Dynamic geometry software within the van Hiele teaching framework

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Dynamic geometry software can facilitate primary school students' development of conceptual understanding in geometry. A teaching sequence involving DGS combined with tangrams is explained.

Introduction

The foundation of geometric knowledge and understanding starts in primary school, and in contemporary mathematics education there is an emphasis on identifying learning progressions and trajectories as a way of moving students forward. The van Hiele (1986) theory is well-documented and provides insights into the progression and differences of individuals' geometric thinking. Van Hiele's theory significantly influences geometry curricula worldwide and contains teaching phases that can help all teachers understand developmentally appropriate ways to facilitate their students' geometric thinking. This article considers previous work by presenting dynamic geometry software (DGS) as a tool for a teaching sequence that is embedded within the van Hiele Teaching Phases and links theory and practice within a primary mathematics classroom.

This article presents a strategy for using DGS within a theoretical framework. DGS was originally designed for high school students but can be used from about Year 4, and it can be engaging and a lot of fun for upper primary students. DGS allows the easy construction of points, lines and geometric figures on a computer or a tablet. Once drawn, geometric objects can be moved and manipulated in a countless number of ways. As geometric objects are changed, the measurements of distances, lengths, areas, angles and perimeters affected are updated instantly. Importantly, when a geometric object is created with a particular relationship to another, that relationship is maintained no matter how either object is manipulated or changed. One of the best-known DGS is *GeoGebra* (free public domain software at www.geogebra.org)

Although there are only a limited number of studies that have researched the use of DGS in primary schools (Sinclair et al. 2016), the majority of them have shown that DGS does facilitate primary school students' development of conceptual understanding in geometry (Kesan & Caliskan, 2013; Lin, Shao, Wong, Li & Niramitranon, 2011). DGS can be used for higher-order pedagogical tasks instead of for activities that resemble print-based learning material, and for tasks aiming at reproducing knowledge or inducing rote learning (Castro Sanchez & Aleman, 2011; Tezci,2011). While pre-constructed resources are readily available, teacher designed resources are better at meeting students' individual needs because careful attention can be given to the introduction of formal mathematical language and concepts (Serow & Inglis, 2010).

Van Hiele teaching phases

According to van Hiele (1986) there are five teaching phases that represent a framework that encapsulates students' progress from one geometrical level of understanding to the next. The phases are sequential, and consist of an information phase, a guided orientation (direction) phase, an explicitation (new ideas) phase, a free orientation (ownership) phase, and an integration phase. According to van Hiele (1986) a teacher's responsibilities during the phases include: lesson planning; scaffolding learning to focus on the geometric qualities of shapes; introducing new language; engaging children in discussions; and promoting problem-solving using geometric shapes. A key element of the phases is that students maintain ownership of their ideas. The intention of the phases is to encourage students to engage in discourse with the teacher or their peers in order to clarify their conceptual understandings. Language plays an extremely important role in this teaching framework. Only after students have identified and described concepts in their own language is more technical mathematical language introduced (Serow & Inglis, 2010).

To follow is a teaching sequence, which incorporates the five phases of the van Hiele framework. The teaching sequence places emphasis on the changing role of language as a student progresses through the phases. Based on similar work by Serow and Inglis (2010), the teaching sequence begins with simple tasks that are scaffolded and directed by the teacher and then moves to tasks that require more student initiative in the form of problem solving and reflection.

A teaching sequence

Serow (2002), argues that teachers need to use more dynamic teaching strategies to assist students in their understanding of geometry to address the hurdles that many children encounter when learning geometry. To make mathematics learning experiences more meaningful, it is essential for teachers to structure geometry teaching and learning in a way that involves children manipulating materials. Two tools that have lent themselves to student centred problem-solving tasks in geometry are DGS and tangrams (a seven piece, dissection puzzle originating from China). The teaching sequence presented here combines the benefits of both of these tools through the creation of a virtual manipulative. A tangram can be used to develop geometric concepts by categorising, comparing and working out the puzzle, and thereby promoting imagination and logical thinking through observation and analysis (Olkun, Altun & Smith, 2005; Russell & Bologna, 1982). Explorations with tangrams deal with shapes and their properties, symmetry, parallelism and area (van Hiele, 1999).

