Choosing the right resources to support the learning of polygons

Rebecca Seah
RMIT University, Vic.
<rebecca.seah@rmit.edu.au>

Using the correct materials and manipulatives appropriately allows the richness of geometrical concepts to be fully investigated to enable students to explore and reason about shapes.

Geometry, from Greek meaning 'earth measure', is a formal study of the properties of shape and objects and their relative position in space. Geometric knowledge is the key to succeeding in science, technology, engineering, and mathematics disciplines (Wai, Lubinski, & Benbow, 2009). It is also needed when pursuing leisure activities such as designing a garden, engaging in craft work, comprehending maps, and making decisions involving measurement situations. An important benefit of learning geometry is the ability to support visualisation, spatial reasoning, critical thinking and deductive reasoning (Jones, 2002). In a world that is bombarded by images, being able to reason spatially and critically is essential to navigate the space we live in and engage in civic affairs. As reflected in *The Australian Curriculum: Mathematics* (Australian Curriculum Assessment and Reporting Authority (ACARA), n.d), numeracy is no longer confined to having number sense but includes data, spatial and measurement sense. This article explores some of the key ideas involved in learning about two dimensional shapes and discusses how materials can be used to support learning.

The language of geometry

Anyone attempting to learn geometry will be confronted with its many terminologies. Words such as rhombus, trapezium, scalene, triangle, parallelogram, and icosahedron are derived from Latin and Greek roots and reflect their properties. For example, the word polygon derives from the Greek where 'poly' means many and 'gon' means angle ('many angled'), so hexagon means six-angled shape and pentagon means five-angled shape and so forth. However, this system of naming does not extend to naming all polygons, for the word 'angle' in triangle is adopted from Latin, referring to a three-angled shape. The anomaly in the way the root words are used can be a challenge for learners. Adding to this is the fact that colloquially, many people refer the term 'triangle' as a three-sided shape. Although there is no visual distinction between seeing a triangle as a three-sided rather than three-angled shape, the lack of attention to the etymology of geometric terms can mar the beauty and usefulness of geometry and hinder later learning when such a distinction is important.

The representation of geometry

The natural world we live in is mostly curved in form, yet we tend to see most things in terms of straight lines and two-dimensional shapes. Thus, the outline of a pine tree may look like a triangle, an orange may look circular, but closer observations reveal them not to be so simple. To understand space, diagrams are used schematically to represent an 'ideal', 'perfect' space with exact relationships. This is then studied in terms of their invariance, symmetry and transformation (Johnston-Wilder & Mason, 2005). Because of the abstract nature diagrams represent, additional tools are needed to help children comprehend the concepts. The use of materials and manipulatives as tools to support mathematics learning is not new. For example, base-ten blocks, place value charts, dice, counters, and Unifix are items commonly used for teaching number and algebra. The relationships they show are often linear and straightforward. When materials are used in geometry however, the relationship is no longer linear but complex. This is because reasoning about geometric relationships is dependent on an individual's ability to visualise. For example, triangles can be constructed by using strings or straws but an ability to reason about the constructed triangles requires one to see that the length of the sides influence the type of angles made, leading to the name given to these shapes (see Figure 1). Equilateral and scalene are taken from Latin words meaning equal and unequal sides respectively whereas isosceles is from the Greek, 'iso' meaning equal or identical and 'skelos' meaning legs.

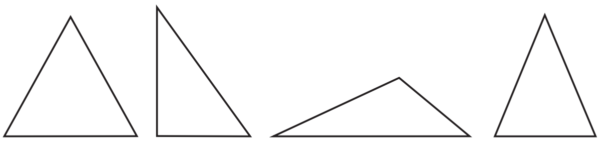


Figure 1. Examples of an equilateral, right-angled, scalene and isosceles triangle.

