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Anthropological analysis of content knowledge of pre-service elementary mathematics teachers' on graphs

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The purpose of this study is to analyse the content knowledge on graphs of pre-service elementary mathematics teachers from an anthropological perspective. 112 pre-service elementary mathematics teachers participated in the study. Concentric mixed pattern research method has been used in the study. The data was collected through Graphic Content Knowledge Scale and interview method in order to examine the content knowledge of primary school math teacher candidates related to graphics anthropologically. This scale which was developed within the frame of determined institutional recognitions includes graphic concept and its usage conditions, graphic types and different display forms, making proper transformations between graphics related to a given context, graphic reading, graphic interpreting and graphic drawing skills. In addition, for the purpose of having a more detailed review of the individual recognitions of the 10 participants, a semi-structured interview scale has been developed and used during the interviews. Data acquired have been analysed by using ecologic and praxiologic approach suggested within the framework of Anthropological Theory of the Didactic. The knowledge scale and interview data have been subjected to content analysis and descriptive analysis to interpret the individual recognitions of the pre-service teachers within the scope of institutional recognitions. As a result, it has been observed that the individual recognitions of pre-service teachers related to column, circle and line graphs conform to the institutional recognitions, however in institutional recognitions the techniques specified for column graphs are being used for histogram, hence they have difficulties in comprehending the differences between histogram and column graph. It has also been concluded that they were not aware of a theory based on graph knowledge.

Key words: Anthropological theory of the didactic, ecological approach, graphic knowledge, mathematical content knowledge, praxiological approach, preservice teachers.

INTRODUCTION

Doing mathematics is not only restricted to knowing mathematical concepts, but also the skills of using concepts and the relations between these concepts in daily life and other disciplines (MoNE, 2013). In this

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direction, when individuals doing mathematics encounter a problem, they reach to a conclusion by using cognitive skills such as classification, reasoning, argument in order to solve the problem and they express these conclusions with various representation forms in mathematics. Representation forms ensure significant mathematical concepts to be comprehended and abstracted (McArthur et al., 1988; National Council of Teachers of Mathematics (NCTM), 2000). Graphics which is a part of the mathematical language is one of this representation forms.

Graphics which has various applications in discipline of mathematics and various discipline fields play a significant role in conceptual understanding by developing problem solving, relating between variables, comparing variables and predicting according to data (Cai and Lester, 2005; Duval, 1999; Friel et al., 2001; Schultz and Waters, 2000; Winn, 1991) while transferring the information to the reader by visualising it simply and essentially (Hiebert and Carpenter, 1992; Roth and Bowen, 2003). Graphics which interpret the information by concreting the abstract thoughts with visual elements (Altun, 2006), is among the essential concepts which is commonly used in reading, science and mathematics literacy (Long, 2000). The usage of graphics in various fields of daily life such as health and economy which concerns society requires the raising of conscious individuals who can comprehend graphics at a basic level.

Even though the graphics have an important role in mathematical thinking; in the conducted studies, it was determined that important problems are experienced in the understanding and usage of graphics in teaching process and students in different levels have various mistakes and concept errors in reading, interpreting and forming graphics (Bayazıt, 2011; Bruno and Espinel, 2009; Capraro et al., 2005; Cavanagh and Mitchelmore, 2000; Clement, 1985; Çelik and Sağlam Arslan, 2012; Egin, 2010; Hotmanoğlu, 2014; Kramarski, 2004; Leinhart et al., 1990; Özçelik and Tekman, 2012; Roth and Bowen, 2001; Tortop, 2011; Turhan, 2015). This situation reveals the truth of there being differences between the knowledge which will be taught about graphic knowledge and the knowledge which student learned. It is stated that prior knowledge of students, their experiences (Dunham and Osborne, 1991), their knowledge about the content of the data in the graphic (Roth and Bowen, 2001), teaching activities performed within the class and basic characteristics of graphic knowledge are effective in the occuring of these problems experienced about the learning of graphic concept (Curcio, 1987; Leinhart et al., 1990; Shah and Hoeffner, 2002). These factors which Brousseau (2002) explained with the concept of obstacle are defined as ontogenetic (progress level of the learner not being sufficient), didactic (teacher originated) and epistomological (originated from knowledge learned) obstacles.

One of the important factors in eliminating the problems encountered in the learning of graphic which is a basic subject and representation form in so many fields, mathematics discipline being in the first place, from statistics to social sciences is the knowledge of the teacher who organises the didactic environments and conducts the teaching. The knowledge required by the teacher in the teaching process has been the subject of teacher education studies and it was emphasized that teacher must have pedagogical content knowledge in order to carry out an effective teaching process. (An et al., 2004; Ball et al., 2008; Fennema and Frankel, 1992; Park and Oliver, 2008; Shulman, 1986). Pedagogical content knowledge which is identified as “content knowledge necessary for teaching” requires a deep content knowledge along with the pedagogy knowledge (Shulman, 1986). A teacher whose subject content knowledge is at an adequate level, due to having a holistic view related to the subject, enhances the learning-teaching activities (Cohen et al., 1993) and accordingly increases the student achievement (Ball et al., 2008; Hill et al., 2005). But if the teacher has an incorrect or erroneous knowledge about the content of the knowledge to be taught; then the knowledge which s/he transfers to the student may become an incorrect knowledge which is not valid scientifically (Käpyla et al., 2009).

In this direction, the knowledge of the teacher related to the subject which s/he will transfer to students being compatible with the related discipline has a great importance. The content knowledge is required to be evaluated with a model which analyses in terms of both scientific theories and implementation within the frame of the institution it is included. In this context, this study examines the content knowledge which the pre-service teachers must have within the frame of Anthropological Theory of the Didactic (ATD) which studies on knowledge structure, function and its various usages.

The Anthropological Theory of the Didactics

The body of knowledge has been analyzed in detail in the framework Theory of Didactic Transposition from the French perspective. According to Chevallard (1991), who introduced the theory of transposition, the knowledge transformed from scholarly knowledge via knowledge to be taught and the actual knowledge taught to learnt knowledge. The first step of transformation, the external didactic transposition, takes place in the noosphere (Bergsten et al., 2010) and the scholarly knowledge becomes information to be taught in which curriculum, textbook, etc. play a role in teaching the information to be taught. The second step of transposition, the internal didactic transformation, the transition between the knowledge taught and the knowledge learned. Chevallard (1991) distinguishes the types of teachable information that live in an institution as knowledge in use and practical knowledge. The Anthropological Theory of the Didactics, which gives an opportunity to make an observable analysis of the transformed body of knowledge in the direction of this division, has emerged as a result of Theory of Didactic Transposition.

In anthropological theory which the knowledge can be evaluated based on actions (Chevallard, 1991), the knowledge structure which individuals have is approached within the frame of institutional recognitions determined by ecological and praxeological approaches based on the conditions of the institution it is in. The institutional recognition of the graphic knowledge within the scope of the study was determined through the examination of the documents used as source in the instituon establishment of Elementary School Mathematics Special Teaching Methods by Akar and Dikkartın Övez (2018). Within the frame of determined institutional recognitions, the places (habitat) and functions (niche) of graphics were found out with ecological approach, task types with praxeological approach (T) and also technique which is the actualisation form of task types (τ), technology which explains why the technic is valid (θ) and theories which explain and assert technology scientifically are found out (Chevallard et al., 2015). Within the direction of ecological approach, it was determined in the related institution that graphics are used in the position of tool, goal and both tool and goal in the learning and teaching of the subjects of ratio and proportion, percentage, slope, equations and inequations, equation systems, function, statistics and probability which take place in the learning fields of numbers and operations, algebra and data processing; in mathematical literacy, problem-solving, communication association and development of psychomotor skills.

Within the frame of praxeological approach, it was concluded that there were three mathematical organisations as graphic reading and interpreting (MO1), graphic creation (MO2) and making appropriate conversions between graphics (MO3); including 11 task types related to bar graph, pie chart, line graph and histogram. In the mathematical organization of graphic reading and interpreting; the technique of calculating the height of bars (τ2) for the task of obtaining information from bar graph (T1); making proportional calculations using the central angle or percentage for the surface area of the pie slice (τ2) for the task of obtaining information from pie chart (T2); determining the value of the relevant point on the line regarding the vertical or horizontal axis (τ3) for the task of obtaining information from line graph (T3) and calculating the height and width of bars (τ4) for the task of obtaining information from histogram (T4). It was ascertained that the technology which explains these techniques (θ1) is the comprehension of graphic displays including cognitive skills of external recognition, internal recognition and perception of correspondence (Bertin, 1967) and Pinker (1990) explains and asserts the determined technology with Theory of Graphs Comprehension (Θ1).

In the organization of graphic creating, it is concluded that there are tasks of creating bar graph, pie chart, line graph and histogram, in order to fulfill these task types; it is used as the techniques of drawing rectangles at the heights equal to the frequency of the data groups (τ5), using technology (τ6), slicing the pie graph using central angles or percentages in proportion to the frequency of the data groups (τ7), connecting the consecutive points that represent the data (τ8) and showing the data groups at certain intervals and with adjacent rectangles (τ9) to fulfill these tasks; descriptions and appropriate uses of graphics (θ2) constitute the technology by explaining these techniques; and Basic Perceptional Tasks Theory (Θ2), constructional components of graphics (Θ3), common standards (Θ4) and NCTM standards (Θ5) explain and ascertain this technology. As for making appropriate conversions between graphics; it was seen that the technique of creating a pie chart upon the required angle, area percentage and proportion calculations (τ10) or the technique of create a line graph by determining the points that represent the data on the axes and consecutively connecting these points to each other (τ11) is used for the task of conversion of bar graphs into other graphs appropriate for the data (T9); the technique of drawing bars at the height equal to the frequency of these data groups (τ12) or (τ11) technique is used for the task of conversion line graphs into other graphs appropriate for the data (T10) and (τ10) or (τ12) technique is used for the task of conversion line graphs into other graphs appropriate for the data (T11). The organizations of graphic creating and making appropriate conversions between graphics are local organizations which have the same technology and theory.

Universities which carries out the pre-service education activities doubtlessly perform the biggest duty in the gaining of professional competence of teachers who affect the raising of forthcoming generations directly. Within the teacher education programme, a pre-service mathematics teacher must have deep, correct and important mathematical knowledge with regard to content knowledge from the institution s/he is educated and s/he must apply this knowledge in teaching environments (NCTM, 2007; Niess et al., 2009; Otero et al., 2018). In this respect, it is required to examine the content knowledge of pre-service teachers before going into profession and the deficiencies and mistakes emerging as a result of these examinations should be eliminated. This study aimed to examine the graphic content knowledge of pre-service elementary mathematics teachers anthropologically. In the direction of the objective of the study, the answers to the below questions were searched;

1) How is the content knowledge of pre-service elementary mathematics teachers related to graphics?

2) What extent does the content knowledge of pre-service elementary mathematics teachers related to graphics coincides with the institutional recognitions?

METHODS

Research model

The model of this study which is aimed to examine the content knowledge of pre-service elementary mathematics teachers related to graphics anthropologically was determined as exploratory consecutive design which is among mixed research designs. In exploratory design, qualitative research follows quantitative research and explains the relations and tendencies inside quantitative data (Clark and Creswell, 2014). Survey model was used in the quantitative aspect of the study. Survey model is a research approach which aims to put forth a situation in the form it exists (Karasar, 2011). And interview was made in the qualitative aspect of the research.

