Mathematical problem-solving and metacognitive skills of 5th grade students as a function of gender and level of academic achievement

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Abstract

This study aimed to investigate differences in the mathematical problem-solving and metacognitive skills of the fifth-grade students in Oman as a function of gender and level of academic achievement. The participants were 90 grade five students randomly selected from one educational governarate in the Sultanate of Oman. They were evenly divided into three groups based on gender and levels of academic achievement. Four instruments were used in the study: a mathematical problem-solving test, a non-verbal metacognitive scale, Raven Coloured Progressive Matrices and a long-term memory test. The results for metacognitive and mathematical problem-solving skills indicate that students with a high level of academic achievement obtained the highest score while students enrolled in a learning disability program obtained the lowest score. In addition, possible interventions were identified that may improve the metacognitive skills of students enrolled in the learning disability program, which could lead to improvement in their mathematical problem-solving skills.

Keywords: Problem-solving, metacognition, learning disability, academic achievement.
1. Introduction

Performance in mathematics relies not only on what we know but also on how, and how efficiently, we employ our knowledge of concepts and facts (Schoenfeld, 2014). Mathematics engages our problem-solving skills: an individual’s understanding of mathematics is affected by the way he or she uses the information provided (Lambdin, 2003). According to Silver (2013), a problem is a situation in which a goal is to be achieved but the direct route to attain the goal is blocked. Courses in problem-solving often refer to mathematical ‘tasks that have the potential to provide intellectual challenges for enhancing students’ mathematical understanding and development’ (Cai & Lester, 2010, p. 1). Mathematical problem-solving therefore helps individuals to build logical reasoning skills that can be useful in many situations (Mullis, Martin, Foy & Arora, 2012). Accordingly, the Ministry of Education in the Sultanate of Oman stipulates that 30% of mathematics assessment in cycle one schools should evaluate mathematical problem-solving skills. However, this percentage drops to 20% of mathematics assessment when students move to cycle two of their education (i.e., fifth grade until the 10th grade).

Mathematical problem-solving tasks become more difficult as students progress through the school grades, and they prove to be a real challenge for students at every level of achievement. However, students at risk of underachieving in mathematics and students with learning disabilities are at a particular disadvantage. According to a number of studies (Geary, Hoard, Nugent & Bailey, 2012; Hessels & Schwab, 2015; Krawec, 2014; Shondrick, Serafica, Clark & Miller, 1992), students with learning disabilities show significantly lower performance on mathematical problem-solving tasks compared to their peers at different levels of academic achievement. These findings indicate that such students need support to enhance their academic performance and to increase their chances of developing more effective thinking behaviours. They also need support to learn how to process efficiently the information contained in problem-solving questions presented as mathematics problems. This requires students to develop an awareness of their abilities, to be capable of managing their knowledge and problem-solving strategies, and to continue to evaluate their own thinking. All of these requirements fall within the category of ‘metacognitive thinking’.

The term ‘metacognition’ was coined by Flavell in the late 1970s to refer to ‘one’s knowledge concerning one’s own cognitive processes, outcomes and anything related to them’ (Flavell, 1976, p. 232). Nine years later, Flavell offered a model of cognitive monitoring (Flavell, 1979) in which he discussed the actions of and interactions among four classes of phenomena related to cognitive monitoring: metacognitive knowledge, metacognitive experience, goals or tasks, and actions or strategies. Each of these classes consists of a number of components that play an important role in monitoring a wide variety of cognitive enterprises.

Between that time and the late 1990s, the term metacognition was subject to a variety of definitions and implementations. However, a review of the literature indicates that it was in the early 1980s that researchers started to focus on the relationship between metacognition and mathematical problem-solving (Schoenfeld, 1992). At the beginning of the present century, Wilson used the term metacognition to refer to the awareness individuals who have their own thinking; their evaluation of that thinking; and the individuals’ regulation of that thinking (Wilson, 2001). Two years later, Wilson and Clarke (2004) suggested a model of mathematical metacognition based on three main domains: metacognitive awareness, metacognitive evaluation and metacognitive regulation.

