Challenging tasks lead to productive struggle!

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Challenging mathematical tasks can be designed to allow students of all abilities to experience productive struggle. It is important for teachers to communicate with students that productive struggle is important and it is what mathematicians do.

> I like doing problems where I have to keep working on them until I finally get it … it's frustrating if it's too easy. (Natalie, Year 6).

Productive struggle, such as that experienced by Natalie, leads to productive classrooms where students work on complex problems, are encouraged to take risks, can struggle and fail yet still feel good about working on hard problems (Boaler, 2016). Teachers can foster a classroom culture that values and promotes productive struggle by providing students with challenging tasks. These tasks are designed in such a way that they are accessible to all students and the expectation is that everyone will persist when solving challenging mathematical tasks.

This article looks at examples of challenging mathematical tasks designed to enable students of all abilities to experience productive struggle, and examines the lesson structure and teacher's role when implementing a challenging task, along with students' responses to being challenged through the tasks. It adds to existing research on challenging tasks, including in previous editions of APMC (e.g., Cheeseman, Clarke, Roche, & Walker, 2016; Roche & Clarke, 2014; Russo, 2016) through challenging tasks being used to evoke productive struggle, and illustrative examples of what productive struggle 'looks like'.

Challenging tasks

Consider the following two fraction tasks that may be suitable for Year 6 students—Figures 1 and 2.

Which of the two tasks would you consider to be 'challenging'?

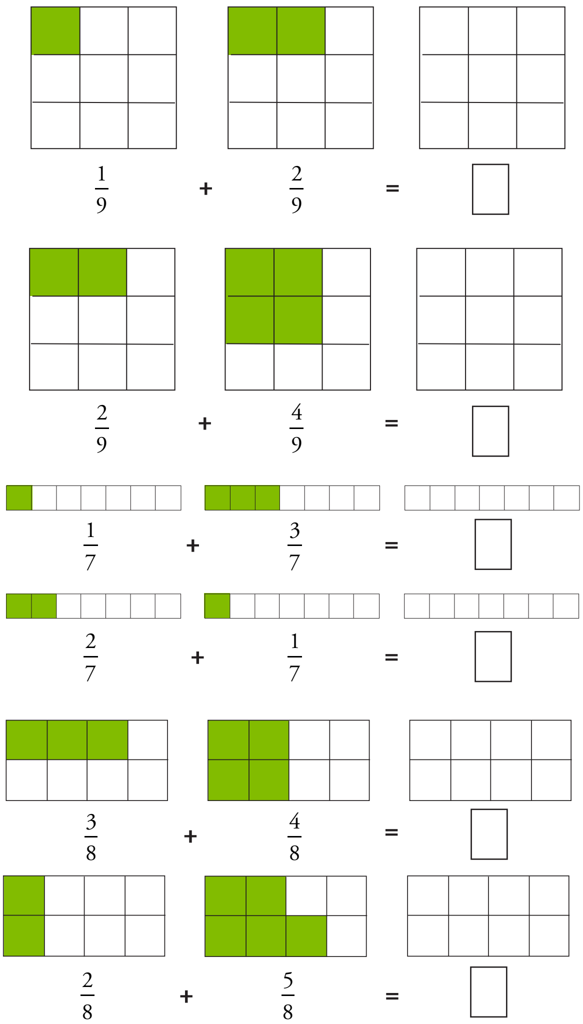
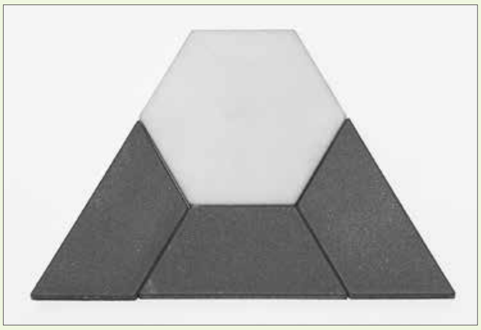


Figure 1. Fraction tasks as found in text books.

What fractions can you make?
Consolidating task



The shape made out of Pattern Blocks is worth 1. Using either the hexagon or the trapeziums, or both, what different fractions of the whole shape can you make? Draw and label your fractions.

Figure 2. Consolidating fraction task from *Encouraging Persistence, Maintaining Challenge (EPMC)* project (Sullivan, 2016).