Study group

112 (91 female, 21 male) pre-service teachers who study at the Elementary Mathematics Teaching Programme of a midsize public university in Marmara region in 2016-2017 academic year constitute the study group of the research. These pre-service teachers are assigned through proper sampling method which ensures the process to be faster and more practical by minimising the time, labor and cost loss. Objective sampling is defined as a probabilistic and non-random method. Also, it is seen as an ideal method in-depth research (Creswell and Clark, 2011). Besides, due to aiming the examination of content knowledge of the pre-service teachers participating in the research related to graphics in detail, interview was made with 10 volunteer pre-cervice teacher (7 female, 3 male) in the study group which quantitative data was collected from.

Data collection tools and the collection of data

The data was collected through Graphic Content Knowledge Scale (GCKS) and interview method in order to examine the content knowledge of pre-service elementary mathematics teachers on graph anthropologically. This scale which was developed within the frame of determined institutional recognitions by Akar and Dikkartın Övez (2018) includes graphic concept and its usage conditions, graphic types and different display forms, making appropriate conversions between graphics related to a given context, graphic reading, graphic interpreting and graphic creating skills. For the content validity of scale, the opinions of an expert group consisting of three academic members who are experts in the field of mathematics teaching and 4 elementary mathematics teachers were received. In the direction of expert opinions, Content Validity Proportion (CVP) was found for each item and Content Validity Indexes (CVI) was calculated by averaging these CVP values (Lawshe, 1975). In the result of the calculations, the CVI value of the scale was found as 0.87. The 9-item scale was applied to 30 different pre-service teachers from the study group. According to the pilot study, required arrangements were made in scale items with regards to incoherency and grammar mistakes. As a result, GCKS which includes 1 question aiming to measure the knowledge of pre-service teachers with respect to habitat and ecological niche of graphic concept, 6 questions aiming to measure the skills of graphic reading, interpreting, creating and conversion and 2 questions aiming to measure the theoretical knowledge in the graphic subject. Developed GCKS (Appendix) was implemented to 112 pre-service elementary mathematics teachers by considering the answer period in the pilot study. Finally, semi-constructed interview technique was used in order to examine content knowledge of the pre-service teachers related to graphics in detail. Within the content of interview form which was developed to be used in interviews, it was given place to graphic concept, graphic types, the relation between features of data and graphic, basic graphical skills and principles and theories used about graphics. The CVI value of the form consisting of 12 questions was calculated as 0.97. Developed interview form was used in semi-structured interviews lasting approximetely 30 min with 10 pre-service teachers. The researcher had a long-lasting interactivity with the participating individuals and interviews were made in an environment which participants can express their opinions and thoughts comfortably. Interviews were recorded during this period in order to prevent time, data loss and the effect of the subjective judgement of the researcher.

Data analysis

In this study which was conducted in order to examine the content knowledge of pre-service elementary mathematics teachers related to graphics anthropologically, obtained data was analysed within the frame of institutional recognitions which were determined through ecological and praxeological approach. In the direction of ecological approach, content analysis was used in the analysing of the answers given to the first question related to the habitat and ecological niche of graphics which was directed to pre-service teachers in GCKS. In the analysis process, the answers given by pre-service teachers to the related question were codified by two field experts and the relation between different coding results were examined. The encoder reliability which was calculated with the formula of Reliability=Consensus/ (Consensus + Dissensus) X 100 (Miles and Huberman, 1994) was found as 92.75%. The data which was separated to codes and themes was presented as tables showing frequencies and percentages. In these tables, sample expressions of pre-service teachers were given place under codes and themes.

In the direction of praxeological approach, the other data which were obtained from GCKS was presented as frequency-percentage tables within the frame of praxeological components of mathematical organizations whose institutional recognitions were determined. In these tables task types which were required to be carried out by pre-service teachers in the scales, usage situation of the techniques which were preferred in these task types and the expressions of the pre-service teachers which reflect theoretical knowledge were quantified. The methods used except the components of mathematical organisations related to determined institutional recognitions (T_i, τ_i, θ_i, Θ_i) were classified as another category. In addition, the determined categories were supported with the explanations which pre-service teachers made in interview and knowledge scale and with graphics they drew.

FINDINGS

Individual recognitions of pre-service elementary mathematics teachers related to ecology of graphics and their relation with institutional recognitions

Within the frame of ecological approach, GCKS related to where (habitat) and why (niche) the graphics were used in the institution was used and various questions were asked to pre-service teachers in the interviews. The distribution of the answers given by pre-service teachers with regards to the question directed to pre-service teachers in GCKS related to what graphic is and for what purposes it is used in mathematics was presented in Table 1.

When Table 1 was examined, it was seen that pre-service teachers answered by considering the definition of graphic and its niche together. In this direction, answers given by pre-service teachers take place under the themes of presenting knowledge, developing skill, providing learning-teaching and developing concept.

It was seen that according to codes determined under the theme of presenting information, pre-service teachers made definitions by considering graphics under the categories of data display, showing the change of data, showing the relation of data between each other, concreting knowledge, showing numerical characteristics of data and explain their function. The answer given by PMT2 who takes place in the category of data display is expressed below:

“Graphic is the display of relation of two dependent, independent variable with each other on (x,y) coordinate. It is used to display the relation of variables with each other, to display to what extent and in which direction is the change.”

As seen, PMT2 defined the graphic as displaying of relation between two variables which have a relation with each other on coordinate system. He stated about the function of graphic in institution (niche) that graphic is used to display the relation of two dependent, independent variables with each other on the coordinate system, to show the change of variables which have a relation with each other and to what extent is this change and to show in which direction is this change. When this statement is related to the functions of graphics determined in the institutional recognition, it was seen that it is compatible with functions of A3. Association line graphs with slope, A4. Determining any proportions between to quantities with given graphics, A5. Calculation of the proportionality constant, in the usage of graphics as tool, B1. Interpreting data on bar graph, B2. Interpreting data on pie chart, B3. Interpreting data on line graph, B4. Interpreting data on histogram, B5. Presenting data on bar graph, B6. Presenting data on pie chart, B7. Presenting data on line graph and B8. Presenting data on histogram which take place in usage of graphics as goal, C1. Problem-solving, association, communication and psychomotor skills development, C2. Interpretation of the data that include linear relations and assciating them with algebraic representation and C3. Presentation on graphic of the data that include linear relations which takes place in the usage of graphics as tool-goal.

It was seen that according to codes determined under the theme of skill developing, pre-service teachers made definitions by considering graphics under the categories of comparison, interpretation, relating, problem-solving, prediction, reasoning and mathematical literacy and explain their function. The answer given by PMT105 who takes place in the category of comparison is expressed below:

“Graphic is the display of certain data with figures like line, form, bar etc. It ensures plenty of data to be comprehended easily by displaying them with figures. It also enables the difference between data to be seen easily and facilitates comparison.”

PMT105 defined the graphic as display of data with forms like line, figure, bar etc. Pre-service teacher stated about the niche of the graphic that graphics are used to display the data with with forms like line, figure, bar etc., visualising data with figures, enable the data to be comprehended easily, displaying the difference between data and to compare data. When the answer given is compared with the functions determined in the institutional recognitions; it was seen that the answer is compatible with functions of A2. Calculation openness/ average/mode/median, A3. Association line graphss with slope, A4. Determining any proportions between to quantities with given graphics, A5. Calculation of the proportionality constant, in the usage of graphics as tool; B1. Interpreting data on bar graph, B2.Interpreting data on pie chart, B3. Interpreting data on line graph, B4. Interpreting data on histogram, B5. Presenting data on bar graph, B6. Presenting data on pie chart, B7. Presenting data on line graph and B8. Presenting data on histogram which take place in usage of graphics as goal.

It was determined that according to codes determined under the theme of learning-teaching, pre-service teachers made definitions by considering graphics under the themes of facilitating learning teaching, ensuring permanent learning, drawing attention, inter-discipline transition, eliminating misconcetion and explain their function. The statement of PMT40 who takes place in the category of comparing learning-teaching is expressed below:

“Graphics develop skills of noticing, understanding and interpreting the relation between data. Because of being a visual display graphics enable students to learn easier and funnier and they appeal to the visual intelligence of students.”

When the expression of PMT40 is examined it was seen that s/he defined the graphic as a visual display; s/he stated the niche of the graphic as correlating between data and providing the interpretation of data, displaying the data by using visuals, ensuring students to understand funnier and easier and evoking their visual intelligence. It was determined that the answer given by the pre-service teacher is compatible with functions of A3. Association line graphs with slope, A4. Determining

Table 1. The distribution of answers related to graphic and its usage purpose in mathematics.

Usage purpose		f	%	Sample expression
Presenting Knowledge	Displaying data	45	12.61	PMT1- Graphic is formedfor the representing of existing data or knowledge with figures and lines. It is used in the display of data.
	Displaying the change of data	36	10.08	PMT2- Graphic is the display of the relation of two dependent independent variables with each other on (x,y) coordinate. It is used to display the relation of variables with each other, to display to what extent and in which direction is the change.
	Displaying the relations of data with each other	15	4.20	
	Concreting knowledge	20	5.60	PMT41- It is a structure which displays various data concretely by drawing them by figure or line. It adds concreteness to the subject to show the relation between data to students.
	Displaying numerical features of data	2	0.56	PMT27- Classifying a set of data numerically according to data is called graphic. It can be used in grouping of data, displaying whether there are some characteristics or displaying their degree of presence. For example, bar graph can be used for instructing categorical data.
Developıng Skıll	Comparison	40	1.20	PMT105- Graphic is the display of certain data with figures like line, form, bar etc. It ensures plenty of data to be comprehended easily by displaying them with figures. It also enables the difference between data to be seen easily and facilitates comparison.
	Relating	33	9.24	PMT111- It is used to express representations which are long and hard to read easily and to classify them according to certain standards. It strengthens relational consideration and it is the step point of function subject.
	Interpretation	21	5.81	PMT58- Graphic is open and short visual display of data which is long and hard to express verbally. It can express complicated data easier because of enabling various data to be seen well-coordinated and we can interpret it.
	Problem-solving	8	2.24	PMT101- Data collected together by means of axes and by naming axes creates graphic. Graphics may be used in problem-solving. It enables student to see the problem more concrete. Besides, it prepares students for the forthcoming subjects psychologically.
	Prediction	4	1.12	PMT73- Graphics are used for making comparison, predicting about future by seeing data, having an opinion by seeing the rates of data, for seeing the difference between two data set and seeing data as a whole in mathematics.
	Reasoning	2	0.56	
	Mathematical literacy	2	0.56	PMT106- Graphic is required in order to develop literacy, to provide a basis for the subjects of ratio-proportion, function and statistics.
Learnıng Teaching	Ensuring permanent Learning	23	6.44	PMT17- Definitely it makes people having different intelligence comprehend. In addition, it supports prior knowledge and it provides understanding because it is more permanent for people to remember what they see.
	Facilitating learning teaching	20	5.60	PMT40- Graphics develop skills of noticing, understanding and interpreting the relation between data. Because of being a visual display, graphics enable students to learn easier and funnier and they appeal to the visual intelligence of students.
	Drawing attention	10	2.80	PMT100- They are used for making a subject more comprehendible and interesting.
	Inter-discipline transition	4	1.12	PMT95- Graphic is one of the subjects which provides the basis of more important subjects such as function which student will face at more advanced levels. He will be unsuccessful in lessons such as science and social studies because graphics are also used in that lessons.
	Eliminating misconcetion	3	0.84	PMT94- It can be used to notice and eliminate student mistakes and concept errors.
Developıng Concept	Rate-proportion	19	5.32	PMT43- In mathematics, graphics are used to teach some subjects.SC21- It is used in subjects such as function, proportion, slope, in production, distribution, trade and management.
	Slope	12	3.36	PMT60- It is used in the teaching of subjects of slope, rate-proportion. SC96- Graphics are benefited in order to pass to advanced subjects such as slope, function.
	Function	10	2.80	
	Equation, inequation	9	2.52	PMT86- It is used to instruct equations to student concretely, to show equation, slope and Cartesian coordinate system, to solve inequation systems.

