The Impact of Different Missing Data Handling Methods on DINA Model

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ABSTRACT

In this study, it was aimed to investigate the impact of different missing data handling methods on DINA model parameter estimation and classification accuracy. In the study, simulated data were used and the data were generated by manipulating the number of items and sample size. In the generated data, two different missing data mechanisms (missing completely at random and missing at random) were created according to three different amounts of missing data. The generated missing data was completed by using methods of treating missing data as incorrect, person mean imputation, two-way imputation, and expectation-maximization algorithm imputation. As a result, it was observed that both s and g parameter estimations and classification accuracies were effected from, missing data rates, missing data handling methods and missing data mechanisms.

Keyword:
Classification accuracy
DINA Model
Handling
Missing data
Parameter estimation

1. INTRODUCTION

Cognitive diagnosis assessments (CDAs) are increasingly becoming a popular research area in the fields of measurement and psychology. Leighton and Gierl [1], pointed that CDA “is designed to measure specific knowledge structures and processing skills in students so as to provide information about their cognitive strengths and weaknesses”. Cognitive Diagnostic Models (CDMs) are psychometric models which were developed to identify the examinees’ ability to master fine-grained skills. CDMs provide a profile of whether the individual has pre-determined skills. By this way, richer, more meaningful and more informative information about the individual can be provided.

The cognitive diagnostic models (CDMs) connect the latent skills with observed behaviors (tasks) which were required by a Q-matrix [2]. The Q-matrix is a format for specifying the underlying cognitive attributes measured by the test items. Creating a Q matrix is one of the most important steps of the CDMs applications. In a Q-matrix items (J) yields in the rows and attributes (K) yields in the columns with the elements of qjk. The elements qjk of Q matrix get values 1 or 0. 1 indicates that mastery of attribute k is required by item j. Contrary, 0 indicates that mastery of attribute k is not required by item j [3]. Before running a CDM to test data, the Q-matrix must to be already determined.

The literature review shows that, CDMs were classified into various ways [4],[5] and several CDMs have been developed to evaluate examinees’ status relative to mastery or non-mastery on each of a set of attributes [6]–[10]. One of the frequently used CDMs is the deterministic, inputs, noisy “and” gate (DINA) [10] model. This study is limited with the DINA Model. A brief description of DINA model is given below:

The DINA model (Haertel, [8]) is a noncompensatory model and it has conjunctive condensation rule [11]. It is easy to estimate DINA model parameters for the item response function which is given by

\[
P(X_{ij} = 1|\eta_{ij}) = \theta_j^{1-\eta_{ij}}(1 - \sigma_j)^{\eta_{ij}}
\]

where \(X_{ij}\) denotes the response of the \(i\)th examinee to item \(j\), with 1 representing a correct and 0 representing incorrect response [12]. \(\sigma_j\) and \(\theta_j\) represent the guess and slip parameters for the \(j\)th item, respectively. The slip parameter is interpreted as the probability that students who possess all the required attributes for an item answer it incorrectly, whereas the guess parameter is the probability that students who lack at least one of the required attributes for an item to answer it correctly [12]. \(\eta_{ij}\) is the latent response and it is given by

\[
\eta_{ij} = \prod_{k=1}^{Q} a_{ik}^{\alpha_{jk}}
\]

\(\eta_{ij}\) assumes a value of 1 or 0. If \(\eta_{ij} = 1\), it indicates that student \(i\) possesses all the attributes required for item, and \(\eta_{ij} = 0\), it indicates that student \(i\) lacks at least one of the attributes required for item \(j\) [12]. \(a_{ik}\) refers to the entry in the \(i\)th row, \(k\)th column of the \(Q\) matrix [12].

In this study, the effect of the missing data on the parameter estimation and classification accuracy of the CDM’s DINA model was investigated. The following section briefly provided information for the missing data:

Missing data and missing data handling methods are important topics in many research fields. Many educational and psychological data frequently have missing values because of several reasons. Since, most of the statistical approaches require full data, missing values threatens the data analysis process. Missing data handling methods are used to deal with missing data. However these methods may have biased estimates like other statistical process. Several missing data handling methods have been developed to overcome this problem.

The first step of missing data analysis is determining the reasons for the missing data and the amount of missing data. Missing data patterns and missing data mechanisms are the reasons for decisions about whether or not missing data will be neglected. The missing data pattern describes the patterns of missing data occurrence of observed data in a data set and the missing data mechanisms describe possible relationships between the measured variables and the probability of missing data.