Visualising shapes is a cognitive activity that requires individuals to see beyond the physical appearance and interpret the visual images using existing networks of beliefs, experiences and understanding. When a quadrilateral is constructed it has additional properties related to parallel sides and diagonals. These properties are not present in triangles and are not always obvious to viewers. Materials do not necessarily carry meaning for students and no single material can be used to represent all geometric concepts. Patterns blocks are good for sorting activities, improving visualisation, learning that shapes can be constructed by combination of shapes, and introducing the concept of symmetry. They are less effective for teaching about quadrilaterals or the concept of regular and irregular polygons.

It is widely known that students and adults tend to have a prototypical view of what a shape should look like (Fujita, 2012). Once an image is formed, the student is less likely to change their view. For example, if a child's image of a square is one that sits horizontally along a plane (i.e. □), they are unlikely to accept that a shape sitting on its axis (i.e. ◇) is the same shape. A lack of exposure to shapes and objects from different perspectives hinders the ability to reason geometrically. Indeed, research has shown that even above average learners, who know the definition of the shapes, are unable to see shapes from different perspectives and comprehend that shapes are grouped according to a hierarchy (Fujita, 2012).

Understanding the properties of shapes and objects, and being able to use diagrams and symbols (e.g. °, ||, ∡) to engage in a logical argument is an essential part of thinking mathematically (Netz & Noel, 2008). Geometry differs from number concepts in that students do not learn all about triangles before moving on to learning about other shapes. All shapes are learned concurrently, becoming increasingly integrated and synthesised (Jones, 2002). Edward Wilson (Wilson, 2014), an American Biologist once stated, "You teach me, I forget. You show me, I remember. You involve me, I understand". The most effective way to teach for conceptual understanding then is to provide opportunities where students can construct and talk about shapes/objects in order to connect their actions with symbols and images.

Teaching for conceptual understanding

Naming Polygons

Two dimensional shapes have only two dimensions, length and width. They can be formed by straight lines, curves or a combination of both. Without depth, they cannot be picked up and handled. This can present a conceptual challenge when using pattern blocks to teach 2D shapes as they are technically three dimensional objects, not two dimensional shapes. As all materials have their limitations, careful scaffolding of the learning process that connects colloquial and everyday language to mathematical terms is needed.

The most common shapes learned in the primary years are those bounded by straight lines. Initially, students should be encouraged to make different shapes on a geoboard and talk about their features. Counting the number of corners leads to an understanding that polygons are named according to the number of angles the shape has (Figure 2). This activity leads to the naming of polygons and helps to prevent students from forming a stereotypical idea of what a shape should look like.

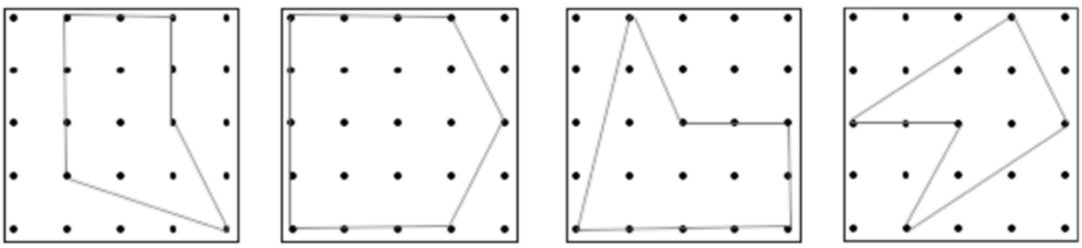
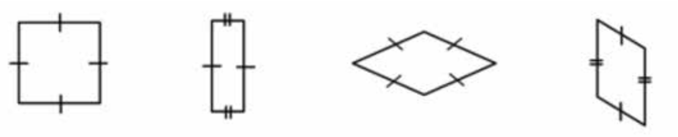
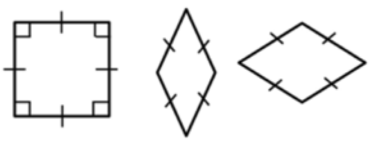
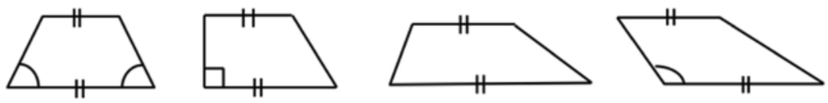
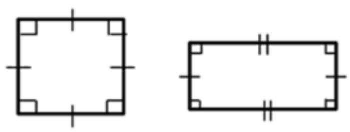
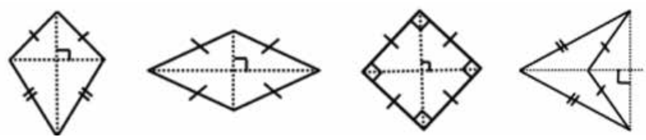
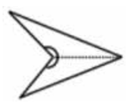


