithm in multiplication to another set of numbers. Put differently, these teachers could apply the systematic action sequences to perform the nonstandard algorithms. This could be interpreted as the prospective teachers' having procedural knowledge about nonstandard algorithm in subtraction and multiplication. On the other hand, Rittle-Johnson et al. (2001) stated that a task that included a nonstandard algorithm or unfamiliar procedures could be used to assess someone's conceptual knowledge. From this point of view, the Andrew task and the Doubling task could also be used to evaluate teachers' conceptual knowledge beside their SMK. As most of the prospective teachers in our study could not explain the underlying meaning of the nonstandard algorithm in these tasks, it can be concluded that their conceptual knowledge is not adequate to teach these concepts effectively. In other words, the prospective teachers could apply mathematical algorithms and perform them accurately and efficiently (National Research Council [NRC], 2001). Similarly, Philipp et al. (2008) reported that the participants in their study had high performance on applying nonstandard algorithm to multiplication even though the algorithms were not familiar to them.

Another point worth discussing is the prospective teachers' knowledge of mathematics on division. The important point regarding division is that the prospective teachers recognize the difference between multiplication and division and between partitive and measurement division. Based on the analysis of the data, only a few prospective teachers provided justification for the distinctions between multiplication, partitive division and measurement division. It is surprising that more than half of the prospective teachers did not explain the difference between multiplication, partitive division and measurement division. Furthermore, a few prospective teachers had the understanding of the distinction between multiplication and division, whereas they do not know the difference between partitive division and measurement division. In other words, they know that multiplication requires finding the total number of objects and division requires finding the number of objects in each group or finding the number of groups. However, they did not realize which quantity is the unknown. In more detail, the prospective teachers did not know the distinction between whether the number of groups is the unknown or the number of objects in each group is unknown. This result lets me conclude that most of the prospective teachers' understanding of division in whole numbers is not adequate to teach these algorithms to students deeply. Similarly, Ball (1990) stated that prospective teachers could not explain the underlying principles and meanings of the division with fraction. According to Ball (1990), prospective teachers had a narrow understanding of division. On the contrary, Philipp et al. (2008) concluded that prospective teachers' mean score in Division Task was high. Thus, they stated that teachers had enough knowledge to explain the difference between multiplication, partitive division and measurement division.

The results of the study revealed that more than half of the prospective mathematics teachers had knowledge about the place value of 1 in addition and subtraction, and also in multiplication. However, most of the prospective teachers could not explain the underlying principle and the meaning of the Andrew's strategy, which contains subtraction. Similar to their knowledge on subtraction, prospective teachers' knowledge on division was narrow. As a conclusion, the prospective teachers' SMK on the algorithms in standard addition, subtraction, and multiplication was found to be high; however, their SMK on nonstandard subtraction and division was found to be low. In other words, it could be stated that most of the prospective teachers have conceptual knowledge related to addition and subtraction, but they have procedural knowledge on nonstandard subtraction and division. Based on these findings, some implications and recommendations could be suggested. As basic algorithms with whole numbers is a central theme in the mathematics curriculum, it is very important to have this knowledge for effective teaching. Therefore, prospective teachers should extend their knowledge on this topic, and instructors should give them opportunities in this regard. In order to do this, the class hours of the lessons containing content knowledge might be increased and the underlying principles of the basic algorithms with whole numbers might be discussed. Moreover, the opportunities might be provided to prospective teachers to gain experience in basic algorithms during school experience and practice teaching in elementary education courses.

Although the present study provided interesting findings that contribute to the literature and teaching practices, there is still more to explore. In the present study, the prospective mathematics teachers' knowledge on basic algorithms with whole numbers was analyzed. When the importance of teachers' knowledge on students' mathematical understanding is considered, it is essential to conduct similar studies with Turkish in-service primary school and mathematics teachers to reveal their knowledge and understanding of the basic algorithms. Apart from the whole numbers, it would be valuable to investigate teachers' knowledge on basic algorithms with different number sets. As a final point, data might be collected via follow-up interviews to gain an in-depth understanding of prospective teachers' knowledge of algorithms with whole numbers.

References

Ball, D. L. (1990). Prospective elementary and secondary teachers' understanding of division. *Journal of Research in Mathematics Education, 21*, 132-144.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389- 407.

Creswell, J. W. (2007). *Qualitative inquiry and research design: Choosing among five approaches* (2nd ed.). Thousand Oaks, California: Sage Publications, Inc.