In light of the above, the teaching sequence that follows attempts to link van Hiele's theory with practice within a technological environment in the mathematics classroom. The teaching sequence utilises technology as a facilitator of students' growth in understanding geometrical concepts, and utilises the van Hiele teaching phases as a way of maintaining student ownership of ideas throughout the learning process (Serow, 2007).

The teaching sequence consists of twelve sessions of approximately 30-40 minutes duration designed for the upper primary school mathematics classroom. The sequence was based on the measurement and geometry strand of the Australian Curriculum, Assessment and Reporting Authority [ACARA], 2014, and is also related to the number and algebra strand, specifically fractions. Some of the target outcomes that can be addressed by the teaching sequence are, "Compare and describe two-dimensional shapes that result from combining and splitting common shapes, with and without the use of digital technologies" [ACMMG088], "Create symmetrical patterns, pictures and shapes with and without digital technologies" [ACMMG091], "describe translations, reflections and rotations of two-dimensional shapes. Identify line and rotational symmetries" [ACMMG114], "Investigate combinations of translations, reflections and rotations, with and without the use of digital technologies" [ACMMG142], "Investigate strategies to solve problems involving addition and subtraction of fractions with the same denominator" [ACMNA103], and "Solve problems involving addition and subtraction of fractions with the same or related denominators" [ACMNA126] (ACARA, 2014).

The learning sequence

Phase 1: Information

> For students to become familiar with the working domain through discussion and exploration. Discussions take place between teacher and students that stresses the content to be used. (Serow, 2007, p. 384).

Activities

The teacher models the construction of a tangram master template using a Netbook computer or iPad, a SmartBoard, and *GeoGebra* software. Students then create their own tangram master template using a Netbook or iPad, and *GeoGebra* software, which they save as a master template that can then be altered/manipulated and resaved in subsequent activities.

Sample 1

Modelled construction of a tangram master template: creating a base tangram layer that can be 'virtually' traced to create the seven individual shapes of the tangram (see Figure 1).

Sample 2

Modelled construction of a tangram master template: individual pieces being created by 'virtually' tracing each of the seven individual shapes of the tangram (see Figure 2).

Sample 3

The individual pieces of the tangram should be named Tan 1, Tan 2, Tan 3, etc. as per the completed tangram

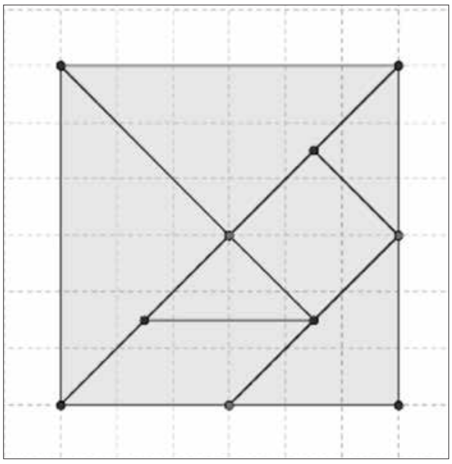

Figure 1. Modelled construction of a tangram master template.

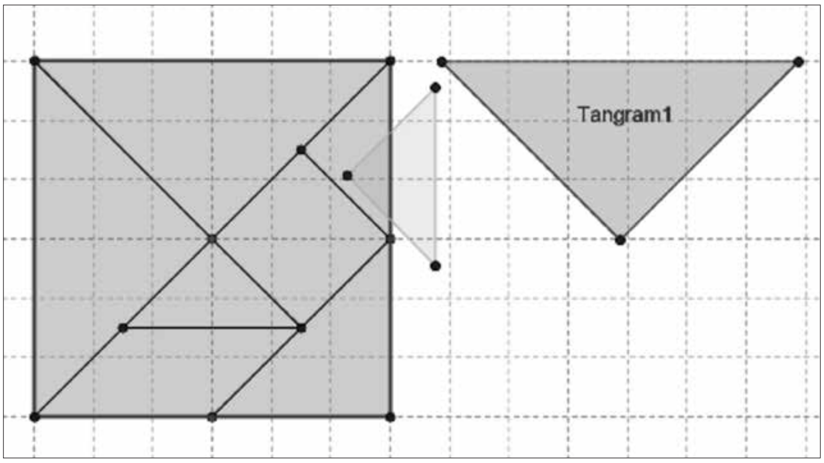

Figure 2. Modelled construction of a tangram master template.

