



## Helping college students claim ownership of their mathematics learning

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### **Abstract**

This is an exploratory study about engaging students in mathematics learning both inside and outside of the classroom of an introductory proof course. The Author utilized the framework of scholarship of teaching and learning as a guide to ensure the research process was carried out systematically. This study was conducted through one cycle of scholarly teaching. The implemented teaching strategies showed positive impacts on students' learning. These strategies can be tested and improved through more cycles of scholarly teaching research in different settings.

**Keywords:** Scholarly Teaching, Active Learning, Reflective Learning, Informative Assessment, Pre-class Preparation, Exit Cards.

### **Introduction**

The research was conducted at a minority-serving state university which has 70% Hispanic student population. In winter quarter 2015, the author taught the general education (GE) course – Critical Thinking through Applications of Mathematics in the university. This GE course is an introductory class on mathematical proofs and serves to help students transition to more advanced mathematics courses. It is intended to help students to build students' critical thinking through applications of mathematical logic. After a quarter of teaching this course, the author found the following problems: many students struggled with basic mathematics notations and operations, most of students did not know how to use textbook to study; Most of students were not able to effectively communicate their mathematical ideas orally and in writing, and writing proofs was very difficult for them. Intending to improve students' learning, the author decided to change the teaching strategies when teaching the same course in winter 2016. The existing literature was reviewed to guide the decision of making the change. Education research shared a finding that active learning was necessary for students to retain knowledge and become critical and creative learner (e.g., Boyer, 1990; Keyser, 2000; Coorey, 2016). However, Richards (1991) argued that "students will not become active learners by accident, but by design" (p. 38) and a structured plan was needed to guide students' learning. We want to design a plan that can engage students in learning both inside classroom and outside classroom and implementation of the plan allows students to take control of their own learning, gain knowledge/wisdom, and develop their skills (Summerlee, 2010). Some research found that pre-class preparation helped students succeed in in-class learning (Moravec, Williams, Aguilar-Roca & O'Dowd, 2010). Pre-class reading assignment was one form of pre-class preparation that could help students learn more deeply through increasing students' engagement in class discussions and their understanding (Crouch & Mazur, 2001; Aldag, 2007). The author hypothesized that pre-class reading assignments would help students learn how to orally communicate their mathematical ideas and how to write mathematical proofs in the introductory course on mathematical proofs.

In addition to engaging the students in pre-class reading, the author also wanted students to engage in reflective learning through the use of exit cards which have become a popular tool for informative assessment in recent years. An exit card requires students' written responses to teacher's questions at the end of a class and provides teachers informative assessments of students' current level of understanding (Cross, 1998; Ross, Hogaboam-Gray, & Rolheiser, 2002; Marzano, 2012). Answering questions on exit cards helps students reflect on what they have just learned in class and facilitates critical thinking; analyzing students' responses on exit cards allows the teacher to adapt to students' inquiries and to adjust teaching based upon students' current level of understanding (Wagner, 2005; Albers, 2006). The author also hypothesized that exit cards would help document students' thinking and catch any misconceptions that need to be addressed.

The author structured the class as follows: Pre-class reading assignments were assigned to students before each class; small group discussions and whole class discussions were conducted by students to solve problems during each class; exit cards were answered by students in a reflective manner in the end of each class to immediately give the teacher important information about students' learning in that specific class. This paper focuses on how pre-class reading assignments and exit cards affected students' learning in the introduction to mathematics proof class. Although collaborative learning was also included in the course design, this strategy was not in the domain of discussion for this study.

### **Guide of the Research**

This study was structured following the framework of scholarship of teaching and learning (Boyer, 1990; Richlin, 2001). Scholarly teaching was brought up by Boyer in 1990. In his book *Scholarship reconsidered: Priorities of the professoriate*, Boyer emphasized the importance of teaching and proposed to add scholarship of teaching to academic life. In 2001, Richlin conceptualized scholarly teaching. He described a scholarly teacher as one who takes the revision process a step further by conducting a logical, systematic process. This systematic approach is significantly different from the intuitive way of process. According to Richlin, scholarly teachers at first observe a problem, then research the available literature before making any changes to a course. After conducting a literature review, they select and implement course changes, and observe effects of the change; they systematically gather and analyze the data collected from their class to discover if the change is effective. Finally, they prepare manuscript about the course change and share their finding(s) by presenting/publishing in peer-reviewed journals. The findings will be added to knowledge base. The cycle of the process of scholarship of teaching/learning continues, teaching improves and teaching knowledge base has been accumulated through the repeated cycle of scholarship of teaching/learning.