Engelbrecht, J., Harding, A., & Potgieter, M. (2005). Undergraduate students' performance and confidence in procedural and conceptual mathematics. *International Journal of Mathematical Education in Science and Technology*, *36*(7), 701-712.

Even, R. (1993). Subject matter knowledge and pedagogical content knowledge: Prospective secondary teachers and the function concept. *Journal for Research in Mathematics Education, 24*(2), 94-116.

Grossman, P. L. (1990). *The making of a teacher: Teacher knowledge and teacher education*. Teachers College Press, Teachers College, Columbia University.

Hiebert, J., & Lefevre P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), C*onceptual and procedural knowledge: the case of mathematics*, (pp.1- 27). Hillsdale, NJ: Lawrence Erlbaum Associates.

Isiksal, M. (2006). *A study on pre-service elementary mathematics teachers" subject matter knowledge and pedagogical content knowledge regarding the multiplication and division of fractions.* Unpublished doctoral dissertation, Middle East Technical University, Turkey.

Lucus, C. A. (2006). Is subject matter knowledge affected by experience? The case of composition of functions. In J. Novotna, H. Moraova, M. Kraka & N. Stehlikova (Eds.) *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 97-104). Prague: PME

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.

Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation*. San Francisco: Jossey-Bass.

National Research Council. (2001). Adding it up: Helping children learn mathematics. Washington, DC: National Academy Press.

Nilsson, P., & Lindstrom, T. (2012, July). *Connecting Swedish compulsory school teachers' content knowledge of probability to their level of education, teaching years and self-assessments of probability concepts.* Paper presented at the 12th International Congress on Mathematical Education, Seoul, Korea.

Philipp, R., Schappelle, B., Siegfried, J., Jacobs, V., & Lamb, L. (2008). The effects of professional development on the mathematical content knowledge of K-3 teachers. In *annual meeting of the American Educational Research Association, New York*.

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology*, *93*(2), 346.

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The Knowledge Quartet and the case of Naomi. *Journal of Mathematics Teacher Education, 8*(3), 255-281.

Siegfried, J. M., Gordon, J. M., & Garcia, J. R. (2007). Barely in STEP: How professional development affects teachers' perspectives on and analysis of student work. In T. Lambert & L. R. Wiest (Eds.), *Proceedings of the 29th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. University of Nevada: Reno.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harward Educational Review, 57*(1), 1-22.

Strauss, A., & Corbin, J. M. (1990). *Basics of qualitative research: Grounded theory procedures and techniques*. Newbury Park, California: Sage Publications, Inc.

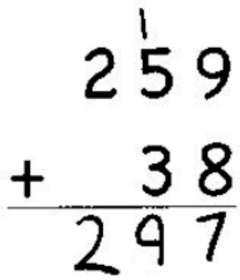
Thanheiser, E. (2009). Preservice elementary school teachers' conceptions of multidigit whole numbers. *Journal for Research in Mathematics Education*, *40*(3), 251-281.

Appendix

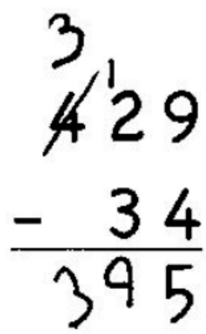
Ones Task

Below is the work of Terry, a second grader, who solved this addition problem and this subtraction problem in May.

Problem A



Problem B

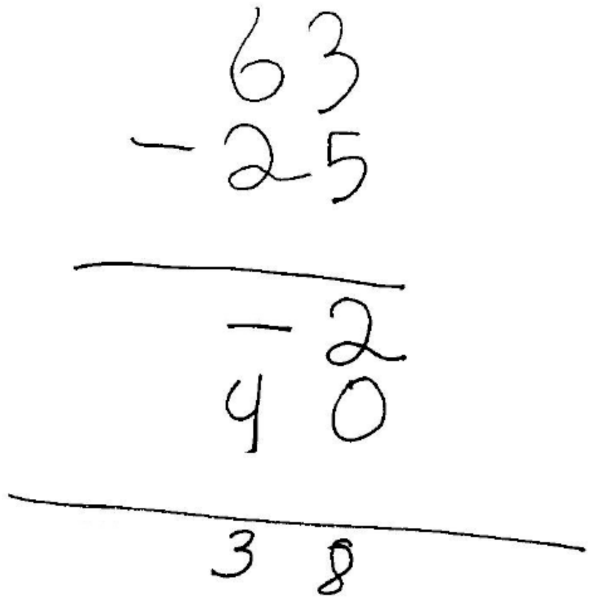


- Does the 1 in each of these problems represent the same amount? Please explain your answer.