Table 1. Cont'd

	Statistical information	8	2.24	PMT24- It is used in interpreting knowledge, sorting, interpreting between statistical information such as mod, median, arithmetic average, standard deviation.
	Coordinate system	6	1.68	PMT77- The coordinate system is used for interpreting slope, line equations, providing transition between each other.
	Pattern and generalisation	2	0.56	PMT109- Graphics are necessary for the subject of pattern. It is possible to pass to a pattern from a graphic, the general rule can be found. Slope calculation maybe seen easily in graphics. Drawing the graphic of an equation enables us to find its slope easier. Otherwise, we have to deal with formulas. We use it widely in analytical geometry. There are certain operations for rate-proportion. But we can see them directly and comfortably by drawing graphic.
	Analytical geometry	2	0.56	
	Field	1	0.28	PMT4- It is a form of sorting and evaluating data according to a certain order. We can use data in ascending sorting, area calculation and distance-time formulas.
Total		357	100	

N=112, (PMT:Preservice Mathematics Teachers).

any proportions between to quantities with given graphics, A5. Calculation of the proportionality constant, A6. Determination of the sollution st for linear equation systems which take place in usage of graphics as tool; B1, B2, B3, B4, B5, B6, B7 and B8 functions which take place in usage of graphics as goal; C1, C2 and C3 functions which take place in usage of graphics as tool-goal of graphics determined by ecological approach.

It was determined that according to codes determined under the theme of concept development, pre-service teachers explained the functions of graphics under the categories of rate-proportion, slope, function, equations and inequations, statistical information, coordinate system, pattern and generalisation, analytical geometry and field. In this direction, the expresion of PMT109 is expressed below:

"Graphics are necessary for the subject of pattern. It is possible to pass to a pattern from a graphic, the general rule can be found. Slope calculation may be seen easily in graphics. Drawing the graphic of an equation enables us to find its slope easier. Otherwise, we have to deal with formulas. We use it widely in analytical geometry. There are certain operations for rate-proportion. But we can see them directly and comfortably by drawing graphic."

As seen from the expression, s/he thinks that graphic is used as means to develop concept. PMT109 stated that graphics are used in the subject of pattern in order to determine pattern rule and to determine the slope of a line in the graphic belonging to its equation and to show rate-proportion in the relations given in graphic. In this direction, it was seen that the answer is compatible with functions of A3. Association line graphs with slope, A4. Determining any proportions between two quantities with given graphics, A5. Calculation of the proportionality constant, in the usage of graphics as tool, B3. Interpreting data on line graph and B7. Presenting data on line graph functions which take place in the usage of graphics as goal, C2. Interpretation of the data that include linear relations and associating them with algebraic representation and C3. Presentation on graphic of the data that include linear relations which take place in usage of graphics as tool-goal of graphics determined by ecological approach.

It was seen that, similar to the answers given by pre-service teachers with regards to function of graphic in the institution (niche), in the research conducted by Şahinkaya and Aladağ (2013); the pre-service class teachers also expressed that graphics provide easier comprehension of data, permanency, visuality and concreteness and facilitate learning. But, it was determined that the answers of usage of drawings for tabulation to display data (0.56%), drawing attention in learning-teaching (2.80%), facilitating making numerical operation in rate-proportion (1.12%) and teaching parabola-hyperbola in analytical geometry (0.28%) which pre-service teachers stated as its function (niche) was not given place in institutional recognitions.

Within the frame of ecological approach, the question of "Where graphics take place in mathematics?" was directed to pre-service teachers in order to examine their knowledge related to where the knowledge is in the institution; in other words, the habitat of graphics. The findings obtained from the answers which pre-service teachers gave to this question is presented in Table 2.

When Table 2 is examined, it was determined that pre- service teachers correlates the place of graphics (habitat) in mathematics with learning field, subject, chapter, its usage in different

Table 2. The distribution of answers related to usage fields of graphics.

Usage field		f	%	Sample expression
Learning Field	Data processing	54	25.96	PMT16- Graphics take place in data processing learning field in mathematics teaching programmes. Graphics are used in this learning field in student's transforming data to table incertain research questions, in student's being able to draw various graphic types on the basis of table, in the facilitation of making comparison between research groups.
	Probability	16	7.69	PMT78- Probability and statistic learning field. It is used in relating probable situations, information in the table according to requested information and transforming them into a graph, in central tendency and dispersion measures.
	Geometry and measurement	15	7.21	PMT80- It is included in the geometry learning field. Drawing figure, usage of coordinate axes, collecting data and displaying them on graphic. It also takes place in data processing learning field.
	Algebra	14	6.73	PMT46- Data processing: It is forming, interpreting, reading of graphic and relating them to other display forms by processing data. It exists in every grade level. Algebra: There are especially linear equations in 7. grade algebra field, equations of line whose slope is known in 8. grade algebra field.
	Number and operations	10	4.81	PMT65-It takes palce in learning fields such as data processing, geometry and measurement, algebra, number and operations.
	Logic	1	0.48	PMT87- Graphics are included in the logic learning field. A student understanding graphics correlates better.
Chapter	Data analysis	7	3.37	PMT94- It takes place in the part of data processing, analysis and interpretation.
	Data collection, organisation, evaluation and interpretation	6	2.88	
Subject	Equations and inequations	15	7.21	PMT22- Graphics of slope of lines, coordinate system area volume relations, distance and speed problems connected to time, function, inequations are related to many other subjects. It is not possible to consider them apart. Meanwhile, benefiting from graphics while instructing the subjects eases our work with regards to mathematical teaching. Teaching by correlating as a whole will also help the development of relational consideration skills of students in front of us.
	Problems	11	5.29	
	Slope	10	4.81	
	Function	10	4.81	
	Coordinate system	8	3.85	
	Rate-proportion	5	2.40	
	Area-volume	5	2.40	
	Central tendency and dispersion measures	5	2.40	PMT4- It exists in data collecting, organising and graphic forming, in the field of statistics. It is used in area calculation, in speed-time-distance relations, in trigonometry, derivative, integral.
	Pattern	4	1.92	
	Derivative-integral	4	1.92	
	Fraction and perceptions	3	1.44	
	Trigonometry	1	0.48	
Chapter	Data analysis	7	3.37	PMT94- It takes place in the part of data processing, analysis and interpretation.
	Data collection, organisation, evaluation and interpretation	6	2.88	
Other		4	1.92	PMT17- Algebra learning is used in the subject of data processing. It develops the skill of graphic reading in questions of interpreting from the graphic in social studies lesson.
Total		208	100	

N=112.

disciplines anddaily life problems and stated in different categories under its other themes. According to institutional recognitions obtained with ecological approach; in addition to geometry and measuring, probability learning fields along with number and operations, algebra and data processing learning fields in which graphics take place, one pre-service teacher showed logic learning field which takes place in high school curriculum as the address of graphics. The 70.9% of 110 answers given as learning field constitutes the learning fields determined in institutional recognition. In this direction, it was stated that graphics take place in data processing learning field at the most (25.96%).

When the subjects in which graphics take place stated by pre-service teachers in Table 2 is examined, it was seen that the subjects of derivative-integral and trigonometry (6.17%) stated by pre-service teachers are different from the determined institutional recognitions. It was concluded that the subject having the higher percentage (7.21%) in the answers given under the subject theme is equations and inequations. In this direction, even though the pre-service teachers show data processing learning field mostly in learning fields as the habitat of graphics, it is seen that they are directed to subjects which take place in number and operations learning field with regards to subjects. While pre-service teachers specify only central tendency and dispersion measures in data processing field; they mentioned the subjects in number and operations learning field by 45.68%, the subjects in algebra learning field by 30.86%, the subjects in geometry and measuring learning field by 17.28 %.

It was determined that 3.37% of the answers of the pre-service teachers under chapter theme is data analysis and 2.88% of them is data collection, organisation, evaluation and interpretation. Besides, different fields except the mathematics discipline were specified as the fields which graphics were used. Within this scope, while 0.96% of pre-service teachers specify that graphics are used in daily life, 0.96% of them stated that they are used in social sciences field aside from determined institutional recognitions.

When the total of 208 answers given by pre-service teachers was examined in general, it was concluded that logic learning field, derivative, integral and trigonometry subjects along with social sciences field (3.84%) are aside from determined institutional recognitions.

Individual recognitions of pre-service elementary mathematics teachers related to the praxeology of graphics and their relation with institutional recognitions

In the direction of institutional recognitions determined with praxeological analysis, questions related to bar graph, pie chart, line graph and histogram aimed at graphic reading and interpreting, graphic creating and making appropriate conversions between graphics were directed to pre-service teachers. In the (a) ve (b) items of the second question of GCKS, the pre-service teachers were asked to fulfill the obtaining information from bar graph (T1) task type. According to determined institutional recognitions, the technique which must be used for T1 task type is τ1 technique which is calculating the height of bars given in the graphic. The classification of techniques which pre-service teachers used in obtaining information from the given bar graph is presented in Table 3.

According to Table 3, in the task of obtaining information from the bar garph (T1) for the item (a) 96.43% of the pre-service teachers reached correct result by using τ1 technique (calculating the height of bars) and 2.68% of them left the question unanswered. As for the item (b) 87.5% of the c pre-service teachers reached correct result by using τ1 technique, 8.04% of them reached incorrect result by using the same technique and 4.46% of them left the question unanswered. Pre-service teachers did not use a technique which is different from τ1 technique determined in institutional recognitions for both of the items. The answers of PMT47 and PMT49 who used τ1 technique in the related question, but gave different answer are presented in Figure 1.

According to Figure 1, PMT47 and PMT49 used τ1 technique by calculating the height of the bars while determining the frequency of categorical data. This situation is seen obviously in the frequency tables formed by PMT47. These pre-service teachers answered “rose” like the other answering pre-service teachers for the item (a); and as for item (b) they reached to different numerical results. It was determined that PMT49 made proportion mistake for the item (b) which was directed about how many times the number of students which like rose is of the total number of students in the classroom.

Consequently, it was seen that pre-service teachers used τ1 technique (calculating the height of the bars) for the duty of T1 (obtaining information from the bar graph). PMT15 who answered the question compatible with institutional recognitions by using τ1 technique specified that he considered the height of the bars while acquiring information from bar graph by his explanation of “Bar Graph which is easier due to being interesting and comprehensive. Besides, it has a simple structure. The number is at the value of the height of the bar.” İn the interview made, although the pre-service teachers correctly answered frequency determining question (a), they made mistake in the question with regards to rate calculation. This situation resembles the research findings about having difficulty in determining the relation between data while interpreting single variable bar graph (Hotmanoğlu, 2014).

In the item (c) of the same question, pre-service teachers were asked to form a pie chart by using τ10 (creating a pie chart upon the required angle, area

Table 3. The classification of the answers belonging to (a) ve (b) items of the second question including obtaining information from bar graph.

Answer		Question items			
		(a) Which is the most liked flower?		(b) How many times is the number of students which like rose of the total number of students in the classroom?	
		f	%	f	%
Calculating the height of bars (т1)	Correct result	108	96.43	98	87.5
	Incorrect result	-	-	9	8.04
Unanswered		3	2.68	5	4.46

N=112.

