MATHEMATICS FLUENCY AND TEACHING SELF-EFFICACY OF TEACHER CANDIDATES

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Abstract: Developing high levels of mathematical skills in students is a concern for teachers worldwide, since most of the fastest growing occupations require significant mathematics or science preparation. Teachers need to be prepared to teach mathematics successfully. If teachers are not fluent in simple mathematical calculations, they may not feel confident in applying complex pedagogical content knowledge. This paper explores the association between mathematics fact fluency and mathematics teaching self-efficacy. Fifty-seven elementary teacher candidates completed two assessments. The first assessment was a mathematics fact fluency assessment evaluating the participants’ fact fluency for the four basic mathematics operations: addition, subtraction, multiplication, and division. The second assessment was the commonly used Mathematics Teaching Efficacy Beliefs Instrument. Person r correlations were completed to examine the relationship between mathematics fact fluency and self-efficacy in teaching elementary mathematics. Positive relationships were found between participants’ scores on the Mathematics Teaching Efficacy Beliefs Instrument and the addition and multiplication fact fluency scores. Personal mathematics teaching efficacy was related to addition fact fluency with higher teacher candidates’ efficacy scores associated with the correct completion of more addition facts. Mathematics teaching outcome expectancy was positively related to addition, multiplication, and total facts completed.

Keywords: mathematics fluency, teaching self-efficacy, teacher candidates, cognitive load

Introduction

Mathematical fluency is defined as the ability to perform a computation with accuracy, speed, and minimal effort (Cates & Rhyme, 2003; Ramos-Christian, Schleser, & Varn, 2008). To date, much of the available research about mathematical fluency has investigated the effects of fluency on elementary students’ abilities, attitudes, and beliefs (Billington & Skinner, 2002; Cates & Rhymer, 2003; Poncy, Skinner, & Jasper, 2007; Ramos-Christian et al., 2008; Therrien, 2004). In general, as students’ mathematical fluency increased, performance in mathematical activities improved and mathematical anxiety decreased. However, there is seemingly less research exploring the relationship between teacher mathematical fluency and performance. Presumably, the same associations between mathematical fluency and beliefs should hold true for teachers as well as students.

Background

In the United States, the Principles and Standards for School Mathematics (National Council of Teachers of Mathematics [NCTM], 2000) emphasized computational fluency for all students. NTCM produced a document titled, Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics (NCTM, 2000). This document stressed, "developing quick recall of basic addition facts and related subtraction facts and fluency with multi-digit addition and subtraction" (p. 14). The Singapore Mathematics Syllabus (Primary one to four) also highlighted achieving mastery of basic addition, subtraction, multiplication, and division.

Researchers have (Poncy, Skinner, & Jasper, 2007; Ramos-Christian et al., 2008; Therrien, 2004) suggested that increasing students’ accuracy and speed (fluency) for basic math facts can increase their performance in mathematical activities. One explanation for increased student performance may be that automaticity of math facts reduces cognitive demands on working memory. Students who lack fluency may spend more of their working memory on recalling basic math facts, and thus, have less space in working memory to focus on other math activities. Kelley (2008) asserted that basic fact fluency is as central to higher-level mathematics as decoding is to reading. Some students have difficulty in their mathematics class because they are not fluent with their basic mathematics facts (Ramos-Christian et al., 2008). Furthermore, students who can perform fluently with basic mathematical skills are more likely to be successful in applying those skills to new and more advanced mathematical concepts and tasks (Poncy et al., 2007; Ramos-Christian et al., 2008). Researchers found that students who are fluent in basic mathematical skills are less anxious and more engaged in math activities than those who lack mathematical fluency (Billington & Skinner, 2002; Cates & Rhymer, 2003).

The complexity of effective mathematics teaching also can place more demands on the working memory of teachers, thus creating higher cognitive load. Cognitive load describes the total amount of mental effort needed to complete a task. Cognitive load is influenced by the nature of the task (i.e., number of elements that need to be processed, task complexity), and learners’ expertise (Antonenko, Paas, Grabner, & van Gog, 2010). The limited capacity of working memory sets the parameters for cognitive load. Since 1956, psychologists have maintained that the brain can only consciously hold between five and nine pieces of information at any one time (Miller, 1956). When working memory also is used to process the information in some manner, the number of items working memory can hold can drop to two or three (Sweller, van Merrienboer, & Paas, 1998). Thus, the more cognitive processing needed, the less ‘space’ available in working memory and conversely, the less taxing the cognitive processing, the more ‘space’ available in working memory. When teachers are able to retrieve math facts easily without calculation, they have more space in their working memory to consider how to modify their instruction so that their students can learn math more effectively.