Various missing data handling methods and analysis were developed for the missing data mechanisms, with different assumptions about missing data. According to Rubin [13], there are three types of missing data mechanisms: missing completely at random (MCAR), missing at random (MAR) and missing not at random (MNAR) [13],[14]. Missing completely at random (MCAR) arises when a subject with incomplete observations are a random subset of the complete sample of subjects [13]. MCAR is defined as if the probability that the data are missing does not depend on any variables, either observed or unobserved [14]. MAR occurs when the probability of missing data on a variable is unrelated with the value of that variable however it may be related with other variables in the data set. NMAR occurs when the probability of missing data on a variable is a function of the value of that variable [13],[15]. If the missing data mechanisms are MCAR or MAR then it is not necessary to model the process that generates the missing data in order to accommodate the missing data. MCAR and MAR mechanism that produces the missing data are ignorable. However MNAR mechanism is non-ignorable. It is necessary to model this mechanism to deal with the missing data in a valid manner.

Missing data introduce an element of ambiguity into data analysis and they can affect properties of statistical estimators. Various methods have been developed to solve the problem of missing data and they can have profoundly different effects on estimation. The problems of analyzing data with missing values have been reviewed extensively in the literature [14],[16]-[18]. Some frequently used missing data handling methods are; deletion methods (pairwise and casewise deletion) and imputation methods (mean imputation, regression imputation, EM imputation, multiple imputation [14],[16]. In this study; treating missing data as incorrect (IN), person mean imputation (PM), two-way imputation (TW), and expectation–maximization (EM) algorithm imputation methods were investigated for DINA model analysis. In CDM applications, treating missing data as incorrect is widely used. In this method, missing responses are scored as incorrect. In person mean imputation method, at first the average of the observed item scores is computed for each respondent then computed average is imputed for the item scores that are missing for that respondent. In two-way imputation [19] method, the imputed value is calculated by adding the person mean to the item mean and subtracts the overall mean of the data from that score. Expectation–maximization (EM) algorithm imputation method is a two-step method based on the completion of missing data using the maximum likelihood estimates. In the E-step, the missing data is completed by the expected values and in the M step, parameter estimation is done using the values estimated in the first step [20].

Literature review shows numerous missing data and missing data handling methods investigations in terms of combinations of factors like, sample size, proportion of missing data, method of analysis, and
missing data handling method [17],[21]-[26]. Also there are many investigations in cognitive diagnostic models which use DINA model [1],[3]-[12],[27]-[33]. However there are limited practical research on CDMs where missing responses were present [32],[34],[35].

In CDMs, parameter estimates might be threatened by missing data, too. While trying to provide more detailed information about the individuals by using CDMs, biased estimates can be made with the presence of the missing data. In this study, it is aimed to determine the performances of different missing data handling methods for CDM estimations. For this purpose several data sets with missing data were analyzed by using CDM in order to determine effective factors which cause biased estimations. Literature review shows very limited study for missing data in CDM applications. This study will contribute this gap with its different manipulation factors and its levels.

2. RESEARCH METHOD

2.1. Simulation Design

2.1.1. Simulation Conditions

Model: The DINA model is used to generate data, to estimate parameters and to calculate the classification accuracy. DINA model is preferred because it is widely used within the CDM studies and it is easier to estimate the parameters.

Sample Size (N): Literature review shows that sample size is an important factor affecting parameter estimation. de la Torre, Hong and Deng [30] used 1000, 2000 and 5000 sample sizes, de la Torre and Douglas [29] used 1000 sample size, and de la Torre and Douglas [6] used 2000 sample size in their studies. In this study three different sample sizes (1000, 2000 and 3000) were used for each condition.

Number of item (NI) and number of attributes (K): Literature review shows that the number of attributes varied between 3 – 8 ranges [3],[5],[6],[29]. In this study, number of attributes was fixed as 4 and number of items for 4 attributes manipulated as 15 and 30.

s and g Parameter Levels: Henson and Douglas [3] used s and g ≈ U(.05 -.40); Rupp and Templin [5] used s ≈ U(.0 -.25) and g ≈ U(.0 -.15). In this study the parameter distribution and range were preferred as s and g ≈ U(.10 -.30).

Missing data mechanisms and Missing Rate (MR): In this study three levels of missing rate (5%, 10%, 15%) and two missing data mechanisms (MCAR and MAR) were investigated. In the literature, rates of missing data ranged from 2% to 50% however most of them yield between 5% - 30%.

Missing data imputation methods: In this study, four missing data handling methods (treating missing data as incorrect (IN), person mean imputation (PM), two-way imputation (TW), and Expectation Maximization (EM) imputation were used missing data handling.