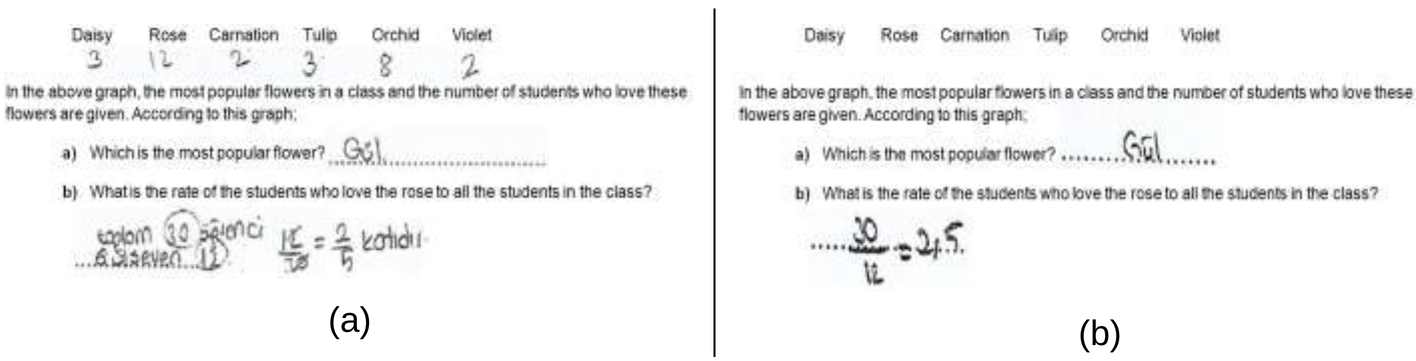


Figure 1. Answer examples of pre-service teachers (a) PMT47 and (b) PMT49 who use т1 technique.

Table 4. The classification of the answers belonging to item (c) of the second question including the task of conversion bar graph into pie chart.

Answer		Question item	
		(c) Draw pie chart showing pie slices with central angle measures.	
		f	%
Slicing the pie graph using central angles or percentages in proportion to the frequency of the data groups (т7)	Correct drawing	69	61.61
	Incorrect drawing	22	19.64
Other		4	3.57
Unanswered		17	15.18

N=112.

percentage and proportion calculations) technique within the frame of conversion bar graph into other graphs proper to data (T9) task type. According to institutional recognitions, pre-service teachers are expected to use т7 technique (slicing the pie graph using central angles or percentages in proportion to the frequency of the data groups) in order to form pie chart. In Table 4, the classification of the techniques which pre-service teachers used for conversing bar graph into pie chart is given.

According to Table 4, 61.61% of the pre-service teachers made correct drawing by using т7 technique, 19.64% of the pre-service teachers made incorrect drawing by using the same technique with 3.5% of them forming pie chart by using the other technique, whereas 15.18% of them did not draw any graphs. 17.86% of the pre-service teachers making incorrect drawing did not consider the area represented by the angle while slicing the pie chart with central angle in proportion to the frequency of data groups, whereas 2.56% of them miscalculated the rates. Also, the pre-service teachers using other techniques were not included in institutional

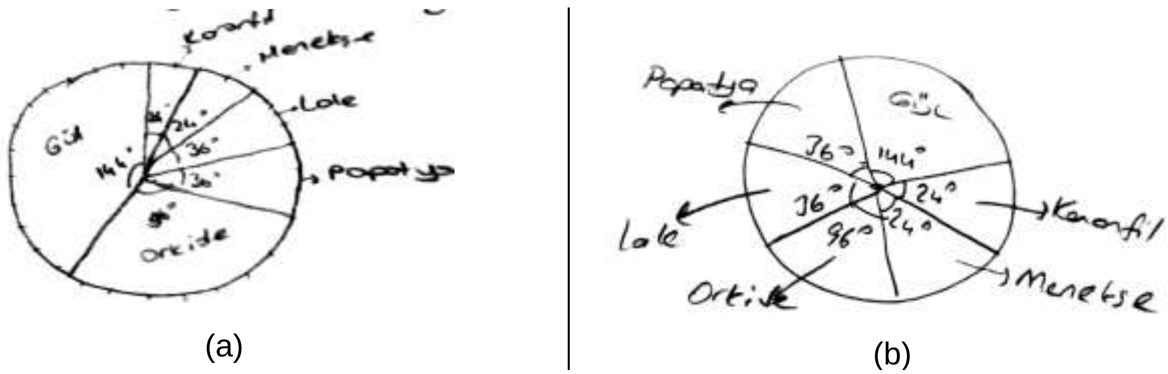


Figure 2. Pie charts drawn by (a) PMT39 and (b) PMT95.

recognitions, they did not specifiy the rates of variables numerically with percentage or central angle, but just wrote the variable names. The graphics which was drawn for second problem by PMT39 and PMT95 who used τ7 technique but made different creating are given in Figure 2.

When the pie chart drawn by PMT39 in Figure 2 was examined, it was seen that the pre-service teacher cut the pie to 30 equal units and specified every number of students liking the related flower kind in a way corresponding to 12°. For example, 2 students liking violet was displayed with an angle of 24° by slicing violet slice into two pieces. PMT95 who calculated central angle measures correctly and used the same technique could not display the area tracked by the angles in his/her drawing correctly. In the pie chart drawn according to institutional recognitions, the total of the number of students which like rose and which like tulip or daisy should correspond to 180° which is half of the pie in such a way showing 15. But PMT95, in the graphic he drew displayed the angle measure corresponding to half of the pie bigger than 180°. The pre-service teacher determined the central angles representing the number of daisies, roses and tulips as 36°, 144° and 24°consecutively, but he failed to draw the pie slice which is 180° corresponding to the total of daisy, rose and tulip flowers in such a way representing the half of the pie. Instead, s/he drew the pie slice having a central angle of 204° which s/he determined as the total of daisy, rose and tulip in such a way corresponding to 180° by cutting in half of the pie.

The graphic drawn for second problem by PMT73 who used another technique aside from technique of slicing the pie graph using central angles or percentages in proportion to the frequency of the data groups (τ7) for the creating pie chart (T6) duty is given in Figure 3.

In the pie chart given in Figure 3, it was seen that PMT73 cut the pie into random slices equal to the number of variables and he did not use an identifier specifying the size of slices with regards to area. According to the theory of structural elements of graphics

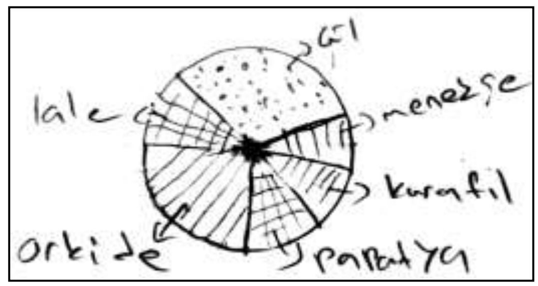


Figure 3. Pie chart drawn by PMT73.

determined in institutional recognitions (Θ3), there should be elements of frame, identifier, label and background in a graphic drawn completely. But the pre-service teacher did not specify central angle or percentage rate representing the size of slices. In this context, the pre-service teacher did not use identifier element of the graphic.

In the interview made with PMT41 who used the technique of slicing the pie graph using central angles or percentages in proportion to the frequency of the data groups (τ7) while forming the pie chart and made the correct drawing, the statement of the pre-service teacher related to the technique used is as below:

"According to me, it is hard to form pie chart. Because we know the data. But it is required to be displayed by proportioning in such a way that the whole of the pie slices should be 360. And also there is the issue of calculating what percentage each slice will be. This requires a long period of time. In my opinion pie chart is inclined to error. Percentage and angle may be mixed. For example, in the past I thought that 25° and 25% were the same. Moreever, since I did not know what 25% is, I was drawing 25% bigger than 30%. Or I was slicing it by giving random values to slices. In bar graph, we can directly transfer data, but in pie chart operations are required."

Table 5. The classification of answers belonging to third question

Answer		Question item: Ali and Veli who have 7 TL and 10 TL successively in their money box,save from their pocket money4 TL and 3 TL successively after the shopping for their money box every week. According to this, draw a line graph showing the amount of money saved in their money box for 6 weeks. f	%
Connecting the consecutive points that represent the data (τ8)	Correct drawing	66	58.93
	Incorrect drawing	27	24.11
Unanswered		19	16.96

N=112.

When the opinions of the pre-service teachers are examined, it is emphasized that knowing angle and percentage concepts is a must as prior knowledge to form pie chart. In this context, PMT41 stated that problem may be experienced in geometric correlation with angle and percentage features of pie chart. In a similar way to the situation stated in the explanation, 17.86% of pre-service teachers participated in the research drew pie charts incompatible with institutional recognitions by failing in correlating between angle and area while forming the graphics (Turhan, 2015). It is considered that these drawings which are erroneous in terms of institutional recognitions originated from lack of knowledge related calculations of area of circle.

Within the frame of graphic creating organization, connecting the consecutive points that represent the data (τ8) techniques was used while fulfilling the creating line graph (T7) task. In the third question of the scale, after giving two data groups pre-service teachers were asked to draw their graphic. In the graphic created according to institutional recognition, they are supposed to draw two lines which do not intersect x-axis and whose starting points are (0,7) and (0,11), which increase and intersects at a point. The classification of techniques which pre-service teachers used for T7 duty is given in Table 5.

When Table 5 was examined, it was seen that 58.93% of the pre-service teachers made correct drawing by connecting the consecutive points representing the data with a line (τ8) technique, 24.11% of them made incorrect drawing by using the same technique and 16.96% of them did not draw any graphs. In Figure 4, the graphics which was drawn for third problem by PMT55 and PMT62 who used line graph by using τ8 technique but made different drawings are given.

When Figure 4 was examined, it was seen that PMT55 formed a frequency table for the amount of money corresponding to every week, marked the points s/he determined in the graphic and connected these points consecutively with a line. It was seen from the figure that the pre-service teacher drew axes as the frame which is among the structural elements of graphics, placed the labels of the graphics by naming the axes and lines, used two different designs for lines as dotted line and continuous line, displayed the identifier of the graphic by using leader line and used the grid which shows matching of points for the background of the graphic. Also, PMT62 used the same technique (τ8) but determined data values incorrectly and started the lines from the origin. Similarly, it was determined that pre-service teachers who drew incorrectly by using the same technique formed line graph by ignoring initial value or drawing the lines which have to intersect in a parallel way. But all pre-service teachers who drew graphic used structural elements of frame, identifier, label and background in the drawn graphic completely.

In the sixth question of the scale, it is expected to fulfill the (T7) task type which is regenerating a line graph by changing the axes of a line graph. In this task, since the data whose graphic is asked to be drawn takes place on line graph, firstly the task of obtaining information on line graph which ensures the data set to be formed (T3) by interpreting the graphic should be fulfilled. The classification of techniques which pre-service teachers used for changing the axes of the graphic is given in Table 6.