Taking these definitions as a starting point, the present paper investigates the differences among the students in terms of both metacognition and mathematical problem-solving. To achieve our aims, we integrated two models: the cognitive monitoring model (Flavell, 1979) and the mathematical metacognition model (Wilson & Clarke, 2004). We will therefore focus on three dimensions of metacognition in particular: metacognitive awareness, self-evaluation and self-regulation.

The first dimension, metacognitive awareness, takes account of the extent to which individuals know themselves as thinkers and problem solvers. There are three subcategories of metacognitive

awareness: person awareness, task awareness and strategy awareness. Person awareness includes intra-individual differences (e.g., knowing which subjects you are best at) and inter-individual differences (e.g., knowing how you learn differently from your peers). Task awareness involves knowing what the demands of a task are, where the task sits in the learning process or the problem situation and the balance required between acquired knowledge and information presented in the task. Strategy awareness is the individual’s awareness of convenient strategies for solving a specific problem and of what support might be needed to acquire relevant experience and strategies.

The second dimension, metacognitive evaluation, results from self-reflection. When individuals monitor their own thinking processes, they monitor their preferences for various thinking behaviours and observe the strategies they use for solving mathematical problems or dealing with specific learning situations. By means of this self-evaluation, individuals build judgments and opinions about how useful their chosen strategies are and whether their study habits are effective. In short, metacognitive evaluation is the individual’s assessment of their thinking ability and their performance in learning situations.

In the third dimension, metacognitive regulation, individuals make use of their self-awareness and self-evaluation to justify employing a specific strategy in a particular learning situation. As Wilson and Clarke (2004) suggested, metacognitive regulation occurs when individuals take advantage of their metacognitive skills to direct their cognition, knowledge, thinking and learning.

We can conclude from this that the three dimensions of metacognition—metacognitive awareness, metacognitive evaluation and metacognitive regulation—are related to each other and may occur reciprocally. The more individuals know about their own thinking processes, and the more they continue to evaluate those processes, the better able they are to make accurate plans, set realistic goals, manage their time, determine adequate strategies and carry out self-correction. Metacognition is therefore central to help students improve and update their study skills, carry out effective self-management activities and increase their academic competence (Gettinger & Seibert, 2002).

In terms of metacognitive skills, one cannot plan any task without implementing cognitive activities, such as generating steps for solving the problem and sequencing those steps (Veenman, Van Hout-Wolters & Afflerbach, 2006). However, cognitive skills include instructional objectives, components in a learning hierarchy and components in information processing, metacognitive skills focus on strategies for dealing with (for example) mathematics (Mayer, 1998) and play an essential role in problem-solving (Flavell, 1979). Both cognitive skills and metacognitive skills are therefore required for successful problem-solving in mathematical situations.

According to Fisher (1998), children differ in their ability to solve problems or to learn from experience. Individual variations of that kind are related to differences in intelligence, learning experiences and use of metacognition processes. A further differentiating factor among the students in solving a mathematical problem is the degree of dependence on memory in order to retrieve relevant concepts, facts or decomposition strategies (Laski et al., 2013). This accounts for the fact that students at risk of falling behind in mathematics show poorer performance than their peers on tasks that require the use of long-term memory to deduce or transform information (Swanson & Beebe-Frankenberger, 2004). However, Kramarski, Mevarech and Arami (2002) found that students who had received instructions in metacognition, chose the best strategy for solving a problem and outperformed their counterparts; their findings applied to both high achievers and low achievers. In addition, a review of the literature reveals that metacognition can improve students’ learning performance partly independent of their intellectual ability or intelligence (see van der Stel & Veenman, 2014; Veenman & Spaans, 2005; Veenman, Wilhelm & Beishuizen, 2004). Consequently, the aim of the present study is to investigate whether differences in metacognitive skills and mathematical problem-solving performance can be found among three groups of students at different levels of academic achievement after controlling for intelligence and memory.
1.1. Objectives

The objectives of the present study were to:

- Investigate the differences in mathematical problem-solving skills among grade five students as a function of gender and level of academic achievement after controlling for intelligence and long-term memory.
- Investigate differences in metacognitive skills among grade five students as a function of gender and level of academic achievement after controlling for intelligence and long-term memory.