Teachers of mathematics need to believe that they can teach mathematics effectively to students, as well as that they can perform mathematical tasks proficiently. “To be effective, teachers must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks” (NCTM, 2000, p. 17). This capacity relates to the “profound understanding of fundamental mathematics” described by Ma (1999, p. xxiii) which was found to be lacking in many elementary mathematics teachers in the United States. The National Research Council (2001) stated that teachers needed to be flexible in their mathematical practice, understanding the full horizon of the mathematics that they teach.

Teacher efficacy may influence teachers’ abilities to teach mathematics effectively (Holzberger, Philipp, & Kunter, 2013). Bandura (1997) defined self-efficacy as “beliefs in one’s capacity to organize and execute the courses of action required to produce given attainments” (p. 3). Efficacy is task specific and describes individuals’ sense of how successful they will be at certain tasks. Pajares (1997) described self-efficacy as “a context-specific assessment of competence to
Because self-efficacy is task specific, there has been considerable interest in developing task specific scales to measure different types of efficacy (Bandura, 2006). Enochs, Smith, and Huinker (2000) considered mathematics teaching efficacy as consisting of two constructs. Personal mathematics teaching efficacy (PMTE) involves teachers’ beliefs about their ability to be effective teachers. Mathematics teaching outcome expectancy (MTOE) involves teachers’ belief that their instruction can bring about student learning. Teacher self-efficacy is related to student achievement and more productive teaching behavior (Heneman, Kimball, & Milanowski, 2006; Woolfolk Hoy & Burek-Spero, 2005).

Poor mathematics teaching self-efficacy is related to mathematics anxiety (Bursal & Paznokas, 2006; Hadley, 2010; Swars, Daane, & Giesen, 2006). According to Raymond (1997), past school experiences can influence mathematic teaching practices. It may be that mathematics anxiety is caused in part by poor fact fluency. Therefore, low mathematics teaching self-efficacy of some teacher candidates may have resulted from struggles with fact fluency in elementary grades. Teacher candidates who struggle with math fluency also may struggle to achieve high mathematics teaching self-efficacy. In this study, we explored the relationship between mathematics fact fluency and mathematics teaching self-efficacy of elementary teacher candidates. We hypothesized that there would be a positive relationship between teacher candidates’ fact fluency and teaching self-efficacy beliefs.

Method

Participants

This study used a correlational design to explore the relationship between 57 elementary teacher candidates’ mathematics fact fluency and mathematics teaching self-efficacy beliefs. The study utilized all the elementary teacher candidates enrolled in a teacher education program during the semester before their student teaching practicum. The teacher candidates were female and ranged between 20 and 60 years of age. The teacher candidates attended a university in Northern Utah, USA.

Instruments

Each of the 57 participants completed two assessments. The first assessment was a fact fluency assessment that consisted of 100 single-digit addition problems, 100 single-digit subtraction problems, 100 single-digit multiplication problems, and 100 single-digit division problems. The assessment was created using a web-based worksheet generator. The problems for each operation were on separate sheets. Participants’ scores were based on the number of correct answers on each part of the assessment.

The second assessment that the participants completed was the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI: Enochs et al., 2000). This instrument is the one most commonly used in the literature to investigate mathematic teaching efficacy. Each item in the original assessment had five response categories; however, for this study each item was modified to consist of four response categories (see Appendix A). A four-point response scale was used to produce an ipsative (forced choice) measure where no indifferent option was available.

The MTEBI consists of two subscales: Personal Mathematics Teaching Efficacy (PMTE) and Mathematics Teaching Outcome Expectancy (MTOE). PMTE refers to teacher candidates’ beliefs about their own knowledge and abilities in
mathematics to become effective mathematics teachers. MTOE refers to teacher candidates’ beliefs about their ability to directly affect their students’ mathematical learning. For data analysis, some items were reverse coded so that higher scores always represented higher teaching self-efficacy. Subject specific and factorial validity for the MTEBI was established by the authors, with Cronbach alpha of .88 for the PMTE subscale and .77 for the MTOE subscale (Enochs et al., 2000). For this administration, the PMTE had a Cronbach alpha of .79 and the MTOE was .76.

**Procedure**

All teacher candidates participating in the current study submitted a signed informed consent form. The participants were assured that their responses would be kept confidential.