When Table 6 was examined, it was seen that 97.32% of the pre-service teachers determined the values of related points on horizontal and vertical axis by using (τ3) (determining the value of the relevant point on the line regarding the vertical or horizontal axis) technique, 2.68% of them left the question unanswered because of not drawing the graphic. For the duty of creating line graph (T7), it was seen that 92.86% of pre-service teachers created line graph correctly by using τ8

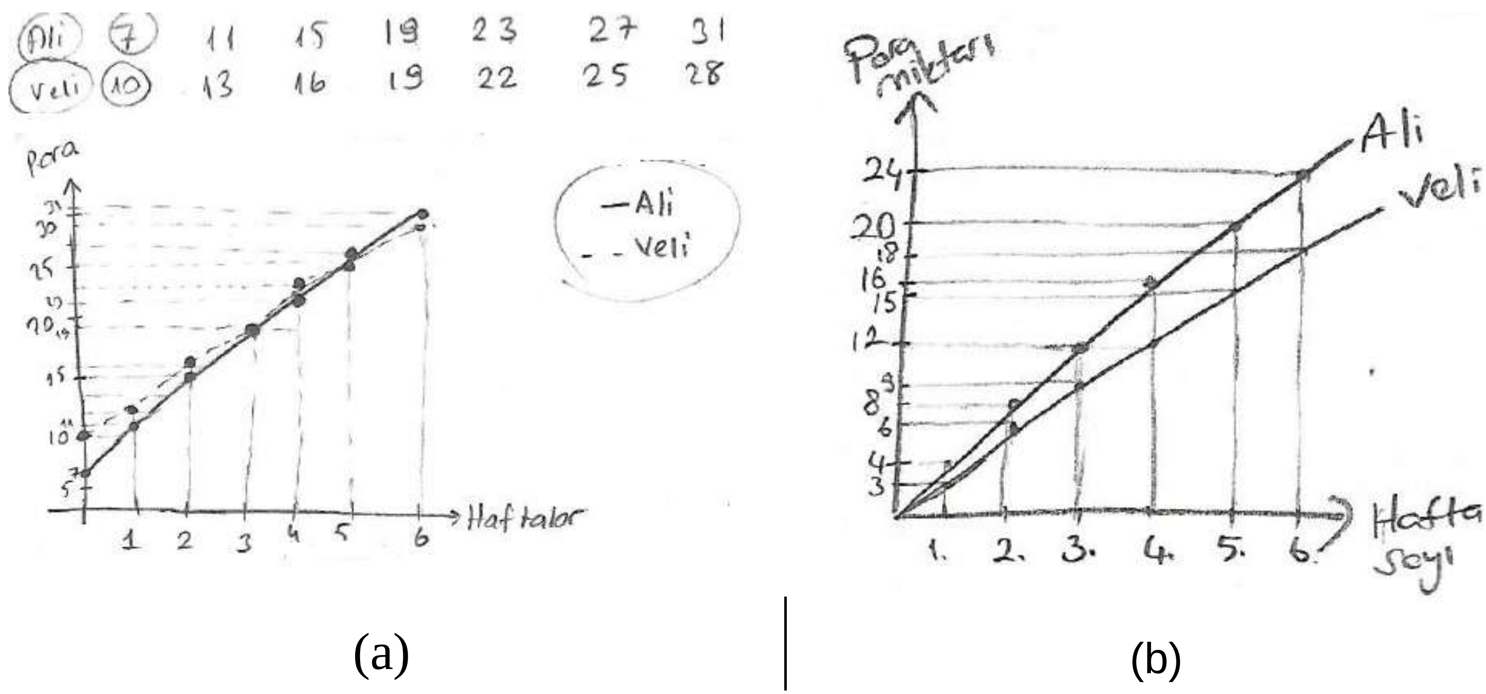
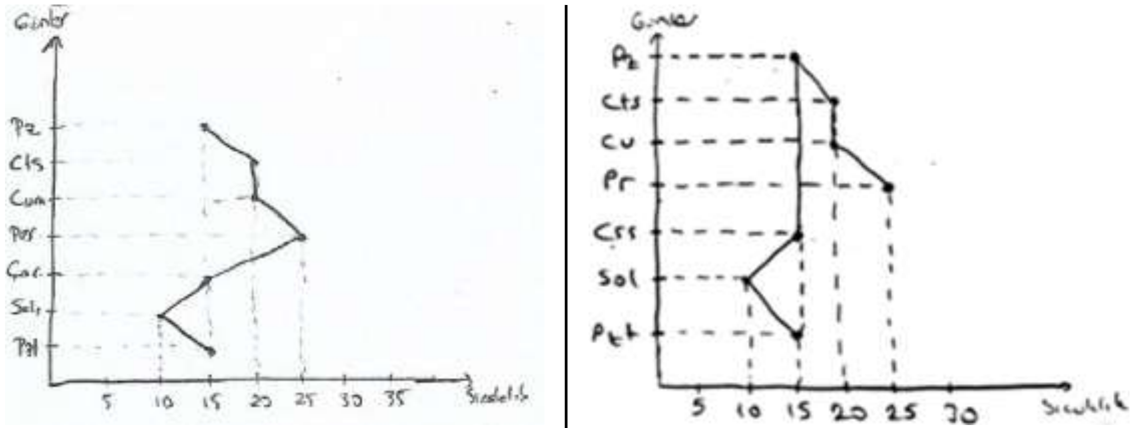


Figure 4. Line graphs created by (a) PMT55 and (b) PMT62.

Table 6. The classification of answers belonging to the sixth question including the task of changing the axes of the line graph.

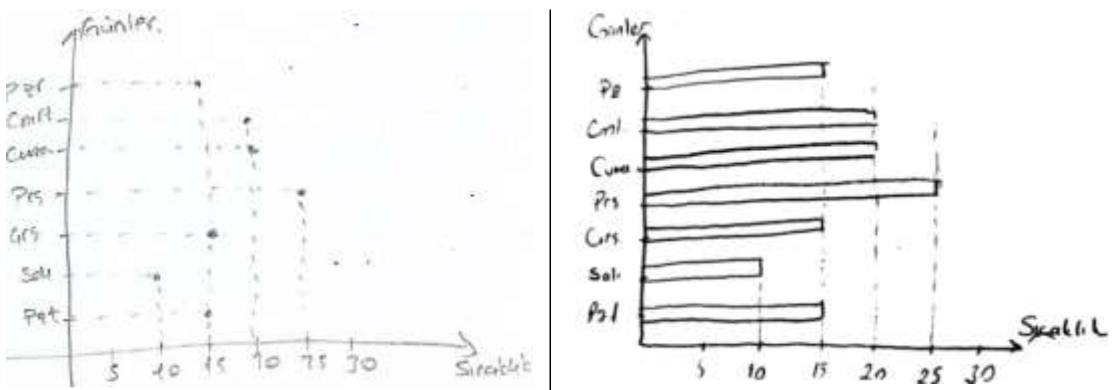
Answer		Question Item			
		In the adjacent graphic, the temperature measurement of A city is given. Redraw the graphic by changing its axes.			
		Obtaining iinformation from line graph (T3)		Creating line graph (T7)	
		f	%	f	%
Determining the value of the relevant point on the line regarding the vertical or horizontal axis (τ3)	Correct result	109	97.32	-	-
	Incorrect result	-	-	-	-
Connecting the consecutive points that represent the data (τ8)	Correct drawing	-	-	104	92.86
	Incorrect drawing	-	-	2	1.79
Drawing rectangles at the heights equal to the frequency of the data groups (τ5)		-	-	2	1.79
Other		-	-	1	0.89
Unanswered		3	2.68	3	2.68

N=112.



(a) (b)

Figure 5. Line graphs drawn by (a) PMT37 and (b) PMT34.



(a) (b)

Figure 6. Graphics drawn by (a) PMT3 and (b) PMT63.

(connecting the consecutive points that represent the data) technique, 1.79% of them made incorrect drawing by failing in connecting the consecutive points, 1.79% of them drew horizontal bar graph by using τ5 (drawing rectangles at the heights equal to the frequency of the data groups) technique and 0.89% of them form dot graph by using other techniques aside from the ones determined in institutional recognitions. The graphics which was drawn for sixth problem by PMT37 who used τ8 technique compatible with institutional recognitions and the graphic drawn by PMT34 who used this technique incorrectly are given in Figure 5.

When Figure 5 is examined, both pre-service teachers draw line graph by using τ8 technique. But, PMT34 neglected that consecutive points should be connected in this technique. In his/her drawing s/he directed from Wednesday to Sunday in day variable whose line proceeds consecutively. The pre-service teacher paid attention to temperature instead of day variable consecutively. The graphics which was drawn for sixth problem by PMT3 and PMT63 who drew dot graph by specifying just the dots representing data or who drew bar graph by using τ5 technique for this question are given in Figure 6.

In the interview made with PMT37 who made drawing by using technique of connecting the consecutive points that represent the data (τ8), the answer of the pre-service teacher about which way s/he had followed while changing the axes and to what s/he paid attention is as below:

"Graphics with axis are generally functional graphics. Parabola graphics etc. Pie chart, graphic without axis, figure, object graphics at lower grades are graphics without axis in a tabulated form. It is required to be careful in changing the axes. Most people make mistake. They change the axes directly and the form of the graphic remains the same. Or even the numbers remain the

Table 7. The classification of answers belonging to the fifth question including the task of obtaining information on line graph.

Answer		Which of the following graphics display the way (distance)which someone going first to the east, then to the north and then again to the east with a constant speed against time covered? f	%
Determining the value of the relevant point on the line regarding the vertical or horizontal axis (т3)	Correct selection	101	90.18
	Incorrect selection	-	-
Other		10	8.93
Unanswered		1	0.89

same, as there are people who just change only the names of the axes. In conclusion, the data in the graphic should change. In fact graphic changes entirely, it does not remain the same."

PMT37 who talked about graphic with and without axis tried to explain what should be done while changing axes with the mistakes made. According to her/him, it is not enough to change only axes and values, also the form of the graphic changes. PMT23 who made the correct drawing by using the same technique explained the method s/he used while changing the axes as below:

"*For example, there is data in the graphic in a way that the number of students is on the horizontal and the point is on vertical. First, I transform them into normal data to prevent confusion. If there are 5 of 10 points, first I write these points side by side. In other words; I tabulate the points first. Then I turn that table into a graphic again. Maybe I lost some time by doing this but I minimize the risk of making mistake."*

When the explanation was made for change of axis; PMT23 stated that the graphic which came out as a result of the change does not remain the same, the values on the axes should be changed, it may be useful to benefit from frequency table for not confusing data.

In the fifth question of GCKS, pre-service teachers are asked to select the graphic representing real life context including direct relation for the task of obtaining information on line graph (T3). Within the frame of institutional recognitions, the usage of the determining the value of the relevant point on the line regarding the vertical or horizontal axis (т3) technique is expected in order to acquire information from line grapht. Pre-service teachers are expected to notice that data constitutes a linear increasing graphic within the frame of the given context.