2. Methods

2.1. Research design

This study employed a descriptive research design. The study aimed at describing differences among fifth-grade students in metacognition and mathematical problem-solving with respect to the level of academic achievement and gender.

2.2. Participants

The population of the study consisted of 6,381 fifth-grade students enrolled in all schools of Al-Batinah South governorate in the Sultanate of Oman (mean age = 9.90 years, standard deviation 0.47 years). The sample included 90 students randomly selected from the schools of the governorate. The selection of the sample was segregated by gender and the level of academic achievement. With respect to the level of academic achievement, the students were classified into three groups: students enrolled in a learning disability program, average achievers and high achievers. As for the students enrolled in a learning disability program, they have been nominated by their teachers as students at risk of underachieving in mathematics, then they have been through diagnostician processes according to a number of procedures that followed by Ministry of Education in the Sultanate of Oman to determine the eligibility for enrolling them in the learning disability program. Average achievers are the students with an overall grade of C across six basic school subjects including Islamic education, Arabic language, English language, mathematics, science and history. High achievers are students with an overall grade of A across the aforementioned six basic school subjects. Each group consisted of 30 students with 15 males and 15 females per group.

2.3. Instruments

Four instruments were used in this study: a mathematical problem-solving test, a non-verbal metacognitive scale, Raven Coloured Progressive Matrices, and a long-term memory test. Following is a description of these instruments.

2.3.1. Mathematical problem-solving test

An achievement test was built by the authors to measure students’ ability to solve mathematical problems. The test consisted of 14 items, all of which were multiple choice questions. The range of possible scores was from 0 to 14. Each item was constructed as a problem in four categories that were: numbers and number theory, operations with whole numbers, geometry and measurement. These categories were selected from the Omani’s mathematical curriculum of grade four. A panel of experts from educational measurement and mathematics education reviewed the items of the test in terms of clarity, relevance to the construct being measured and appropriateness for the target participants. Their feedback was used for the refinement of the test items. Pearson product-moment
correlation coefficients between total test score and subscales’ scores ranged between 0.68 and 0.79. Internal consistency reliability coefficient was 0.73 as measured by Cronbach’s alpha.

2.3.2. Non-verbal metacognitive skills test

Two cognitive–metacognitive models were utilised in the development of this scale. These were Flavell’s (1979) and Wilson and Clarke’s (2004) models. The scale consisted of 12 items divided into three categories: metacognitive awareness, self-regulation and self-evaluation. Each item described a learning situation with five possible solutions reflecting various metacognitive behaviours expressed in pictures. The students were asked to select the most appropriate solution applicable to them in their real life. The range of possible scores for the scale was from 12 to 60. A panel of experts from educational psychology and mathematics education reviewed the items of the test in terms of clarity, relevance to the construct being measured, and appropriateness for the target participants. Their feedback was used for the refinement of the test items. Pearson product-moment correlation coefficients between total test score and subscales’ scores ranged between 0.79 and 0.90. Internal consistency reliability coefficient was 0.80 as measured by Cronbach’s alpha.

2.3.3. Long-term memory test

The sixth episode of a television series called *wamahyaya* (Community Outreach Program, 2014) was implemented for constructing the memory test. The main subject of the video show was about tongue. The test consisted of 11 items all of which were multiple choice based on the presented information. The range of possible scores for the test was from 0 to 11.