### **Implementation of the Teaching Strategies in the Introduction Course of Mathematical Proof**

#### *Motivation to Make a Change*

When teaching the course - Critical Thinking through Applications of Mathematics in winter 2015, the author observed problems that hindered students learning. Many students came to class without knowing basic mathematical notation. They did not know how to use textbook and did not know how to communicate their mathematical ideas orally and in writing. After reviewing the literature, the author decided to implement some new strategies when teaching the same course in winter 2016. First of all, before each class, reading materials and reflective questions were assigned to students in order to motivate them to conduct self-directed learning; during class time, students were required to present the process of writing proofs for given problems after small group (3-4 students) discussions in order to help students build skills of communication and collaboration; at the end of each class, students were asked to answer questions in exit cards for the teacher to assess students' learning.

*Pre-class Reading Assignments*

Pre-class reading and questions were assigned to students one week early before each class. Students were required to finish reading and answer questions before a class started; reading assignments were discussed and collected at class. Average score of reading assignments were counted toward 10% of final grade. No late reading assignments were accepted intending to push students to finish the reading assignment before class if they wanted to receive the credit. The questions in the pre-class reading assignments were designed based on the following principles: provoking thoughts (how and why questions); helping understand concepts/main ideas; stimulating deep thinking; varying at difficulty levels. The following is a sample reading assignment.

Please read Lesson 1: Solving Logic Problems and then answer the following questions:

1. Describe the strategies to solve logic problems.
2. What is direct information? What is indirect information?
3. How can you justify if your answer makes sense?
4. Can you use the process of elimination to solve the famous Muppets problem? Show your process.

Students must read the lesson 1 material thoroughly, and then think about the main ideas of this lesson and try to understand the process of elimination method in order to answer the four questions.

*In-class Discussions and Problem-Solving*

During each class, for the first 20 minutes, students discussed the reading assignments in small groups to understand/clarify main ideas/concepts and to get ready to solve related problems later in the class; some students were asked to explain/articulate their thinking regarding the questions asked in the reading assignments. After discussion of reading assignments, class moved on to solving problems – writing proofs. For about 50 minutes, students conducted problem solving in small groups. Then some groups were selected to present their process of writing proofs to the whole class on the blackboard. Sample of logic problems and proof problems are added here in the following.

*Sample Logic Problems*

- Nancy, Alex, Virginia, and Toby are novelists who each write in a different genre: mystery, biography, science fiction and romance. Each of writers favors a different sport. One of the favorites is soccer. Use the following clues to determine each writer's genre and favorite sport.
  - a) Alex writes biographies or mysteries.
  - b) Nancy's daughter and the biographer's son are in the same 3<sup>rd</sup> grade class.
  - c) The baseball fan and the romance novelist have no children yet.
  - d) Alex's favorite sport is swimming.
  - e) The baseball fan, the tennis fan, and the Toby all have book signings this weekend.
  - f) The mystery writer's mother is looking forward to becoming a grandmother some day.
- In a certain small high school the courses in biology, economics, English, French, History and mathematics are taught by just 3 men: Myers, Smith, and Thomas, each of whom teaches 2 subjects.

The economics teacher and the French teacher are next-door neighbors.

- a) Smith is the youngest of the three.
- b) The men ride to and from school together. The biology teacher, the French teacher and Myers each drive one week out of three.
- c) The biology teacher is older than the mathematics teacher.
- d) When they can find a fourth player, the English teacher, the mathematics teacher, and Smith usually spend their lunch hour playing tennis.

What subjects does each man teach?

(The logic questions were selected from *Problem Solving in Mathematics*, Fischman and MacMurren, 2011)

#### Sample Proof Problems

1. Suppose  $a, b \in \mathbb{Z}$ . If  $a^2(b^2 - 2b)$  is odd, then  $a$  and  $b$  are odd.
2. Suppose  $x, y \in \mathbb{Z}$ . If  $x^2(y + 3)$  is even, then  $x$  is even or  $y$  is odd.
3. Suppose  $n \in \mathbb{Z}$ . If  $3 \nmid n^2$ , then  $3 \nmid n$ .
4. Prove that  $\sqrt{6}$  is irrational.
5. Given an integer  $a$ , then  $a^3 + a^2 + a$  is even if and only if  $a$  is even.
6. Suppose  $a, b$  and  $c$  are integers. If  $a|b$  and  $a|(b^2 - c)$ , then  $a|c$ .
7. Suppose  $x \in \mathbb{R}$ . If  $x^5 + 7x^3 + 5x \geq x^4 + x^2 + 8$ , then  $x > 0$ .