We believe that Figure 2 provides an example of a challenging task because the task has been designed to promote rich student-centred thinking including productive struggle. When exploring the task by themselves without input from the teacher, it is anticipated that students will engage with productive mathematical struggle because they are attempting a task they do not yet know how to solve. When considering responses to the pattern block task there is also more than one solution and more than one method for finding the range of answers.

As a comparison, Figure 1 provides a typical example of a task that is found in many textbooks. It is unlikely to create productive struggle in students as it is essentially a counting and colouring exercise. The example in Figure 2, however, requires a deeper application of fraction knowledge, including the recognition that a shape is cut into unit fractions and that the parts must be the same size. The task is also designed in such a way that the answer is not obvious and the solver is required to process multiple pieces of information and is likely to take some time to solve the question.

The task in Figure 2 also meets the requirements of a challenging mathematical task as identified by Sullivan (2016, p.5) in that it requires students to:

- plan their approach, especially sequencing more than one step;
- process multiple pieces of information, with an expectation that they make connections and see mathematical concepts in new ways;
- choose their own strategies, goals and level of accessing the task;
- spend time on the task and record their thinking;
- explain their strategies and justify their thinking to the teacher and other students.

Why is productive struggle important?

In addition to providing challenging tasks, teachers should communicate frequently that struggle is important, stimulates brain growth and helps develop a growth mindset (Boaler, 2016). The National Council of Teachers of Mathematics (NCTM) recommends that teachers need to provide opportunities for productive struggle as it is significant and essential to learning mathematics with understanding (NCTM, 2014). Challenging tasks can be used to stimulate productive struggle when learning mathematics, but this struggle needs to be facilitated and encouraged by the teacher. This requires the teacher to avoid reducing the cognitive load of the task, such as providing routine instructions for completing the task and over-modelling how to approach the task (Clarke, Roche, Cheeseman, & van der Schans, 2014/2015). According to Boaler (2016), we, as teachers, need to resist valuing "effortless achievement" (p. 178) and instead value persistence and hard thinking.

An Australian project designed to help teachers engage students in productive struggle through attempting challenging mathematical tasks was the Encouraging Persistence Maintaining Challenge (EPMC) project. The rationale for the project involved the belief that while it is possible for everyone to learn mathematics, it takes concentration and effort over an extended period of time to build connections between topics and to understand the coherence of mathematical ideas. When solving tasks students are encouraged to persist, believe they can succeed and appreciate that learning mathematics takes effort. In order to foster this behaviour, students need to engage in challenging tasks and lessons that allow for sustained thinking, decision making, and some risk taking by the students (Sullivan, et al., 2014). The teachers that were involved in the EPMC project were provided with support in the form of professional learning that enabled them to implement appropriately challenging tasks and experiences with their students. One of the key messages of the project was that challenging tasks were important for all students and that 'controlled floundering' was essential for students when extending their thinking at higher levels (Pogrow 1988 as cited in Clarke et al. 2014/2015). Figures 3 and 4 provide two other examples of challenging tasks that were used in the project (in addition to that shown in Figure 2). Both tasks provide for a variety of strategies and responses and require a planned approach and commitment of time to solve efficiently.

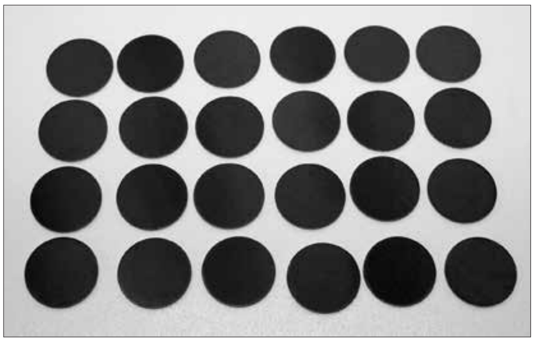
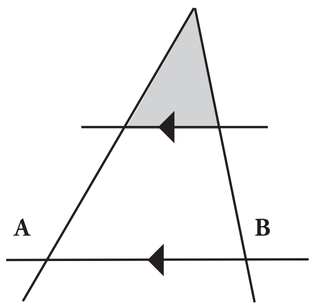


Figure 3. Write as many number facts as you can.



Angles A and B are both multiples of 5 and add to be 240 degrees, but they are not equal. Work out what might be each of the angles in the triangle. (Give at least two different answers.)

Figure 4. Challenging angle task.