According to Sousa (2006), if students are examined for a memory after 24 hours without informing them about the test and they have remembered the concept, then this means that the information had travelled to student working memory, which is one step closer to entering the long-term memory. In this context, the students were required to watch the video show for 13 minutes and pay good attention to the presented information, but without informing them about the test. On the next day, they were asked to retrieve a specific information from that show. A number of considerations have been ensured before selecting the show’s material such as the appropriateness of the information to the intellectual development of the participants (mean age = 9.9 years). According to Piaget’s cognitive stage of development, the participants were in the concrete operational stage (7–11 years) and close to the formal operational stage (Shaffer & Kipp, 2013). Also, the content of the show’s material was aligned with the curriculum of the fifth grade and was new to the participants by not being taught in the class. A panel of experts from educational psychology reviewed the items of the test in terms of clarity, relevance to the construct being measured and appropriateness for the target participants. Their feedback was used for the refinement of the test items. Internal consistency reliability coefficient was 0.77 as measured by Cronbach’s alpha.

2.3.4. Raven coloured progressive matrices

Raven Coloured Progressive Matrices is a non-verbal group test brought by John Carlyle Raven and his colleague Penrose in 1939. The test consisted of 36 items used in measuring abstract reasoning and regarded as a non-verbal estimate of fluid intelligence. It consisted of 36 multiple choice questions, listed in order of difficulty in three dimensions: A, Ab and B, with 12 items in each one. In each test item, the student is asked to identify the missing element that completes a pattern that consisted of six parts. The test was standardised to groups ranging from 5 to 11 years old (Kazem et al., 2007). Pearson product-moment correlation coefficients between total test score and subscales’ scores ranged between 0.71 and 0.86. Internal consistency reliability coefficient was 0.82 as measured by Cronbach’s alpha.
2.4. Procedures

Permission was obtained from Ministry of Education to collect data for the purposes of this study. Three steps were followed in the data collection process. First, the Raven Coloured Progressive Matrices test was given to the students. Second, the Mathematical Problem-Solving test was administered. The students in each group were assessed together. Ten minutes were used to organise the classroom to be a suitable testing room and to read test’s instructions. Then, the examiner gave the students 5 minutes for each question. During these 5 minutes, the examiner read the question for the students and asked the students to think and justify their answers in an empty space in the left side of the question before they choose the answer. After they finished the 5 minutes for each question, the examiner gave the students an extra 10 minutes to check their answers. Therefore, the total time of the test was 80 minutes, in addition to the 10 minutes at the beginning of the testing process. Third, the Non-Verbal Metacognitive Skills test was implemented. The students were assessed individually. Efforts were considered to ensure that each student understands every single non-verbal item and its choices. Finally, the Long-Term Memory test was administered. The students watched the video show without a prior warning about any kind of test following the show. After 24 hours, they were tested based on some information from the show.

2.5. Data analysis

A two-way analysis of covariance was used to examine the differences in mathematical problem-solving skills and metacognitive skills among fifth-grade students as a function of gender and level of academic achievement after controlling for intelligence and long-term memory. The analysis was run for each dependent variable separately. There were no violations in the data with respect to the assumptions of linearity, normality, homogeneity of variance, independence of covariates and independent variables and homogeneity of regression slopes.

3. Results

3.1. Metacognitive skills

Table 1 presents means and standard errors of the students’ scores on the non-verbal metacognitive skills test after controlling for intelligence and long-term memory. Results of the two-way analysis of covariance showed no statistically significant gender differences on the non-verbal metacognitive skills after controlling for intelligence and long-term memory; $F(1,82) = 2.63, p = 0.109$. However, there were statistically significant differences on the non-verbal metacognitive skills among the students with respect to the level of academic achievement after controlling for intelligence and long-term memory; $F(2,82) = 14.63, p = 0.000$, partial $\eta^2 = 0.26$. Post-hoc comparisons using LSD showed that high achievers tended on average to outperform both the students enrolled in a learning disability program and average achievers on the non-verbal metacognitive skills test after controlling for intelligence and long-term memory. Also, average achievers tended on average to outperform the students enrolled in a learning disability program on the non-verbal metacognitive skills test after controlling for intelligence and long-term memory. There was no statistically significant interaction effect between the gender and the level of academic achievement on the non-verbal metacognitive skills after controlling for intelligence and long-term memory; $F(2,82) = 0.882, p = 0.418$. 