(The proof problems were selected from *Book of Proof*, Hammack, 2013.)

#### Exit Cards

During the last 5 to 10 minutes, students were required to answer three questions in the exit cards and then turn them in. Three questions are: 1. What mathematical concept(s)/idea(s) did you learn from class? 2. What was the muddiest point for you to understand? 3. Do you have any questions? The exit cards provided the author information about the level of students' understanding and any common misconceptions. In the beginning of following class, questions/problems were addressed. Usually it took the author about half an hour to review students' responses for the three questions in Exit Cards after class. Reviewing Exit Cards helped the author assess students' understanding, recognize misconceptions and some difficult parts for students needed to be addressed. The problems found from Exit Cards were addressed in beginning of the following class. Exit Cards added more helpful interactions between students and teacher through written communication in a timely manner.

#### Methods

In order to evaluate the effects of the changes, the following data were collected and analyzed during the process of the teaching cycle: pre-class survey (math background, perceptions on mathematics, and expectations); post-class survey (feedback on reading assignments and exit cards); exit cards, teaching evaluations collected by the university; students' work (pre-class reading assignments, homework, in-class work, and tests); the instructor's observations and reflections. Additionally, in an attempt to better understand the students' responses in the Post-class surveys, in the end of the quarter the author sent an email to all the students in the class and asked for volunteers to be interviewed. Three students responded and were interviewed in the beginning of the following spring quarter. The semi-structured interview lasted about half an hour, including the questions: What do you think the overall structure of this class? What do you think the weekly reading assignments? Do you have some suggestions for change for the weekly reading assignment? What do you think exit cards? Do you have suggestions for changes for exit cards? Which part of the class do you like the most and why? Which part of the class do you dislike the most and why? Do you feel like you have learned as the result of the class?

Data collected was analyzed qualitatively and quantitatively. Survey questions were first drafted by the author, then the draft was discussed between the author and a colleague who is an expert of educational research from another university. The surveys were finalized after several revisions. Survey results were analyzed numerically using Excel spread sheet. Qualitative analysis was also utilized following the grounded theory approach (Corbin & Strauss, 1990, Strauss & Corbin, 1994), involving constant comparative analysis.

## Results and discussion

Data analysis revealed that the effects of the strategies are very positive. In the post-class survey, most of students agreed that pre-class reading assignment was very helpful: 83% of students strongly agree or agree that reading assignments help them understand the course content better; 79% of students strongly agree or agree that reading assignments helped them prepared for class; 71% of students strongly agree or agree that reading assignments helped them do well in class; 70% of students strongly agree or agree that reading assignments helped them deepen their thinking on the content. See the table below.

Students' Feedback on Pre-reading Assignment (Post-class survey)

Percentage of students Strongly Agree/Agree	
83%	It helped me understand the course content better
79%	It helped me prepare for class
71%	It helped me do well in class
70%	It helped me deepen my thinking on the content

Quoting some of students' comments on pre-class reading assignment, "The reading assignments helped me learn what materials and information we were going to learn"; "The questions on the reading assignments are usually about the main points/ideas that I need to know of a certain topic. It is very helpful when discuss the reading assignments together in class"; "The reading assignments gave me a preview on the upcoming lesson ... in class the teacher would clarify the points I didn't understand clearly"; "The reading assignments were very helpful and necessary. The combination of the reading assignments and class time really helped me understand the content".

Students' feedback on exit cards showed that exit cards helped them learn: 79% of students strongly agree or agree that teacher's feedback on exit cards helped them understand the part(s) of class they did not understand in the previous class; 75% of students strongly agree or agree that teacher feedback on exit cards helped them recognize some misconceptions. See the table below.