The role of the teacher

Having selected a challenging task, teachers in the project adopted a three-stage process for teaching with challenging tasks: launch, explore, and summarise (Sullivan et al. 2015). The lesson is launched with minimal introduction from the teacher and students are expected to attempt and explore the task first by themselves without any instruction from the teacher (Sullivan 2016). In this phase, students would typically read the task quietly and could ask clarifying questions, but the teacher would not show students how to do the task. During the explore phase students might experience struggle. In the project, the term "zone of confusion" was used to encourage students to persist with solving the problem individually and to help them recognise that struggle was a positive experience and helped to develop a growth mindset (Dweck, 2000). Teachers were also encouraged to foster persistence through using affirming statements such as:

> You did not give up even though you were stuck … you tried something different … you tried more than one answer …

All the suggested lessons in the project included consolidation tasks, enabling prompts (for students experiencing difficulty) and extending prompts (for students who completed the task quickly). It was important that each student had the opportunity to attempt the original task before being given an enabling prompt. To demonstrate how a task was adapted through enabling and extending tasks, consider the following multiplicative task (suitable for Year 5).

The task

> I did a multiplication question correctly for homework, but my printer ran out of ink. I remember it looked like 2_ × 3_=_ _ 0
>
> What might be the digits that did not print? (Give as many answers as you can).

After allowing for some confusion, the teacher might then give individuals the following enabling prompt in order for them to access the task:

> What might be the missing numbers?
> 1 _ × _ = _ 0

For those students who quickly complete the task, the following extending prompt could be given:

> Convince me that you have all the possible solutions.

Using this approach, all students have access to a challenging task where the cognitive demand is not compromised by over-modelling or over-simplifying the task. Students would generally spend a whole mathematics lesson completing the original task and additional consolidating, enabling and/or extending tasks. They would then be expected to explain and justify their thinking in the 'summarise' part of the lesson, where again the teacher plays a key role in terms of selecting, sequencing and connecting the mathematics to be shared (Smith & Stein, 2011). While we recommend that the teacher follows this approach to maximise the learning experiences for the students, it is not a 'recipe' and its enactment would still require the teacher to be flexible in implementing appropriate pedagogies that are responsive to individual students' needs, including consideration of students' affective domain.

Classroom vignette 1: Productive struggle in the 'explore' phase

In order to illustrate how productive struggle can be manifested through this approach, we provide the following example, involving Debbie, a Year 6 student who considered herself to be successful at mathematics and generally found mathematics easy especially when the teacher explained how to solve problems during the lesson introduction.

Debbie's teacher usually explained the mathematics before the lesson, but today was different.

As part of the study, the teacher gave Debbie's class a task (see Figure 4) and asked the students to attempt the task by themselves, without any prior instructions or revision of the topic. Debbie struggled to start the task and became noticeably frustrated at not being able to generate a solution (possibly because the lesson structure was different, she was expected to attempt the task by herself and/or she did not want to record an incorrect response or make a mistake). The teacher observed Debbie and noticed she was unable to break down the task into smaller pieces and was therefore unsure how to begin. Debbie became upset and frustrated as others around her appeared to be able to solve the task. Later Debbie explained, "I was not able to think logically."

The teacher then chose one student to share her strategies and thinking as part of the 'summarise' phase of the lesson, providing Debbie and other students with an example of how to commence the task. Debbie watched and nodded, "I know that … I just didn't read the question properly."

The teacher allowed the students to continue with the problem before stopping the lesson again to share further student thinking and strategies (Figure 5).

After seeing and listening to her class mates share their thinking Debbie had another attempt at the challenging task (Figure 6). By the end of the lesson Debbie had recorded a correct response to the task labelling the angles in the diagram (while the reflex angle has been incorrectly labelled as 120, it does not affect the solution).

While Debbie initially struggled to complete the task, the supportive learning environment and encouragement from her teacher, enabled her to 'regroup' and learn from her peers, emphasising the importance of the need to consider students' affective domain. Finding a balance between knowing when to allow students to struggle and knowing when to provide support is not easy (Roche & Clarke, 2014).