Figure 2. Types of pentagon constructed on a Geoboard.

Table 1. List of quadrilaterals and their definitions.

Parallelogram	A quadrilateral with two pairs of parallel sides.  Rectangles (Square, Oblong) Rhombus Parallelogram
Rhombus	From Greek ‘rhombos’ to mean a spinning top, a magic wheel used by sorcerers. A quadrilateral with four equal sides. 
Trapezium	A quadrilateral with at least one set of parallel lines.  Isosceles trapezium Right angled trapezium Trapezium Obtuse angled trapezium
Rectangle	From Latin, meaning right angles. 
Kite	A quadrilateral where the adjacent sides are equal and diagonals bisect perpendicularly to each other. 
Arrowhead/Chevron	The only quadrilateral with one angle greater than 180° and one internal diagonal line. 

Paper folding activities can be used in a similar fashion, where students fold different shapes and count the number of corners. Using square and A5 paper, children can investigate the shapes made with one, two and multiple folds. Providing young children with cut out triangles of different sizes and shape, can lead to talk about their similarity and differences, to compare and order the size of their corners, the length of the sides and the surface covered. Initially, informal language (such as longer, shorter, and corner) is used to describe these shapes; formal terms (e.g., isosceles) can be slowly introduced when the students are ready.

Quadrilaterals

An ability to classify things into a taxonomy is crucial for cognitive development (Tucker, Singleton, & Weaver, 2002). Through the process of classification, identifying similarities and differences among things, developing efficient ways of organising information and generalising ideas that support the development of deductive reasoning skill can be established. Classifying 2D shapes can be quite complex chiefly due to the way the shapes are defined. Quadrilaterals derive from Latin to mean four sides (whereas the Greek name tetragon focuses on four angles). They are a major group of shapes that many students and adults find difficult to comprehend. The key to understanding this family group of shapes is to

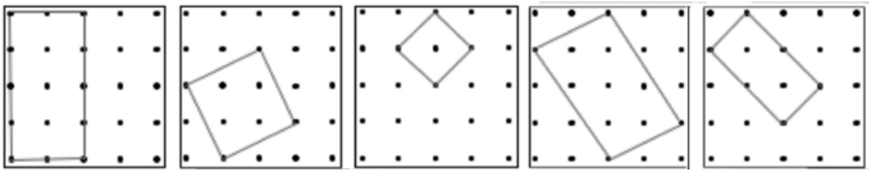


Figure 3. Which of these is a rectangle?

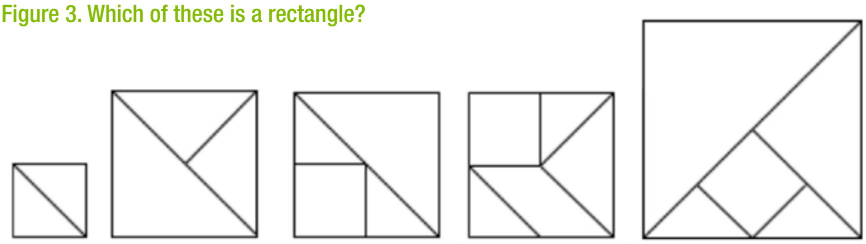


Figure4. Squares made with two, three, four and five tangram pieces.

determine the minimum number of attributes that the general set must possess (Booker, Bond, Sparrow, & Swan, 2014).