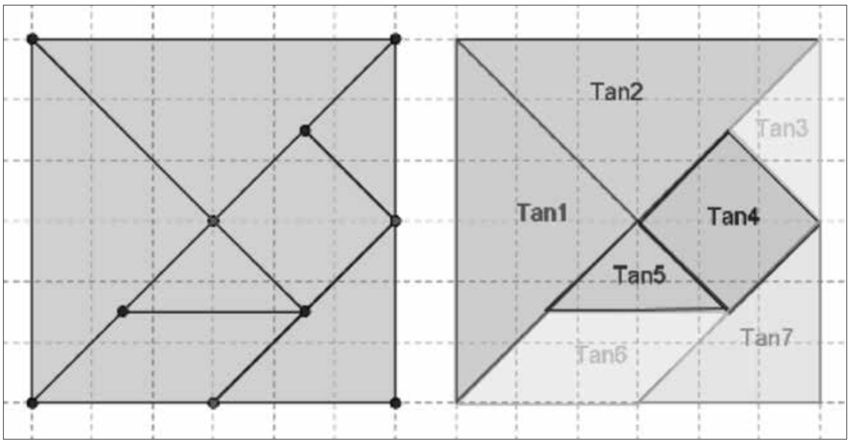

Figure 3. Completed tangram master template.

master template in Figure 3. The correct naming of the tangram pieces is important for activities and tasks in the later phases.

After creating their master template, students are then asked to explore and get used to working with the pieces of the tangram by trying to complete the following three activities. Students should be encouraged to use the appropriate terminology when talking about the different geometrical figures.

Create a horse

Use all your tangram pieces to make a horse. (See Figure 4).

Create a fox

Use all your tangram pieces to make a fox. (See Figure 5).

Create a dancer

Use all your tangram pieces to make a dancer. (See Figure 6).

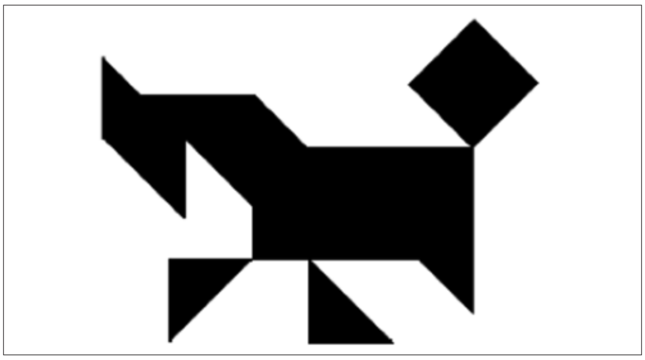

Figure 4. Create a horse.

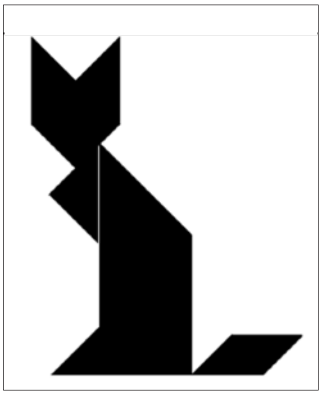

Figure 5. Create a fox.

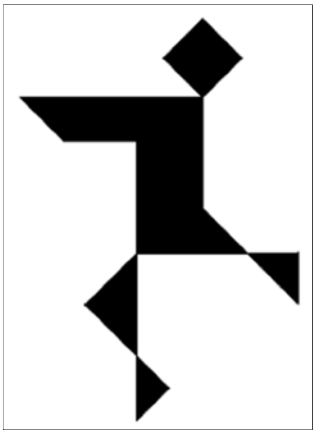

Figure 6. Create a dancer.