When Table 7 was examined, it was seen that 90.18% of the pre-service teachers considered the values of the points on the line graph on horizontal and vertical axis by using т3 technique, 8.93% of them made selections which is not compatible with institutional recognitions by focusing the form of the way within the frame of other techniques and 0.89% of them did not answer the question. Pre-service teachers using other technique selected the graphic which is the same with the form of the way. These pre-service teachers may have picture-like graphic misconception because of preferring the same graphic with the way. In the picture-like graphic concept error, the graphic displaying the same of the way is perceived as the correct one by ignoring the relation between variables (Clement, 1985; LeinharDt et al., 1990; Roth and Bowen, 2001). In the interviews, pre-service teachers are asked with regards to line graphs about how it is drawn and how they interpreted the line graph. In this direction, the opinions of PMT15 about line graph is as follows:

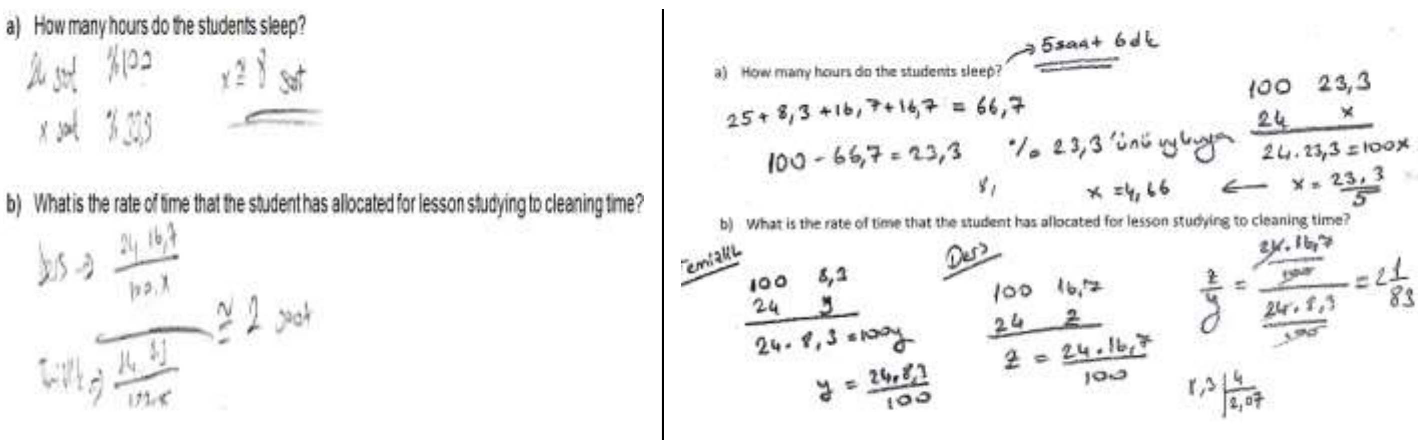
"Line graph, from my point of view, is the hardest graph to be comprehended. Because the visuality of the line graph is at a lower level. We determine the points. We form the graphic by connecting these points. When we take a look to a drawn line graph it is required to see what the points represent and to which values they correspond in both axes. The line being increasing or continuous changes the meaning, meaning of the graphic. Moreover, as far as I know, there are misconceptions about this graphic. The mistakes such as drawing continuously increasing in any condition, displaying the data with line graph even though it is discontinuous, starting always with 0 or perceiving the graph as picture may be made."

In the explanation, it is stated that line graph was drawn by connecting the consecutive points that represent the data namely, т8 technique is used in drawing. And in the interpretation of the graphic, it is stated that the values of the points in the graphic on both axes are taken into consideration. 60% of the pre-service teachers who were interviewed showed line graph as the graph type which people have the most difficulty due to having less visuality. In addition, they stated that there are misconceptions originating from not comprehending the meaning of the line in the graphic. In the (a) and (b) items of the fourth question of GCKS; pre-service teachers are asked to fulfill the task type of obtaining information from the given pie chart (T2). According to institutional

Table 8. The classification of answers belonging to (a) and (b) items of the fourth question including the task of obtaining information from the given pie chart.

Answer		Question ıtem			
		(a) How many hours does the student reserve for sleeping?		(b) What is the rate of the time s/he reserved for studying to the time s/he reserved for cleaning?	
		f	%	f	%
Making proportional calculations using the central angle or percentage for the surface area of the pie slice (τ2)	Correct result	83	74.11	100	89.29
	Incorrect result	9	8.04	2	1.79
Unanswered		20	17.86	10	8.93

N=112.



(a) (b)

Figure 7. Example answers of (a) PMT66 and (B) PMT81 which used τ2 technique.

recognitions, the technique which should be used for T2 task type is the technique of making proportional calculations using the central angle or percentage for the surface area of the pie slice (τ2). The classification of techniques which pre-service teachers used in obtaining information from the given pie chart is given in Table 8.

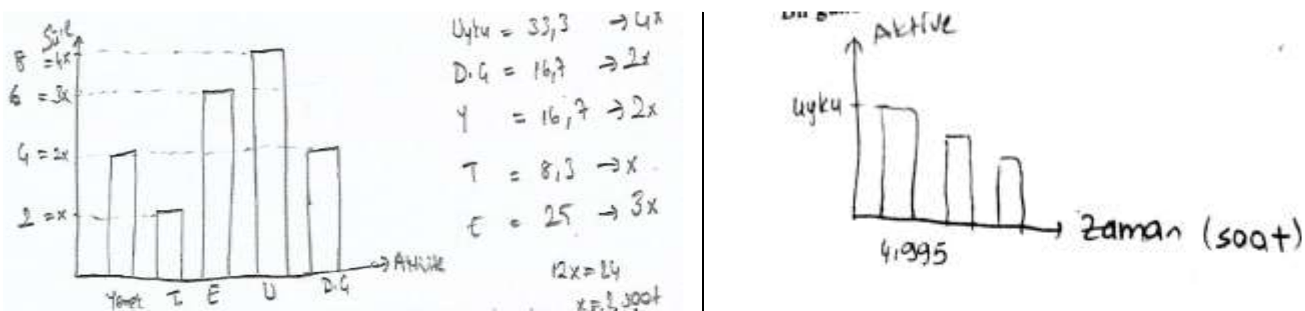
According to Table 8, in the task of obtaining information from the given pie chart, it is seen that for the item (a) 74.11% of the pre-service teachers which used τ2 (making proportional calculations using the central angle or percentage for the surface area of the pie slice) technique answered correctly, 8.04% of them gave unexpected answers and 17.86% of them left the question unanswered; for the item (b) 89.29% of the pre-service teachers which used τ2 technique gave the correct answer, 1.79% of them gave incorrect answer and 8.93% of them left the question unanswered. The rate of the answering of item (a) which requires knowing that the whole of the pie (circle) is 100% and operations should be done on this basis is lower than item (b) which can be concluded by proportioning the slices only. The answers of PMT66 and PMT81 who answered correctly and incorrectly by using τ2 technique is given in Figure 7.

When the answers given by the pre-service teachers on Figure 7 are examined, it is seen that in item (a) all pre-service teachers tried to determine the percentage rate corresponding to area of pie slice in order to find the time reserved for sleeping, even though they used the same technique, due to the mistakes made in arithmetic operations they reached to different conclusions. While PMT66 calculated the time roughly, PMT81 made calculation error. And pre-service teachers who reached correct conclusion generally proportioned the percentages of related pie slices like PMT62 instead of calculating the frequency of two data group like PMT81. The rate of answers given compatible with institutional recognitions being correct is above 70%. In this direction, it may be said that in general the knowledge of pre-service teachers obtaining information from pie chart is compatible with institutional recognitions.

In the interviews, pre-service teachers put emphasis on the concepts of circle, angle, rate and percentage while stating their opinions about pre-knowledge required for pie chart. In this context, PMT47 talked about his/her

Table 9. The classification of the answers belonging to item (c) of the fourth question including the task of convertion pie chart into graphs proper to data.

Answer		(c) Draw a graph in another type displaying the time spent for each activity in a day f	%
Drawing rectangles at the heights equal to the frequency of the data groups (τ5)	Correct drawing	78	69.64
	Incorrect drawing	1	0.89
Showing the data groups at certain intervals and with adjacent rectangles (τ9)		14	12.5
Connecting the consecutive points that represent the data (τ8)		2	1.79
Unanswered		19	16.19



(a) (b)

Figure 8. Bar graphs drawn by (a) PMT32 and (b) PMT56.

experiences in his/her educational life related to pie chart.

"While pie chart was taught me, teacher draw pie chart after instructing angle, rate and circle first. While using the information given in the graphic, for example while looking for the biggest amount, we were getting results by proportioning the area with the number in the big pie slice. We were correlating the angles in the sliced pie with 360° and judging about data. The operations we do are the same. We are reaching a conclusion while interpreting pie chart by proportioning angles with 360°."

In the (c) item of the fourth question, the pre-service teachers are asked to fulfill conversion of pie graphs into other graphs appropriate for the data (T10) task type. Within the frame of determined institutional recognitions, τ12 (creating a bar graph by placing the variable values of the relevant data groups on the axes and drawing bars at the height equal to the frequency of these data groups) or τ11 (creating a line graph by determining the points that represent the data on the axes and consecutively connecting these points to each other) techniques in order to convert pie chart into other proper graphs. In the context of the question, activity data given in pie chart, because of being categorical and discrete, is more convenient to be converted into bar graph. The classification of the techniques which pre-service teachers used for converting pie chart into other graphs proper to data is given in Table 9.

When Table 9 was examined, it was seen that 69.64% of the pre-service teachers who drew bar graph by using τ5 technique draw the graphic correctly, 0.89% of them made incorrect drawing, 12.5% of them drew histogram by using τ9 (showing the data groups at certain intervals and with adjacent rectangles) technique; 1.79% of them formed line graph by using τ8 (connecting the consecutive points that represent the data) technique and 16.19% of them did not answer this question. In Figure 8, the graphics which was drawn by PMT32 and PMT56 who drew bar graph by using τ5 technique are given.

When Figure 8 was examined, it was seen that PMT32 proportioned the quantiles and benefited from algebraic statements in order to draw the graphic and find the time reserved for each activity. According to this, s/he solved the equation which s/he set up by equating 24 h to the total time s/he found out, he calculated the time reserved for each activity by finding the rate and drew equidistant bars at the same height with this time periods. As for PMT56, s/he drew the graphic by using the same technique (τ5), but while s/he was drawing bars at the same height with the frequency of data, s/he placed the bars to incorrect axes and did not complete the graphic. Even though the graph drawn is a bar graph the drawing of the pre-service teacher did not coincide with the

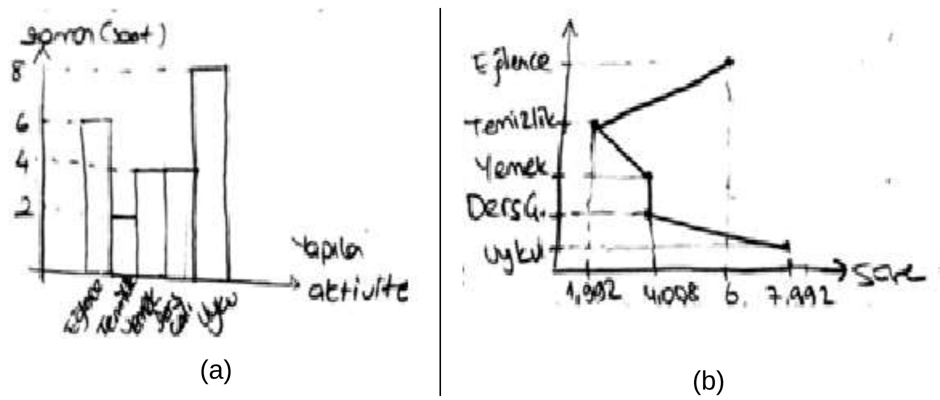


Figure 9. The histogram and line graph drawn by (a) PMT74 and (b) PMT100.

Table 10. The classification of the answers belonging to (a) and (b) items of the seventh question including the task of obtaining information from histogram.

Answer		Question items			
		(a) Which class interval has the highest frequency?		(b) Is there any class interval having the same frequency? If there is, how did you determine these intervals?	
		f	%	f	%
Calculating the height and width of bars (т4)	Correct result	18	16.07	18	16.07
	Incorrect result	-	-	-	-
Calculating the height of bars (т1)	Correct result	-	-	63	56.25
	Incorrect result	63	56.25	-	-
Other		16	14.29	14	12.5
Unanswered		15	13.39	17	15.18

N=112.

determined institutional recognitions. Because while the height of the bars should represent the scalar quantity, in this graph the heights of the bars were matched with activity variables. In addition, the scaling made on horizontal axis being erroneous attracted attention. The height of the bars started to decrease through right. Yet, in institutional recognitions, it was specified that the graphic proceeded in a way increasing to the right (American Statistical Association, 1915).