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Table 1. Means and standard errors of the students’ scores on the non-verbal metacognitive test after controlling for intelligence and long-term memory

<table>
<thead>
<tr>
<th>Variable</th>
<th>n</th>
<th>M</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Males</td>
<td>45</td>
<td>43.63</td>
<td>0.82</td>
</tr>
<tr>
<td>Females</td>
<td>45</td>
<td>45.53</td>
<td>0.82</td>
</tr>
<tr>
<td>Level of academic achievement</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Students enrolled in a learning disability program</td>
<td>30</td>
<td>40.13</td>
<td>1.21</td>
</tr>
<tr>
<td>Average achievers</td>
<td>30</td>
<td>43.36</td>
<td>1.01</td>
</tr>
<tr>
<td>High achievers</td>
<td>30</td>
<td>50.24</td>
<td>1.20</td>
</tr>
</tbody>
</table>

3.2. Mathematical problem-solving skills

Table 2 presents means and standard errors of the students’ scores on the mathematical problem-solving skills test after controlling for intelligence and long-term memory. Results of the two-way analysis of covariance showed statistically significant gender differences on the mathematical problem-solving skills test after controlling for intelligence and long-term memory; $F(1,82) = 8.26$, $p = 0.005$, partial $\eta^2 = 0.09$. As shown in Table 1, male students tended on average to outperform female students on the mathematical problem-solving skills test after controlling for intelligence and long-term memory. Also, there were statistically significant differences on the mathematical problem-solving skills among the students with respect to the level of academic achievement after controlling for intelligence and long-term memory; $F(2,82) = 14.77$, $p = 0.000$, partial $\eta^2 = 0.27$. Post-hoc comparisons using LSD showed that high achievers tended on average to outperform both the students enrolled in a learning disability program and average achievers on the mathematical problem-solving skills test after controlling for intelligence and long-term memory. Also, the average achievers tended on average to outperform the students enrolled in a learning disability program on the mathematical problem-solving skills test after controlling for intelligence and long-term memory. There was no statistically significant interaction effect between gender and level of academic achievement on the mathematical problem-solving skills after controlling for intelligence and long-term memory; $F(2,82) = 2.85$, $p = 0.064$.

Table 2. Means and standard errors of the students’ scores on the mathematical problem-solving skills test after controlling for intelligence and long-term memory

<table>
<thead>
<tr>
<th>Variable</th>
<th>n</th>
<th>M</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Males</td>
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<td>7.60</td>
<td>0.35</td>
</tr>
<tr>
<td>Females</td>
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<td>6.18</td>
<td>0.35</td>
</tr>
<tr>
<td>Level of academic achievement</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The students enrolled in a learning disability program</td>
<td>30</td>
<td>5.21</td>
<td>0.51</td>
</tr>
<tr>
<td>Average achievers</td>
<td>30</td>
<td>6.16</td>
<td>0.42</td>
</tr>
<tr>
<td>High achievers</td>
<td>30</td>
<td>9.30</td>
<td>0.51</td>
</tr>
</tbody>
</table>

4. Discussion

This study aimed to investigate the differences in mathematical problem-solving and non-verbal metacognitive skills among fifth-grade Omani students with varying levels of academic achievement after controlling for age and intelligence. The results show that there are significant differences in the non-verbal metacognitive skills among the three groups of students. Students in a learning disability program exhibited the lowest score in these skills, while the high achieving students showed the highest score. This result corroborates the previous findings of a study we conducted (Al Shabibi, 2017), which suggested that academic achievement was responsible for these differences. Accordingly, students in special education curricula (e.g., students with learning disabilities) tend to
perform at a lower level of metacognitive knowledge than other students (Handel, Lockl, Heyrdich, Weinert & Artelt, 2014). This is because metacognitive skills are related to several enablers of achievement (Kitsantas, Steen & Huie, 2017), such as study habits, study orientation and study attitudes of students with different academic levels (Ozsoy, Memis & Temur, 2017). In addition, metacognitive skills are also related to motivational variables (Van Kraayenoord & Schneider, 1999). Therefore, motivated students tend to show the best metacognitive self-regulation skills of all students (Dekker et al., 2016), which may explain the differences in metacognitive skills among the three groups of students with the respects to their differences in academic achievement.