Phase 2: Direction

For students to identify the focus of the topic through a series of teacher-guided tasks. At this stage, students are given the opportunity to exchange views. Through this discussion there is a gradual implicit introduction of more formal language (Serow, 2007, p. 384)

Activities

Students work through a series of activities using their master template (see Figure 7) where they are required to manipulate the pieces of the tangram in order to make new shapes.

Making new figures part 1

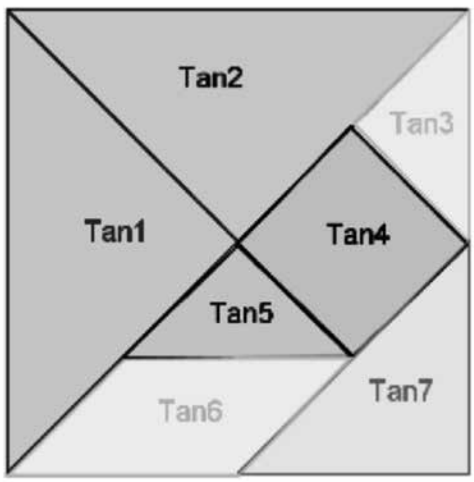

Figure 7. Master template for making new figures.

Teacher-guided tasks can include students working through a series of questions such as the ones below. Students can work individually or in groups, but should be given the opportunity to exchange views because through this type of discussion there will be a gradual implicit introduction of more formal language.

Sample 1

1. Use Tan 3 and Tan 5 to make a square the same size as Tan 4. How do you know that the figure you have made is a square?
2. Use Tan 3 and Tan 5 to make a triangle the same size as Tan 7.
3. Use Tan 3, Tan 5 and Tan 7 to make a square. How do you know that the figure you have made is a square?
4. Use Tan 3, Tan 4 and Tan 5 to make a rectangle. How do you know that the figure you have made is a rectangle?
5. Use Tan 3, Tan 4 and Tan 5 to make a triangle the same size as Tan 2. Is there only one way?
6. Now use Tan 3, Tan 5 and Tan 6 to make a rectangle. How do you know that the figure you have made is a rectangle? Is there more than one way?

Sample 2

1. Use Tan 3 and Tan 5 to make a parallelogram the same size as Tan 6. Is there more than one way? How do you know the new figure is a parallelogram?
2. Use Tan 3, Tan 5 and Tan 6 to make a parallelogram. Is there more than one way? How do you know the new figure is a parallelogram?
3. Use Tan 3, Tan 5 and Tan 6 to make a rectangle. Is there more than one way? How do you know the new figure is a rectangle?
4. Use Tan 1, Tan 3, Tan 4 and Tan 5 to make a rhombus.
5. Use Tan 1 and Tan 7 to make a trapezium.
6. Use Tan 1, Tan 2, Tan 3, Tan 5 and Tan 6 to make a rectangle.
7. Use Tan 1, Tan 3, Tan 4 and Tan 5 to make a five-sided shape. What do we call this shape?
8. Use smaller figures to make each of the following Tans. In each case explain how you know you have made required figure.
   (a) Tan 1
   (b) Tan 7
9. Tan 1 and Tan 2 can be combined to make a square. John, a student, says that there is no way to make a congruent square using the other pieces from the tangram. Do you agree with John?
10. Now use your tangram pieces to make as many different geometrical figures as you can. Try to give each shape a name.

Making new figures: Notes

The tangram is a very rich tool for exploring a range of geometrical figures and concepts, for example:

- In creating the required figures and justifying the figures they have made, learners have to focus on the properties of figures.
- In creating figures that are the same size and shape as others (as in Question 1), learners are creating congruent figures. This also provides an opportunity for comparing the area of the different figures.

- In creating larger figures learners are required to transform the figures and can be encouraged to describe these movements.

The aim of this activity is to focus the learners on the properties of the different geometrical figures, this is important in preparing learners for thinking on the van Hiele analysis level. It is also a useful context to reinforce the correct mathematical terminology for the different figures and geometrical concepts. In each case learners should be challenged to consider whether there is more than one way of making the required figure.