Table 1 shows how quadrilaterals can be grouped differently based on their properties. There are several interesting facts when tracing the etymology of the names of these shapes. For example, the word trapezium can be traced to the Greek word 'trapezion' meaning a little table. Depending on where you live, the term trapezium and trapezoid can have totally different meaning. In Australia and the UK, the former refers to a quadrilateral with one pair of parallel lines whereas a trapezoid is a quadrilateral with no sides parallel. In North America, the meanings of both terms are reversed. This confusion can be dated back to how the term is defined in *Hutton's Mathematical Dictionary* used in the United States in the 18th century. This confusion persists and reflects how mathematics is a human invention,

Another shape that can cause a lot of confusion is rectangle. Most people think of a rectangle as an elongated quadrilateral with right angles rather than a quadrilateral with right angles (Fujita, 2012). This stems from the way most people learn about shapes. Once a label is given to a thing, an image or an object, few would see the need to define the term. Yet in geometry, definitions of terms govern how shapes are grouped together and is an important aspect of deductive reasoning.

To teach for conceptual understanding, students can explore all possible shapes within each classification by making them on a geoboard, including shapes with different orientations and sizes. Students can then discuss why a shape is or is not a parallelogram, rectangle and so forth (Figure 3). Students can also experience making a particular shape without looking. This can be done by placing a Geoboard on their lap underneath the table. Making shapes without visual cues heightens geometric sense, training students to visualise the necessary properties of the shapes in their mind, reinforcing the concepts.

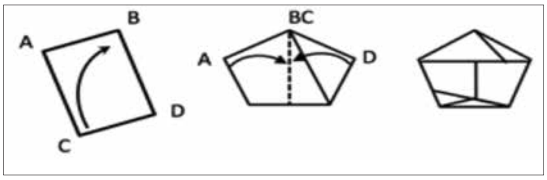
Another visual activity can be based on a tangram where the possibility of using two, three, four and all pieces to make a triangle, a square, a parallelogram, a rectangle that is not a square and a trapezium can be investigated (Figure 4). This form of activity cements students' understanding of the shapes and develops an awareness that shapes can be formed by combinations of other shapes.

The concept of regular, similar and congruent

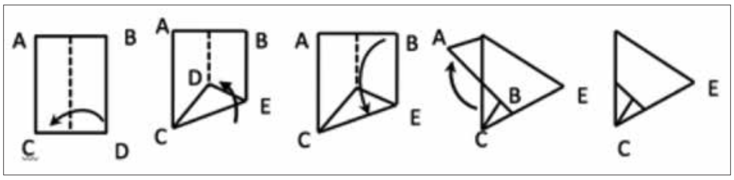
When common vocabularies such as similar, regular and bisect are used in geometry, they have very specific meanings. For example, colloquially the word bisect means divide into two parts. In geometry, it means divided into two equal parts. The word 'regular' in everyday conversation may mean the size of a beverage one purchases or being common or predictable. In geometry 'regular' refers to a shape that has sides of equal length and angles of the same magnitude. Hence, squares or platonic solids are regular shapes and objects whereas an oblong or a hexahedron formed by six equilateral triangles is not. If the word is not properly introduced and explained, learners are likely to mistake oblongs and rectangular prisms as regular because they are more likely to see them in their daily life.

Paper folding activities are more effective than the Geoboard for introducing the concept of 'regular', since the only regular shape students can make on the Geoboard is a square. Using different sized rectangular papers (for example A4 or A5), students can learn to fold different sized squares, equilateral triangles, regular pentagons and regular hexagons (Figure 5).