However, the results demonstrate that there are no statistically significant differences in metacognitive skills between male and female students. This finding agrees with those of previous studies (see Al Shabibi, 2017; Siswati & Corebima, 2017). In this regard, Golzadeh and Moiinvaziri (2017) determined that although male and female students may use different metacognitive strategies in learning, there are no gender differences in the overall score on metacognitive strategies. On the other hand, Baas, Castelijns, Vermeulen, Martens and Segers (2015) found that girls outperformed boys in the application of metacognitive strategies in learning situations, product evaluation and process evaluation. However, these differences may be attributed to the specific characteristics of individuals and to their learning environment.

Similarly, as for metacognition results, the findings of the present study reveal that there are significant differences in mathematical problem-solving skills among the three groups. Students in the learning disability program achieved the lowest score, while students with a high level of academic achievement tended to score higher than the two other groups of students in these skills. A review of the literature reveals that metacognitive skills improve student learning performance somewhat independently of their intelligence level (see Van der Stel & Veenman, 2014; Veenman & Spaans, 2005; Veenman et al., 2004). Hence, we controlled for a number of factors (e.g., intelligence and long-term memory) which may positively affect students’ performance in mathematical problem-solving tasks. However, Guven and Cabakcor (2013) found that there is a relationship between the academic success and mathematical problem-solving skills, as we mentioned previously, and between academic success and metacognition skills. Therefore, we suggest that metacognitive skills played an essential role in our results; student performance can be enhanced by cultivating mathematical problem-solving skills.

Accordingly, students with a high level of academic achievement outperformed both students with an average level of academic achievement and students in the learning disability program in these skills. This is because students with a high level of academic achievement tend to activate their metacognitive skills in mathematical problem-solving situations by using effective strategies that lead to active thinking processes comparing with other students (Garcia, Betts, Gonzalez-Castro, Gonzalez-Pienda & Rodriguez, 2016). This is because metacognitive skills help students to successfully direct their learning processes, leading to an increase in mathematical problem-solving achievement level (Ozsoy et al., 2017). Moreover, previous studies found that instruction on self-regulation of metacognitive skills influences student performance in solving mathematical problems (Ozsoy et al., 2017; Tzohar-Rozen & Kramarski, 2014), and this intervention has a positive effect on students with both low and high levels of academic achievement (Kramarski et al., 2002). Therefore, the results of this study support previous findings which suggest that metacognition skills help students succeed at mathematical problem solving.

The present study also determined that there were statistically significant gender differences on mathematical problem-solving skills, indicating that male students tend to outperform female students. This result is in agreement with a number of previous studies that revealed gender differences favouring males over females in mathematical problem solving. Nevertheless, there were no gender differences found in our study between boys and girls in metacognitive skills. However, a review of the literature revealed that there are differences between males and females in the type of strategies they use (see Golzadeh & Moiinvaziri, 2017). Therefore, we suggest that although male and
female students may have similar metacognitive skills, they tend to apply these skills differently in learning situations or in tasks that require them to solve mathematics problems. As a result, these performance differences were reflected on these tasks.

5. Recommendations and conclusion

The results of the present study may draw educators’ attention to the need for further research, such as investigating differences among groups in metacognitive skills and mathematical problem solving after controlling for a number of factors related to student success in mathematical tasks (e.g., working memory or motivation). Moreover, the outcomes of our study may help educators focus on interventions that develop the metacognitive skills of students which accommodate student strengths and ability levels. Such interventions may improve student performance specifically in mathematical problem-solving tasks and help them succeed more generally in their academic endeavours.

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References