Students' Feedback on Exit Cards (Post-class Survey)

Percentage of Students Strongly Agree/Agree	
79%	Teacher feedback on exit cards helped me understand the part(s) of class I didn't understand at previous class
75%	Teacher feedback on exit cards helped me recognize some misconceptions

Taking two typical students' comment on exit cards, "Exit cards are a great way of reflecting back on what we learned in class and also a great way for me to ask questions if I get confused"; "Exit cards helped in the way that I could ask misunderstanding I had of the lesson that day."

University Teaching Evaluations from students shows a significant change between winter 2015 teaching without implementation of the new strategies and winter 2016 teaching with implementation of the new strategies. In winter 2015, the class enrollment was 38 and the number of evaluation responses were 26. In winter 2016, the class enrollment was 38 and the number of evaluation responses were 28. For the question “professor contribution to student learning”, scoring from 0 = unsatisfactory to 6 = excellent, 90% of students rated 4 and above in winter 2016 comparing to 62% of students rated 4 and above in winter 2015; 35% of students rated 6 out of 6 in 2016 comparing to 4% of students rated 6 out of 6 in winter 2015 (See the table below). The positive change in students’ feedback in university teaching evaluations indicates that students appreciated the new teaching strategies.

Quarter	Percentage of rated 6	Percentage of rated 5	Percentage of rated 4	Percentage of rated 4 & above in Total
Winter 2015	4%	46%	12%	62%
Winter 2016	35%	30%	25%	90%

(Scoring: From 0 = Unsatisfactory to 6 = Excellent)

One interesting finding was that 83% of students admitted that the reading assignment helped them better understand the course content while only two students indicated they enjoyed the reading assignments and none would have done the reading if they were not graded.

Analysis of the process of teaching and learning throughout the whole quarter indicates that it is very important to have students answer the questions on exit cards every class. After a few classes, students had remembered the three questions on exit cards and sometimes even reminded the author of exit cards before the author asked them to do so. Exit card effectiveness requires the instructor’s consistent and timely follow-up. It is necessary to address the questions and common mistakes/misconceptions identified through exit cards in beginning of the following class in order to increase students’ understanding of course content discussed in the previous class and correct any student misconceptions in timely manner. The practice of addressing student feedback from exit cards motivated students to answer exit cards more thoughtfully and with a positive attitude, as it signaled to them that their feedback and questions were valued and useful. Most students in the post survey expressed that exit cards were an effective way of helping them learn and they appreciated the instructor’s timely follow-up to exit cards throughout the quarter. Some students added that exit cards allowed for more effective interactions between students and the instructor for some quiet and shy students to raise questions and speak out their thinking. Interviews with students also confirmed the positive feedback on exit cards.

Engaging students in pre-class reading assignments creates a path for students to explore and inquire through self-directed learning which may lead to deeper understanding of the material. However, pre-class reading assignment requires incentives in order to be implemented well. Without giving course credit for pre-class reading, students are not motivated to do the assigned reading or answer the important reflective questions that provoke high order of thinking. The author’s observations on this class assured that students’ learning gain through pre-class reading assignments definitely deserved the credit points (10% of course grade) to be rewarded.

This study provides evidence that the implemented teaching strategies were effective tools in teaching the introductory course on mathematics proofs. However, the author believes that these strategies can also be applied to teaching practices in other disciplines other than mathematics. Selective pre-class reading assignments would lead students to think more deeply and stimulate their self-directed learning, while exit cards can help students learn reflectively in classes of disciplines

other than mathematics. The author looks forward to seeing the studies of the strategies in other discipline teaching.

### Future Research

This study was conducted through one cycle of scholarly teaching research. The effects of the teaching strategies implemented were positive. The teaching strategies can be tested and improved through more cycles of scholarly teaching research in different settings. In next cycle of teaching, the following research questions could be investigated: 1. To what extent does this teaching approach affect students' achievement in a follow-up study for two to three years? 2. To what extent does pre-class reading assignments affect students' communication of mathematics orally and in writing? 3. How does the system of pre-class reading assignment - class discussions – exit cards work to affect students' learning outcomes? 4. To what extent does this teaching approach help knowledge retention? 5. How does this teaching approach lead to students' learning success? 6. What strategies can be developed for instructors to analyze exit cards efficiently in order to access to students' thinking and understanding levels? 7. How do these strategies affect students' learning in disciplines other than mathematics?

Future research would help constantly improve the teaching approach and develop more effective strategies. Additionally, research can test the teaching strategies at classes of other subjects in different settings.

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