Histogram and line graph is more convenient for discrete, continuous data. But, according to context given in pie chart, activity variable is a categorical variable. In this direction, the graphics of PMT74 for fourth problem who used т9 technique and PMT100 who used т8 technique are given in Figure 9.

PMT74 who drew histogram wrote the time reserved for each activity by calculating the percentages given in pie chart approximetely and drew the graphic by connecting the bars. Also, PMT100 calculated the activity times in the form of decimal numbers; he drew the graphic on a form that activities are on vertical axis and time is on horizontal axis considering data as continuous. In these drawings which are not compatible with institutional recognitions, the rate of pre-service teachers drawing histogram are quite higher than the ones drawing line graph. In this direction, it is thought that pre-service teachers confuse bar and line graph. In the conducted studies, it was stated that the confusing of bar and line graph originates from not paying attention to the differences between these two graphs (Capraro et al., 2005; Tairab and A-Naqbi, 2004). As seen from the histogram drawn, histogram resembles bar graph structurally. But, two graphs have different characteristics with regards to displaying of numerical and discrete data, stating frequency as area and drawing of bars adjacently (Friel et al., 2001; Lee and Meletiou, 2003).

In the seventh question of the scale, pre-service teachers are asked to fulfill the task of obtaining information from inequally spaced histogram (T4). According to institutional recognition, the technique of calculating the height and width of bars (т4) is used while obtaining information from histogram. In Table 10, the classification of the techniques used in this task is presented.

When Table 10 was examined, for the item (a) which the class interval having the highest frequency is asked, it was seen that 16.07% of them used calculating the height and width of bars (τ4) technique, 56.25% of them used calculating the height of bars (τ1) technique, 14.29% of them used other technique by calculating the width of bars and 13.39% of them did not answer this question. And for the item (b) it was seen that 16.07% of them answered correctly by using τ4 technique, 56.25% of them answered correctly by using τ1 technique, 12.5% of them used other technique by calculating the width of bars and 15.18% of them did not answer the question. For the item (b), it was determined that since the calculation of the areas of the bars or the heights of the bars did not change the result, the answer of the pre-service teachers which used either τ4 or τ1 technique is correct. The answers given by PMT110 used τ4 technique, PMT100 used τ1 technique and PMT62 used other technique for the task of obtaining information from histogram. When the answers given by pre-service teachers are examined, it was seen that PMT100 used τ4 technique by calculating the areas of bars and answered both items correctly according to institutional recognitions. PMT100 used τ1 technique by calculating the heights of bars and specified that the frequency is 12 for item (a). These pre-service teachers who used two different techniques (τ1 ve τ4) stated that there is no equal-frequency class interval which is the correct answer for item (b). PMT62 who used the other technique by calculating the widths of bars which was not determined in institutional recognitions interpreted the histogram according to the width of bars in both items. When the answers of the pre-service teachers for the task of obtaining information from histogram (T4) were examined; it was concluded that the individual recognitions of most of the pre-service teachers (70.54%) are not compatible with institutional recognitions. While pre-service teachers read histogram, they mix histogram with bar graph by calculating the height of bars instead of their areas (τ1). Also in the task of creating bar graph (T5), it was seen that the drawings which are not compatible with institutional recognitions are histograms. In this context, it is considered that pre-service teachers participating in the research experienced serious problems about distinguishing bar graph and histogram. This situation shows that pre-service teachers could not interpret the concepts of group width and group number in histogram (Ulusoy and Çakıroğlu, 2013).

30% of the interviewed pre-service teachers specified that histogram is the most troubling graph type. In this direction, the opinion of PMT17 about histogram is as below:

"*Histogram may be the most troubling graph type. Because histogram can be confused with bar graph. Histogram may be perceived as bar graph and the area may be neglected. Namely, we can read histogram like bar graph. After all, they are quite alike, the only difference between them is with regards to gaps between bars.*"

As can be understood from the expression of PMT17, it was seen that the answers of teacher pre-service teachers about histogram coincide with the institutional recognitions of bar graph. Within this scope, it is considered that there are deficiencies in theoretical knowledge of pre-service teachers related to differences between two graphics and the usage of these graphics for which data is more suitable. In the eight question of GCKS, there are seven items measuring the theoretical knowledge related to basic features of bar graph, pie chart, line graph and histogram within the scope of the research, in which cases it is proper to use them and how they are drawn. The classification of the answers given by pre-service teachers about these items is given in Table 11.

According to Table 11, for the item (a) related to graph type displaying the rates of variables in a whole with percentage or central angle measurements, 96.43% of the pre-service teachers answered as pie chart and 3.57% of them did not answer. θ2 which is the common technology of mathematical organisations of creating graphics according to institutional recognitions and making proper convertions between graphics explains graphic definitions and proper usage conditions. In this direction, the graph type which is suitable for displaying the rates of variables in a whole is pie chart. In addition, in the item, τ7 technique which is used in the drawing of pie chart is stated with regards to creating of graphic. Pre-service teachers, while drawing pie chart, made drawings which are incompatible with institutional recognitions at the rate of 23.21%. In this context, it was seen that pre-service teachers are more successful in cases requiring theoretical knowledge than requiring practice.

It was seen that for the item (b) asking the proper graph to display the change of data by time 75.89% of the pre-service teachers answered as line graph, 8.93% of them bar graph, 3.57% of them histogram, 0.89% of them frequency table and 10.71% of them did not answer. Within the frame of related technology (θ2) in institutional recognitions, it was specified that the suitable graph type for displaying the change of data by time is line graph. Even though, it was answered as different graph types (bar graph, histogram) apart from line graph, the rate of these is quite low (12.5%). In this direction, it can be said that pre-service teachers are aware of theoretical knowledge related to intended use of line graph in general.

According to Table 11, it was seen that for the item (e) which straight line grah is correlated with graph types, 59.82% of pre-service teachers preferred line graph, 0.89% of them preferred frequency polygon and 6.25% of them mentioned different concepts such as coordinate system, axis, slope, equation. While line graph is formed within the frame of mathematical organisation of

Table 11. The classification of answers related to graph types.

Answer	Figure graph		Bar graph		Line graph		Pie chart		Histogram		Frequency Polygon		Frequency/ scoretable		Other (equation, slope, etc.		Unanswered	
	f	%	f	%	f	%	f	%	f	%	f	%	f	%	f	%	f	%
a) The graphic displaying the rates of variables in a whole with percentage or central angle measurements ……..							108	96.43									4	3.57
b) …is proper to display the change of data by time.			10	8.93	85	75.89			4	3.57			1	0.89			12	10.71
c) Bar graph and …….are different from each other in terms of gaps and area.					15	13.39	7	6.25	68	60.71	5	4.46					17	15.18
d) …. The frequency of each class is represented by the areas of adjacent bars whose vertical axis is parallel.			17	15.18	1	0.89			64	57.14	11	9.82	1	0,89			19	16.96
e) …… is also the line graph.					67	59.82					1	0.89			7	6.25	37	33.04
f) …….is formed by the marking of the values of data on horizontal and vertical axis and connecting the determined points.			1	0.89	92	82.14			2	1.79	2	1.79			1	0.89	14	10.71
g) ….. is more suitable for the comparison of different classified data sets.	1	0.89	43	38.39	6	5.36	6	5.36	11	9.82	5	4.46	5	4.46			35	31.25

creating graphic (MO2), it was specified that line graph which was created in case of data being continuously increasing, decreasing or fixed is the straight line graph. In this context, all of the answering pre-service teachers except 7.14% of them answered as line graph. But, since the question requires reasoning, its rate of being answered is lower than other line graph questions. According to Table 11, it was seen that for the item (f) which stands for line graph creating technique, 82.14% of pre-service teachers preferred line graph, 0.89% of them bar graph, 1.79% of them histogram and frequency polygon, 10.71% of them did not answer the question, whereas 1 pre-service teacher answered polygonic graphic in the category of "other". In institutional recognitions, line graph is formed with connecting the consecutive points that represent the data (τ8). Pre-service teachers could implement this item representing theoretical data related to creating line graph in the third and the sixth questions. In this direction, it can be said that the individual recognitions of most of the pre-service teachers are compatible with institutional recognitions.

When Table 11 was examined, it was seen that for the item (g) which the more suitable graphtype for the comparison of different classified data sets is asked, 38.39% of pre-service teachers answered as bar graph, 9.82% of them histogram, 5.36% of them pie chart and line graph, 4.46% of them frequency polygon and frequency table, 0.89 of them figure graph and 31.25% of them did not answer the question. In determined institutional recognitions, the comparison purpose of the graphics is emphasized in technology element of the organisations in which the definitions and usage situations of graphics are stated. In this context, the rates of answering as each category being close to each other except bar graph show that the opinions of the pre-service teachers are compatible with usage of graphics generally for comparison purposes in institutional recognitions. In this context, pre-service teachers expressed that figure graph, frequency polygon and frequency tables are also used for data comparison along with all graph types within the scope of the research.

For the item (c) about graph types specifying that bar graph and histogram are different from one another with regards to gap and area, 60.71% of pre-service teachers preferred histogram, 13.39% of them line graph, 6.25% of them pie chart and 4.46% of them frequency polygon and15.18% of them did not answer the question. The graph types differ from each other in terms of area and gap within the frame of theoretical block of organizations of graphic creating and making appropriate conversions between graphics. In this direction, it was seen that more than half of the pre-service teachers distinguish histogram and bar graph in terms of theoretical knowledge. But, in the seventh question of the scale directed for obtaining information from histogram, 56.25% of the pre-service teachers confused bar graph with histogram; they used for histogram the valid technique for bar graph according to institutional recognitions. Within this scope, the rate of correct answering for theoretical knowledge and the usage of techniques which are incompatible with institutional recognitions in implementation, in using knowledge are nearly the same.

According to Table 11, for the item (d) in which the technique used related to creating of histogram which class frequency is represented by the areas of adjacent bars parallel to vertical axis is specified, 57.14% of pre-service teachers answered as histogram, 15.18% of them bar graph, 9.82% of them frequency polygon, 0.89% of them line graph and 16.96% of them did not answer the question. Within the organization of graphic creation, it is known that in order to creat histogram, adjacent rectangles at regular intervals are drawn (τ9). In this context, half of the pre-service teachers know how histogram is drawn according to institutional recognitions. In the formed mathematical organisations, the explanation of how the graphics are drawn constitutes the technology element of graphic reading and interpreting organisation. In this context, it is considered that pre-service teachers not knowing the histogram forming technique compatible with institutional recognitions affects their graphic interpretations in a negative way. In addition, giving answers which are not compatible with institutional recognitions to other questions of GCKS related to histogram supports this situation.

In the scale, branched tree including theoretical knowledge about the graph types within the scope of the research is given in order to describe their general judgement about the characteristics of graph types. In this context, pre-service teachers are asked to reach a point of exit by deciding whether the sentences are true or false. According to this, the classification of the answers of the pre-service teachers are presented in Table 12.

When Table 12 is examined, it was seen that 80.36% of the pre-service teachers reached to correct exit door by evaluating the general statements given in the question correctly. It was seen that 7.14% of the pre-service teachers reaching the incorrect exit have the opinion of "The biggest number marked on the axes represents the maximum value reached.", 5.36% of them have the opinion of "Graphics must always intersect two axes", 3.57% of them have the opinion of "In pie chart, the variables are only represented with percentage" and 0.89% of them have the opinion of "In bar graph, there is the condition of sorting name data."