The basic math fact assessments were administered first. Since mathematical fluency is defined as the ability to perform mathematical tasks with accuracy and speed and minimal effort (Cates & Rhyme, 2003; Ramos-Christian et al., 2008), a time limit of one minute for each of the four assessments was chosen. The participants were instructed that they had one minute to complete as many problems as possible for each basic math facts assessment. They were informed that there would be insufficient time to complete all of the computational questions, but that they should complete as many items as possible. The participants were allowed to rest for one minute between each of the operations. Immediately after completing the four basic math facts assessments, the participants completed the MTEBI assessment. The MTEBI was administered through ChiTester, an online testing program. No time limit was imposed on the MTEBI assessment. It is unlikely that completing the math fact assessment immediately prior to completing the MTEBI biased the results as the two assessments measure very different abilities: math facts ability and perceived mathematical teaching ability. Furthermore, mathematical teaching efficacy is a task specific construct, not an overall feeling of confidence.

**Results**

Descriptive analyses were generated using the Statistical Package for the Social Sciences (SPSS) and presented a range of scores across the variables (see Table 1). The teacher candidates tended to score in the mid-to-high range with an average of 3 (agree) for both measures of mathematics teaching self-efficacy. Most of the participants were novice teachers of elementary mathematics, with some having taught only two math lessons at this point in their education. These ratings were a bit higher than expected based on their lack of experience and may indicate that they were metacognitively unaware.

For the correlational analysis, the mathematics teaching self-efficacy means were used as both were based on the same 1 to 4 scale rather than the scale scores that consisted of unequal numbers of items across the subscales. Pearson product-moment correlation coefficient was used and correlations were considered statistically significant if the p value was less than .05. As expected, the two self-efficacy subscales were positively associated to each other ($r = .51$) as they are both distinct parts of the larger construct. PMTE was positively associated to addition fact fluency ($r = .30$), with higher efficacy scores related to increased completion to addition facts.
Table 1
Descriptive Statistics for Mathematics Teaching Self-Efficacy and Fact Fluency Measures

<table>
<thead>
<tr>
<th>Measure</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMTE</td>
<td>31</td>
<td>50</td>
<td>39.75</td>
<td>4.34</td>
</tr>
<tr>
<td>PMTE mean</td>
<td>2.38</td>
<td>3.85</td>
<td>3.06</td>
<td>.33</td>
</tr>
<tr>
<td>MTOE</td>
<td>18</td>
<td>29</td>
<td>23.40</td>
<td>2.40</td>
</tr>
<tr>
<td>MTOE mean</td>
<td>2.25</td>
<td>3.63</td>
<td>2.93</td>
<td>.30</td>
</tr>
<tr>
<td>Addition facts</td>
<td>30</td>
<td>83</td>
<td>57.37</td>
<td>12.58</td>
</tr>
<tr>
<td>Subtraction facts</td>
<td>23</td>
<td>68</td>
<td>42.33</td>
<td>9.26</td>
</tr>
<tr>
<td>Multiplication facts</td>
<td>23</td>
<td>61</td>
<td>41.19</td>
<td>8.04</td>
</tr>
<tr>
<td>Division facts</td>
<td>17</td>
<td>58</td>
<td>33.98</td>
<td>9.18</td>
</tr>
<tr>
<td>Total facts</td>
<td>103</td>
<td>264</td>
<td>174.88</td>
<td>33.44</td>
</tr>
</tbody>
</table>

N = 57

*MTOE was positively associated with the number of addition (r = .28), multiplication (r = .31), and total facts (r = .30) completed correctly. Total self-efficacy was positively associated with addition (r = .33) and total facts completed (r = .30). All of the four fact fluency operations were positively related to each other (see Table 2).

Table 2
Pearson’s r Matrix for Mathematics Teaching Self-Efficacy and Fact Fluency Measures

<table>
<thead>
<tr>
<th>Variable</th>
<th>PMTE</th>
<th>MTOE</th>
<th>EFF</th>
<th>ADD</th>
<th>SUB</th>
<th>MULT</th>
<th>DIV</th>
<th>TOT</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMTE mean</td>
<td>1.00</td>
<td>.506*</td>
<td>.937**</td>
<td>.300</td>
<td>.182</td>
<td>.130</td>
<td>.198</td>
<td>.249</td>
</tr>
<tr>
<td>MTOE mean</td>
<td>1.00</td>
<td>.776**</td>
<td>.280*</td>
<td>.214</td>
<td>.310*</td>
<td>.222</td>
<td>.300*</td>
<td></td>
</tr>
<tr>
<td>Total efficacy</td>
<td>1.00</td>
<td>.333**</td>
<td>.786**</td>
<td>.219</td>
<td>.221</td>
<td>.235</td>
<td>.304*</td>
<td></td>
</tr>
<tr>
<td>Addition</td>
<td>1.00</td>
<td>.786**</td>
<td>.585**</td>
<td>.629**</td>
<td>.907**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subtraction</td>
<td>1.00</td>
<td>.527**</td>
<td>.664**</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiplication</td>
<td>1.00</td>
<td>.589**</td>
<td>.768**</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Division</td>
<td>1.00</td>
<td>.837**</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fact Total</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