Sample 1 focuses on figures that will be familiar to most learners, for example, squares, rectangles and triangles. Although learners might not recognise the name of a figure, drawings can be given to show what figure must be made as explained in Sample 2. This can also be used as an opportunity to reinforce terminology.

Phase 3: New ideas

> For students to become conscious of the new ideas and express these in accepted mathematical language. The concepts now need to be made explicit using accepted language. Care is taken to develop the technical language with understanding through the exchange of ideas (Serow, 2007, p.384).

Activity

Building on the previous activities, students express and exchange their emerging ideas about the tangram by discussing their answers and solutions to Sample 1 and Sample 2. As explained by Crowley (1987) the teacher's role in this phase is minimal and mainly involves assisting students in using accurate and appropriate language.

Phase 4: Ownership

> For students to complete activities in which they are required to find their own way in the network of relations. The students are now familiar with the domain and are ready to explore it. Through their problem solving, the students' language develops further as they begin to identify cues to assist them (Serow, 2007, p.384).

Activities

Students design a table/spreadsheet which has the seven shapes of the tangram and each shape's properties. The spreadsheet should also contain information about how each individual shape can be combined to make new shapes. Students also complete another worksheet that combines work they have done with the tangram with fractions so that "many relations between objects of study become explicit to the students" (Hoffer, 1983 quoted in Crowley, 1987, p.6)

Tangrams and fractions part 1

(See Figure 7)

1. If Tan 1 is one quarter of the whole tangram, what fraction of the whole tangram is Tan 2?
2. What fraction of the whole tangram is Tan 7? Why do you say so?
3. Why is Tan 3 called one sixteenth of the whole tangram?
4. What fraction of the whole tangram is Tan 4?
5. What fraction is the parallelogram of the whole tangram?
6. Can you write a fraction name for each tangram piece and show that if you add all the fractions you will get 1?

Students are placed into groups, and asked to design a short lesson/activity which they will present to their peers. The guidelines for the presentation are as follows:

- Should be no more than 15 minutes in length (including their activity).
- Must involve a tangram and incorporate DGS (either as part of the actual presentation or as part of the peer activity).
- The lesson goal is based on a target outcome from the syllabus, which is "manipulates, classifies and draws two-dimensional shapes, including equilateral, isosceles and scalene triangles, and describes their properties" (Board of Studies, 2012).

Phase 5: Integration

> For the students to build an overview of the material investigated. Summaries concern the new understandings of the concepts involved and incorporate language of the new level. While the purpose of the instruction is now clear to the students, it is still necessary for the teacher to assist during this phase. (Serow, 2007, p. 384)

Activities

Students print out and collate all of their *GeoGebra* files, worksheets, lesson plan/presentation and handouts, and put them into their maths workbooks. Students are then instructed to write a summary of what they have learnt about tangrams and the properties of the shapes that make up a tangram. Students also write a reflection about their experiences with DGS and tangrams and should be asked to describe their learning regarding shapes and mathematical language. Writing reflections will help student language learning in a manner conducive to Pegg and Davey's (1989) claim that, "students would benefit from practice in expressing themselves, writing grammatical sentences, spelling mathematical words…the very act of reflecting and having to express an opinion that has not been rote learnt, students were forced to bring their ideas together—laying the groundwork for growth to the next level." (p. 26).

Conclusion

The teaching sequence presented in this article uses DGS in conjunction with the van Hiele teaching phases as a means for developing higher-order skills and understandings in geometry. Additionally, the van Hiele phases also facilitate language development since, through the manipulation of materials and the completion of tasks set by the teacher, the need to talk about the subject matter becomes important. In the early stages, students use their own language; however, over time, teachers and peers assist students to refine language and gradually incorporate, where appropriate, the correct mathematical register (Pegg, Gutierrez, & Huerta, 1998). Some further implications for teaching and learning include that research has shown that: DGS improves student motivation towards geometry (Stumbles, 2018); DGS assists English as a second language (ESL) students develop geometric understanding (Stumbles, 2018); and DGS has a positive impact on classroom interactions and the co-construction of knowledge (Abramovich & Connell, 2014).

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