In the interviews, pre-service teachers are asked whether there is a theory about graphics or not. Within this scope, 20% of the pre-service teachers answered "I have never heard a theory about graphics", 30% of them "I think there is no theory. And if there is, I do not know about it." and 40% of them "I do not know." In the interview made with PMT17, the pre-service teachers expressed his/her opinion about the theory as below:

"*Probably there is theory. But we don't know that. In fact, before coming to the university, we did not even know what was theorem, what was proof. But during our educational life, graphics are always instructed based on examples. Generally, we were reading the graphic drawn on the board or we were rarely drawing graphics. Because drawing graphics were taking time. Moreover, I think we started learning graphic drawing in the analysis lessons.*"

In his explanation, PMT17 stated that they have never been confronted with knowledge structures which take place in theoretical block about graphics in their educational life and they reached to bachelor's level despite their insufficiency of knowledge. In this context, it is thought that pre-service teachers embrace knowledge without interrogating its reasons.

DISCUSSION

In the research conducted for examining the content knowledge of pre-service teachers participating in the research related to graphics anthropologically, content knowledge of pre-service teachers was examined based upon institutional recognitions related to graphics determined by Akar and Dikkartın Övez (2018). The answers of the pre-service teachers given in GCKS and interviews were analysed within the frame of praxeologic elements of mathematical organisations of habitat and niche of graphics, graphic reading and interpreting, creating graphic and making appropriate conversions between graphics which were determined in institutional recognitions.

In the direction of ecological approach, it was seen that in general, pre-service teachers who participated in the research stated the habitat of graphics as data processing, learning field, equations and inequations, problems, slope and functions subjects and data analysis, data collection and organisation, and its function (niche) as comparison and correlation, facilitating learning-teaching and the teaching of the subjects of ratio-proportion, slope, function, equations etc. It was determined that along with these answers compatible with institutional recognitions, in the habitat of graphics, they also answered logic learning field and subjects of derivative, integral and trigonometry; and in its niche they answered tabulating for displaying data, drawing attention in learning-teaching, facilitating making numerical operation and teaching parabole and hyperbole curves. It was determined that the rate of these answers which

Table 12. The classification of the answers belonging to the ninth question.

Answer	f	%
Correct	90	80.36
The biggest number marked on the axes represents the maximum value reached.	8	7.14
Graphics must always intersect two axes.	6	5.36
In pie chart, the variables are only represented with percentage.	4	3.57
In bar graph, there is the condition of sorting name data.	1	0.89
Unanswered	3	2.68

were incompatible with institutional recognitions is 3.84% for habitat and 4.76% for niche. In this direction, it was concluded that generally the individual recognitions of pre-service teachers related to habitat and niche of graphics in the institution is compatible with institutional recognitions.

When the answers given by pre-service teachers in order to fulfill the task types determined within the frame of praxeological approach were examined, it was considered that in the task of obtaining information from graphics, graphic creation and convertion between graphics which requires the usage of techniques related to practical block of mathematical organisations, in general, the individual recognitions of pre-service teachers are compatible with institutional recognitions except for histogram. It was seen that in mathematical organisations of graphic reading and interpretation related to histogram within the frame of institutional recognitions and graphic creation, more than half of the pre-service teachers experienced difficulty in distinguishing task types related to bar graph and histogram. Within this scope, it was determined that in the task of obtaining information from histogram (T4), they used calculating the heights of bars (τ1) technique which is valid for bar graph in institutional recognitions instead of using calculating the height and width of bars (τ4) technique. A similar case was seen in mathematical organisations of making appropriate conversions between graphics and graphic creation. In this direction, for the task of creating bar graph (T5), instead of using drawing rectangles at the heights equal to the frequency of the data groups (τ5) technique, pre-service teachers used showing the data groups at certain intervals and with adjacent rectangles (τ9) technique which was determined for histogram creation (T8) task. It was determined that, in the answers given to the questions directed in the scale and interviews related to theoretical blocks of mathematical organisations, the correct answering rate of items related to histogram is low and the pre-service teachers are directed to bar graph in their answers. In this direction, it was concluded that pre-service teachers have deficient and incorrect knowledge which are not compatible with institutional recognitions in their individual recognitions related to histogram and bar graph.

In the other studies conducted about graphics (Bruno and Espinel, 2009; Capraro et al., 2005; Lee and Meletiou, 2003; Tairab and Al-Naqbi, 2004), it was seen that bar graph and histogram were used instead of each other without considering the differences between them, similar to the results obtained from the research. It was concluded that institutional infrastructure of education situations given to individuals about histogram is an important factor in the learning of the individuals. It must be taken into consideration that the teacher understandings (Ulusoy and Çakıroğlu, 2013) which regards histogram unnecessary by not interpreting the histogram knowledge during their education and by thinking that it has no difference from bar graph dominated the teaching processes (McLeod, 1992; Thompson, 1984). In this context, it can be said that the curriculums including graphic knowledge which changes according to social and cultural needs (Chevallard, 1991) led to conceptual knowledge insufficiency (Baki and Kartal, 2004) of pre-service teachers who are in the position of student in didactic system about histogram. This graphical type, which has recently entered the curriculum in the elementary years of teacher candidates has been exited from the curriculum again in which the candidates have not yet completed their education life (MoNe, 2013, 2017). In this direction, it is suggested that knowledge of the teacher be examined by taking institutional recognitions determined by ecological and praxeological analysis of the documents such as textbook, curriculum which are used in teaching as a reference and curriculums modified due to educational policies should include the concepts required for mathematical literacy. Anymore, studies in recent years have focused on how to teach better than what to teach.

In cases where individual and institutional recognitions are not consistent, related individual is not regarded as a good subject of the institution. In this direction, it is considered that the individuals which will undertake the task of instructor cannot conduct qualified teaching related to the institution. It is predicted that the differences between the knowledge in the institution and the knowledge which the individuals who will transform this knowledge to the students have will cause the students to diverge from the institution in learning knowledge. From this point of view, the differences between the knowledge of the individuals in the role of instructor and the knowledge in the related institution should be determined and eliminated. So minimization of failure situations which students will experience in the future and their developing mathematical concept compatible with institutional values can be provided.

Another important finding related to content knowledge of pre-service teachers about graphics is that the theoretical knowledge of the pre-service teachers related to graphic reading, interpreting, creating and transforming is not at an adequate level. While pre-service teachers use the techniques in mathematical organisations they are not aware that these techniques are valid or not. In the interviews, it was seen that pre-service teachers explain the method (technique) of fulfilling task types related to graph types, but they could not explain why they use that technique. Within the frame of determined organisations, the pre-service teachers using mathematical knowledge only in practical sense prevents the development of level of mathematical thinking (Po-Hung, 2003) which exists with skills such as guess, reasoning, proving etc. in the field of mathematics which has a unique structure (Alkan and Bukova Güzel, 2005). In this direction, it is thought that teaching which is conducted by individuals which have low mathematical thinking capacity transferring to others the information he/she got by memorising without a reason will not be in the requested level. Since taught knowledge is not the simplified version of academical knowledge (Artigue and Winslow, 2010), it is emphasized that knowledge to be learned is to be learned by experience and forming new knowledge from the prior one in an active way like a mathematician (Altun, 2006; NCTM, 2000). In this context, in the teaching of graphic subject practical knowledge and theoretical knowledge should be given in a balance. The validity of the methods used in the implementation and applicability of different methods should be interrogated by benefiting from the historical development of graphics along with graphic reading, interpreting and drawing activities in the teaching process. In the teacher education, how to teach the knowledge structure of the target group of the teacher is discussed within the framework of special teaching methods lessons, it is applied in schools within the framework of the lessons such as school experience and teaching practice. Even though pre-service teachers take the theoretical part of the information required in the content knowledge courses, they do the theorems and proof studies at a higher level than the theory of the knowledge to be taught. The theoretical knowledge should be consistent with its application. It may be appropriate that how information is to be taught and the history of science including the theory target approach the knowledge to be taught in a common way. The differences can be examined by performing longitudinal investigations covering all of the knowledge that have been transformed in further studies.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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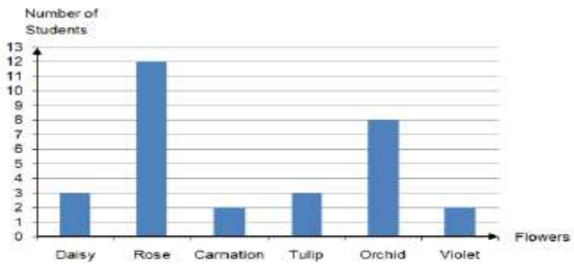
APPENDIX

GRAPHIC CONTENT KNOWLEDGE SCALE

1. Please give information about the concept of graph and its usage in mathematics. Where are the graphics located in mathematics?

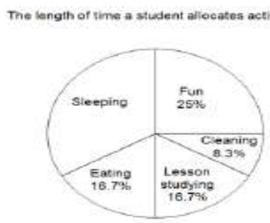
2.



In the above graph, the most popular flowers in a class and the number of students who love these flowers are given. According to this graph;
a) Which is the most popular flower?
b) What is the rate of the students who love the rose to all the students in the class?
c)Draw circle chart showing circle slices with center angle measurements.

3. After shopping, Ali and Veli's coins are 7 TL and 10 TL respectively. Ali and Veli are saving 4 TL and 3 TL each week for their moneybox. Draw a line graph showing the amount of money accumulated by Ali and Veli in their moneybox during 6 weeks.

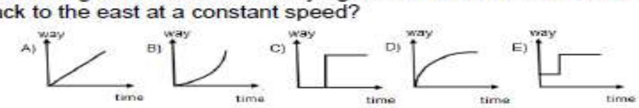
4.



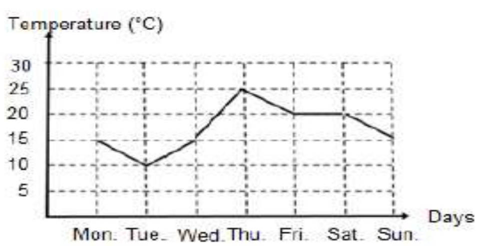
The pie chart shows how a student evaluates a day. According to the chart;

a) How many hours do the students sleep?
b) What is the rate of time that the student has allocated for lesson studying to cleaning time?
c) Draw a different type of graph that shows the time each activity is spent in a day.

5. Which one of the following charts shows the way against time that someone who travels first to the east, then to the north, then back to the east at a constant speed?

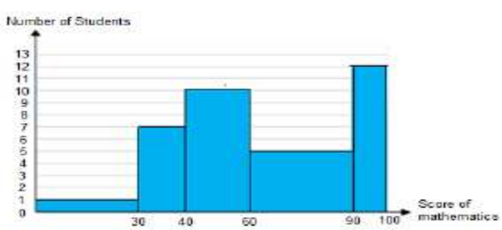


6.



In the graph on the side, a weekly temperature measurement of city A is given. Redraw the graph by changing its axes.

7.



Look at the graph, the scores of the students in a class are shown in the mathematics exam. According to this graph,
a) Which class interval has the highest frequency?
b)Are there class intervals with equal frequencies? If so, how do you specify these intervals?

8. Fill in the following blanks with the appropriate graph type (s).
a). ....The graphic displaying the rates of variables in a whole with percentage or central angle measurements
b). ....is proper to display the change of data by time.
c). Bar graph and ....are different from each other in terms of gaps and area.
d)..... The frequency of each class is represented by the areas of adjacent bars whose vertical axis is parallel.
e)....... is also the line graph.
f)........is formed by the marking of the values of data on horizontal and vertical axis and connecting the determined points.
g)..... is more suitable for the comparison of different classified data sets.

9. The following are true / false statements about the bar graph, pie chart, line graph and histogram. Specify the exit number that you reached by deciding whether the judgments in these statements are true or false