- Explain why in addition (as in Problem A) the 1 is added to the 5, but in subtraction (as in Problem B) 10 is added to the 2.

Andrew Task

In March, Andrew, a second grader, solved 63 – 25 = □ as shown below.



- Explain why Andrew's strategy makes mathematical sense.

- Please solve 432 – 162 = □ by applying Andrew's reasoning.

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  432
– 162
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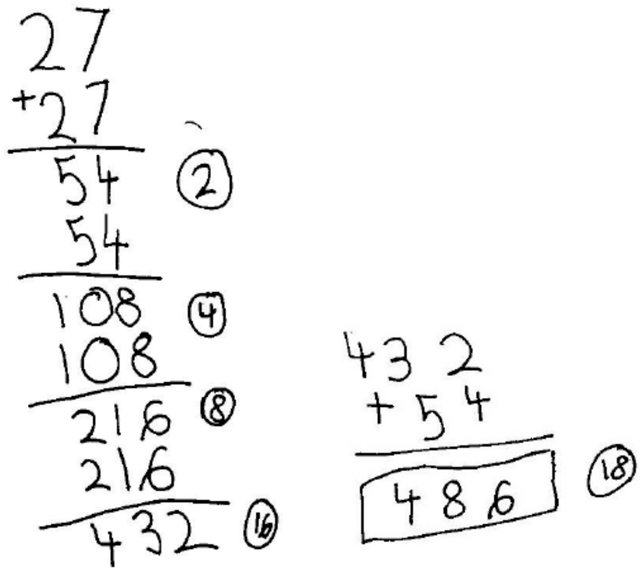
Doubling Task

In November, Todd, a third grader, solved the following problem as shown below:

The teacher bought 18 boxes of stickers with 27 stickers in each box. How many new stickers does she have?



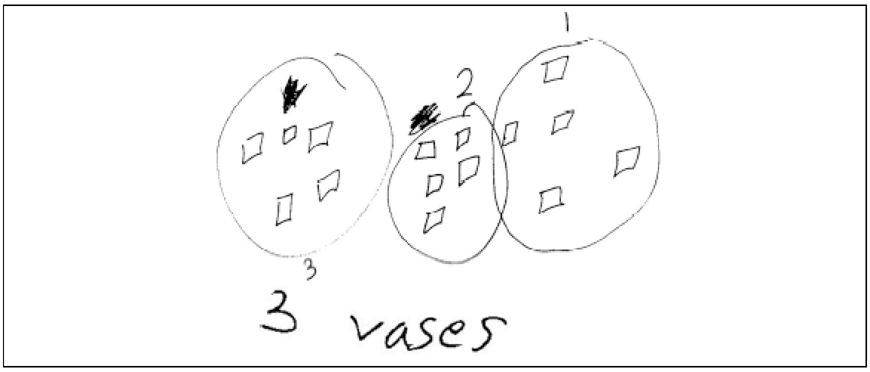
- Please solve 33 x 19 = □ by using Todd's reasoning.

Division Task

(Sufficient space was provided for full written responses.)

The teacher needs to put 15 flowers in vases. Each vase can hold 5 flowers. How many vases does she need?

Russ, a first grader, solved this problem in February. To solve the problem, he counted out 15 linking cubes. He pulled out a group of 5 cubes and then another group of 5 cubes and then another group of 5 cubes, so all the cubes were gone. Then he counted the number of groups, "1, 2, 3," and said that 3 vases were needed. (Below is his record of his work.)



Please think about the following problems and whether Russ is likely to use similar reasoning when solving them.

Part 1 There are 20 children on the playground. The teacher wants to play a game with 4 teams. How many children will be on each team if each team has the same number of children?

• To solve this problem, is Russ likely to use reasoning similar to the reasoning he used to solve the vases problem? ☐ yes ☐ no Why or why not?

• If you answered no, describe one strategy a young child would be likely to use to solve this problem.

Part 2 On Monday, 10 children went to the library and each checked out 3 new books. How many new books did they bring back to class altogether?

• To solve this problem, is Russ likely to use reasoning similar to the reasoning he used to solve the vases problem? ☐ yes ☐ no Why or why not?

• If you answered no, describe one strategy a young child would be likely use to solve this problem.

Part 3 A class of 18 children is going to the zoo. Each car has seatbelts for 6 children. How many cars will be needed to transport all the children to the zoo?

• To solve this problem, is Russ likely to use reasoning similar to the reasoning he used to solve the vases problem? ☐ yes ☐ no Why or why not?

• If you answered no, describe one strategy a young child would be likely use to solve this problem.