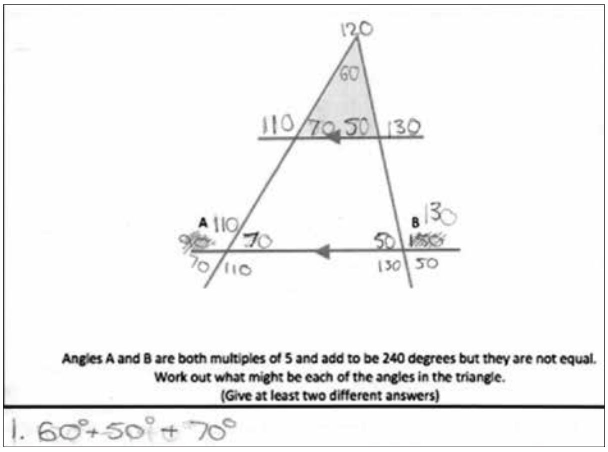
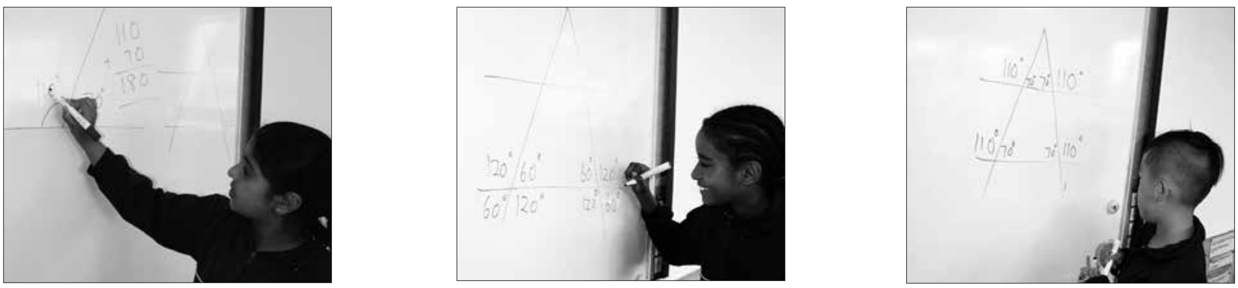


Figure 6. Debbie's response to the geometric task.

The lesson may have had a negative impact on Debbie but her personal reflection after the lesson suggests that it did not:

> The maths lesson today was very challenging … I struggled with understanding the questions about the triangle … [I need to] keep trying, never give up and try to think … outside your comfort zone.

While many students, such as Natalie, embraced the opportunity to engage with challenging tasks, others, like Debbie, found the approach initially confronting. This seemed to be particularly the case with students who were considered very capable and confident mathematicians. This may be attributed to their prior experiences with learning mathematics; it may well be that they were exposed to a performance culture, where struggle and failure have never been valued (Boaler, 2016) or explored. When students inevitably meet struggle, they can then question their ability, having perhaps been successful in the past through "effortless achievement" (Boaler, 2016, p. 178). As teachers, we need to be



1. Knowledge that a straight line equals 180˚.

2. Filled in a lot of numbers to help solve the problem. Knew opposite angles were equal. (Another student shared that the four angles in the quadrilateral added to 360˚.

3. Shared knowledge of the relationship between parallel lines and corresponding angles (same location).

Figure 5. Students sharing during the summarise phase.

sensitive to these students whose mathematical 'identity' may be at risk, and respond accordingly.

Classroom vignette 2: Productive struggle in the 'launch' phase

A different problem, involving another challenging angle task, was implemented with the same Year 6 class. The following vignette demonstrates how productive struggle can occur within the 'launch' part of the lesson. The original task was:

> I know that the minute hand on the clock is on the 2. The hands make an acute angle. Give me as many answers as you can.

The task was considered challenging in that it required more than one step to solve, it had a number of possible correct solutions and could be solved by using a range of methods.

As with the previous example, the teacher gave the students individual copies of the problem with minimal instructions on how to solve the task. Students were asked to attempt the problem without assistance. Most students were willing to have a go at the problem and were able to formulate a correct response. Figure 7 provides an example of an illustrative example of a correct answer to the problem.

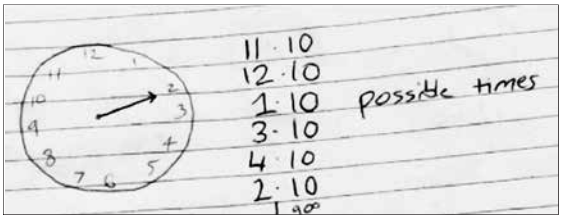


Figure 7. Possible solutions for the angle task.

For those students who could not generate a solution after showing some persistence, the following enabling prompt was provided:

> On a clock, show a time at which the hands make an angle less than a right angle.

For those students who required an additional challenge, the following extending prompt was made available:

> Are there always six possible acute angles no matter what number the minute hand is pointing to?