*p < .05, **p < .01, N=57

PMTE mean (PMTE), MTOE mean (MTOE), Total efficacy (EFF), Addition (ADD), Subtraction (SUB), Multiplication (MULT), Division (DIV), Fact total (TOT)

Discussion

A positive association between teacher candidates’ addition and multiplication fact fluency and mathematics teaching self-efficacy was found in this study. Addition is the first basic math operation learned in school, with all other operations learned by linking one problem to a related problem (Garnett, 1992). Since multiplication is the most closely related operation to addition (being repeated addition), it makes sense for addition and multiplication to demonstrate a greater association with mathematics teaching self-efficacy relative to the other operations.

Teacher candidates who scored higher on the fact fluency tests reported higher total mathematics teaching self-efficacy. This was particularly so in outcome expectancy. It may be that teacher candidates who scored higher on the fact fluency tests used less of their working memory to recall math facts, thus making more working memory space available to think about the teaching process. Teaching mathematics can be a challenging cognitive task that creates a high cognitive load in teacher
candidates. If teacher candidates can recall math facts fluently, their cognitive load may be lessened and they may be more focused on mathematics teaching and thus, be confident in their ability to teach mathematics effectively. As teacher candidates’ math facts scores increased, so did their reporting of mathematics teaching self-efficacy. Further research is needed to discern whether increasing teacher candidates’ math fluency results in higher mathematics teaching self-efficacy, or whether higher mathematics teaching self-efficacy facilitates math fluency. Although previous research has found a correlation between mathematics teaching self-efficacy, effective instruction, and student achievement, further research also is needed to establish whether increased fact fluency in teacher candidates results in reduced math anxiety.

This research suggests it might be wise to have teacher candidates practice their basic math facts. As teacher candidates become more fluent in their math facts, the demands on their cognitive loads may lessen and allow them to think more critically and effectively about the complex task of teaching mathematics. Practice also may increase their mathematics teaching self-efficacy. Teachers’ sense of efficacy repeatedly has been found to impact student achievement (Woolfolk Hoy & Burke-Spero, 2005; Woolfolk Hoy, Hoy, & Davis, 2009). It may be that improving teacher candidates’ math fluency positively impacts their sense of efficacy and thus improves future student math achievement as well.

References


Appendix A

Mathematics Teaching Efficacy Beliefs Inventory
(Adapted from Enochs, Smith, & Huinker, 2000)

Please indicate the degree to which you agree or disagree with each statement as a teacher. (1=strongly agree, 2=agree, 3=disagree, 4=strongly disagree)

1. When one of my students does better than usual in mathematics, it is often because I exerted a little extra effort.
2. I continually find better ways to teach mathematics.
3. Even if I try very hard, I do not teach mathematics as well as I teach most subjects.
4. When the math grades of students improve, it is often due to my having found a more effective teaching approach.
5. I know how to teach mathematics concepts effectively.
6. I am not very effective in monitoring mathematics activities.
7. If students are underachieving in mathematics, it is most likely due to ineffective mathematics teaching.
8. I generally teach mathematics ineffectively.
9. The inadequacy of a student’s mathematics background can be overcome by good teaching.
10. When a low-achieving student progresses in mathematics, it is usually due to extra attention given by me.
11. I understand mathematics concepts well enough to be effective in teaching elementary mathematics.
12. I am generally responsible for the achievement of my students in mathematics.
13. Students’ achievement in mathematics is directly related to my effectiveness in mathematics teaching.
14. If parents comment that their child is showing more interest in mathematics at school, it is probably due to my performance.
15. I find it difficult to use manipulatives to explain to students why mathematics works.
16. I am typically able to answer students’ questions.
17. I wonder if I have the necessary skills to teach mathematics.
18. Given a choice, I do not invite the principal to evaluate my mathematics teaching.
19. When a student has difficulty understanding a mathematics concept, I am usually at a loss as to how to help the student understand it better.
20. When teaching mathematics, I usually welcome student questions.
21. I do not know how to turn students on to mathematics.

Notes: Items for the two scales were arranged randomly in the MTEBI.
The items designed to measure PMTE are:
   2, 3, 5, 6, 8, 11, 15, 16, 17, 18, 19, 20, and 21.
The items designed to measure MTOE are:
   1, 4, 7, 9, 10, 12, 13, and 14.

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