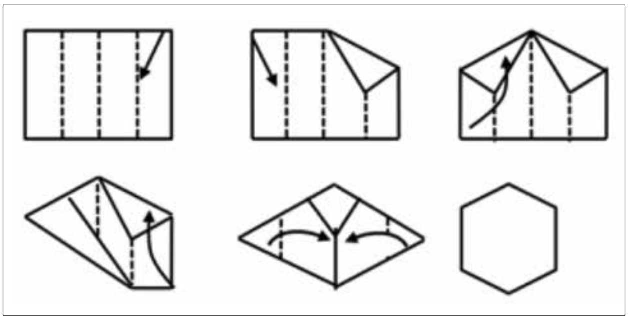
This activity provides opportunities for students to compare the angles and length of the shapes directly without needing to know how to use a scaled measurement. It cultivates an understanding that the side length and angles of a regular shape are equal regardless of the size of the shape. Once an understanding of 'regular'



Regular pentagon



Equilatarel triangle



Regular hexagon

Figure 5. Steps for folding regular shapes using A4 paper.

is established, students can then compare regular versus irregular shapes. Scale instruments can be used in the later years to measure the angles of these shapes. Although it is possible to fold a regular heptagon and octagon, the steps are too complex and beyond what is needed to introduce the idea in the primary years.

The word similar in everyday language means resemblance in appearance. Using this definition, one may assume that an ellipse and a circle are similar since they both have curves or that a rectangle and a rhombus are similar because they have 4 sides. In geometry, 'similar' refers to geometric figures that are the same shape, with the same angles and proportions but not the same length and therefore a rectangle is dissimilar from a rhombus. Congruent refers to shapes that are exact copies of each other. Both ideas are used when comparing shapes. Geoboards can certainly be used to teach both concepts. Getting students to fold similar shapes also provides opportunities to directly compare the length and angle of the shapes. Investigating folding similar shapes using the same size paper provides further opportunities for students to link geometric ideas with concepts of fraction and proportion. For example, a square paper can be dissected into different fractions and be used as a basis for folding parallelograms (Figures 6 & 7). Comparing shapes made using Geoboard and paper folding activities further allow students to comprehend that similarity is determine by the size and angle, not by the different medium used.

Conclusion

As one of the oldest disciplines, geometry is a wonderful subject to teach and learn as it appeals to our visual and aesthetic sense. Using materials and manipulatives appropriately allows the richness of the concepts to be fully investigated and its full meaning be realised. Only then can students be given the opportunities to explore and reason about these shapes and construct meanings that are shared by the mathematics community.

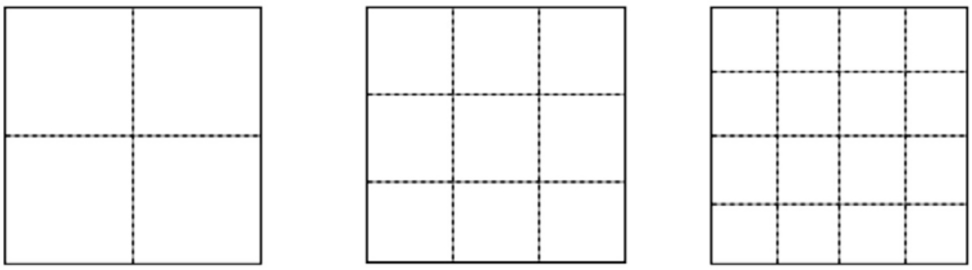


Figure 6. Dissecting square papers into fourths, ninths and sixteenths.

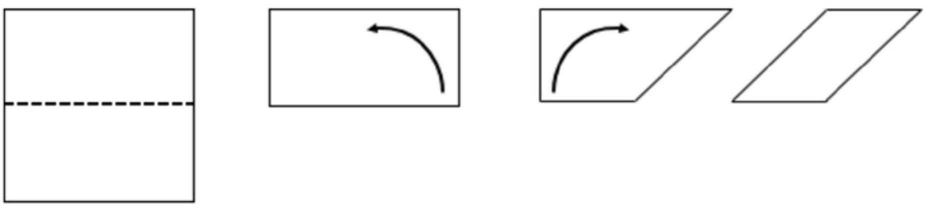


Figure 7. Folding a parallelogram.

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