Did the task provide for productive struggle? All students in the class were able to attempt the task. The task itself was not a straightforward one, with student responses showing some common errors. Some students, for example, thought that the clock hands would be directly on top of each other at ten past two, while others thought that at ten past five, the angle of the hands would be 90 degrees.

Classroom vignette 3: Productive struggle in the 'summarise' phase

While many students were successful with achieving a correct response, the following classroom exchange illustrates how successful problem solvers can still experience productive struggle, particularly when asked to explain their reasoning.

In the summary phase of the lesson when labelling angles of pattern blocks, Maria's solution was displayed on the interactive whiteboard for the class to see (Figure 8). Her solution shows that she correctly labelled the angles in the green parallelograms. Next, she pointed to the opposite angle or the interior angle in the hexagon and said "This is 120 degrees" (see arrow in Figure 8).

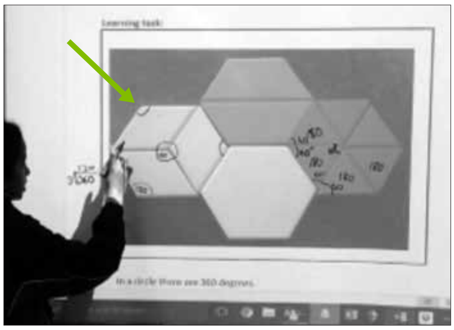


Figure 8. Maria pointing to the angle that equals 120 degrees.

The teacher asked, "How do you know that?" Maria stated, again correctly, that the answer was 120 degrees. She repeated herself and said that she divided 360 by 120 but she was not able to justify her thinking and seemed to be struggling to provide a convincing explanation. She may have known that opposite angles were equal, but never stated this, and it was not clear how Maria knew the angle was equal to 120 degrees.

The teacher continued to question Maria as the class listened.

Teacher: Where did you get 360 from?

Maria: It is because a whole circle is 360 degrees.

[She still did not however use this knowledge to add to her explanation.]

The teacher continued to probe for a more detailed explanation:

Teacher: I am still not convinced how you got the answer 120.

[At this point, another student tried to help].

Helen: It is like the red triangle, because two of those [angles] make one of those [angles].

Teacher: Was that your thinking, Maria?
Maria: It is sort of like my thinking but sort of isn't.
[Another student offered his strategy]
John: [at the board] See—here I think she divided each of these angles [pointing to the centre of the green hexagon] by three, and three times 120 is 360 and these angles would all be the same.
Teacher: Why would they be the same? [referring back to the same problem]
John: I am not quite sure.

At this point, Maria is looking rather concerned and later said that she thought she had the wrong answer.

Teacher: We are trying to work out why Maria's thinking is correct [reassuring Maria that she is correct but still needs to clarify].

A third student, Chloe suggested cutting the green parallelogram in half and then they would be the same as two red triangles and "we would be able to add 60 and 60 together to get 120 because the angles in one triangle equal 180 degrees".

[Finally, Maria was able to justify the answer with understanding after listening to Chloe].

Maria: If you were able to draw a line like that it would be 60 and 60 and if you add these up [pointing to the two angles in the triangle] you get 120.

Thinking about how students respond to a task and guiding students' thinking is an important part of the teacher's role when promoting productive struggle. During the 'summarise' phase the teacher questioning provided an opportunity to promote productive struggle because she did not tell Maria the answer, but kept questioning in order to assist student understanding. Maria was required to justify her thinking, explain her reasoning, and showed persistence in finally being able to do this. Throughout the process, the teacher (and students) questioned and responded to Maria in a sensitive and respectful manner, maintaining the focus on Maria's mathematical thinking, with no judgements being made about her mathematical ability—a practice which is likely to promote a growth, rather than a fixed mindset.

Conclusions

Engaging students in mathematics learning that is purposeful and leads to productive dispositions, requires the teacher to select suitably challenging tasks, appropriately implement these tasks and then expect students to explain and justify their responses to the tasks. Presenting students with tasks they do not yet know how to solve, allowing students to explore the task for themselves, employing effective questioning techniques and providing opportunities for observing how other class members might respond to a task, are important pedagogical practices that teachers should consider when promoting productive struggle. While we acknowledge that there is no 'recipe' to follow to achieve productive struggle, we have illustrated how the challenging tasks detailed in this article, along with the teacher's approach, fostered productive struggle in students as evidenced by their willingness to persist in not only solving the problem but in justifying their responses to the tasks.

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