<table>
<thead>
<tr>
<th>Table 1. Simulation Design Factors and Levels</th>
</tr>
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<tbody>
<tr>
<td>Factors</td>
</tr>
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<tr>
<td>Number of Item</td>
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<tr>
<td>Missing Mechanism</td>
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<tr>
<td>Missing Rate</td>
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<tr>
<td>Missing Imputation</td>
</tr>
</tbody>
</table>

2.1.2. Data Generation

The data were generated according to the DINA model. The parameter values of s and g were ranged between 0.1 and 0.3 (s and g ≈ U(.10 -.30)), the number of attributes were fixed as 4. The number of items were set to 15 and 30, and the sample sizes were preferred as 1000, 2000 and 3000. 100 replications were conducted for each crossing condition. R 3.0 was used for data generation and data management procedures.

Data deletion was performed according to MCAR and MAR for each experimental cell of condition crossings. In order to achieve a missing completely at random condition, the probability of missing of any data is equal to the probability of missing of data in another cell, and these probabilities must be independent from each other. For this purpose, firstly the number of cells to be deleted in the data set was determined according to the amount of missing data. After that, the data of the determined number of cells is deleted from the data set randomly with the written program. In order to achieve a missing at random (MAR) condition, the probability of missing of any data should be related to another variable with a complete error, and the conditional probabilities for this variable must be equal. For this purpose, firstly the examinees were sorted in ascending by their total test scores. The 80% deletion was performed for examinees' responses who
were in the first quantile and 20% deletion was performed for examinees' responses who were in the fourth quantile regarding to the amount of missing data. Finally, the generated missing data was completed using four missing data handling methods (treating missing data as incorrect (IN), person mean imputation (PM), two-way imputation (TW), and Expectation Maximization (EM)).

2.1.3. Analysis of Data
To analyze the data, the results of the item parameters and attribute profiles obtained from the completed data were compared with estimates which were obtained from each missing data handling methods. The root mean square error (RMSEA) was computed for the consistency of the item parameter estimates and the pattern-wise classification accuracies were computed for the classification accuracy for each experimental cell.

3. RESULTS
3.1. Results for s Parameter Estimation
The effects of different missing data mechanisms, sample sizes, number of items, and missing rates on the average RMSEA of s parameter estimation were given in Table 2 and the results of the interaction effects were given in Figure 1 and Figure 2. When Table 2 is examined, it was seen that average RMSEA values obtained in both MCAR and MAR conditions were low. It was also seen that both in MCAR and MAR conditions, the average RMSEA values obtained from missing handleings methods in different sample sizes and numbers of item did not change much. Sample size and number of item manipulation was not affective for the s parameter estimations. However, in both MCAR and MAR conditions, the average RMSEA values tended to increase as missing rates were increased.

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>1000</th>
<th>2000</th>
<th>3000</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.0167</td>
<td>0.0154</td>
</tr>
<tr>
<td>IN</td>
<td>0.0807</td>
<td>0.0796</td>
<td>0.0789</td>
</tr>
<tr>
<td>PM</td>
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<td>0.0146</td>
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<td>TW</td>
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</tr>
<tr>
<td>PM</td>
<td>0.0048</td>
<td>0.0037</td>
<td>0.0034</td>
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<tr>
<td>TW</td>
<td>0.0049</td>
<td>0.0039</td>
<td>0.0035</td>
</tr>
</tbody>
</table>

Table 2. Average RMSEA of s Parameter for Simulation Conditions

All methods, except the IN method, performed similarly under all the missing rate conditions. Especially, it has been observed that the RMSEA values obtained from the IN method at the 15% missing rate condition got the maximum values. RMSEA values obtained from the methods with different missing rates seem to be higher for MCAR than MAR.

When the factor interaction effect graph for the s is examined, it was observed that in both MCAR and MAR conditions, different sample sizes and number of item did not change the average RMSEA values too much. However, it is also observed that RMSEA values were increased with the increase of missing rates.
3.2. Results for g Parameter Estimation

The effects of different missing data mechanisms, sample sizes, number of items, and missing rates on the average RMSEA of g parameter estimation were given in Table 3 and the results of the interaction effects were given in Figure 3 and Figure 4. When Table 3 is examined, similar to the s parameter estimation, it was observed that average RMSEA values obtained for both MCAR and MAR conditions were low. It was also seen that both in MCAR and MAR conditions, the average RMSEA values which obtained from missing handlings methods for different sample sizes and numbers of items didn’t change much. For MCAR and MAR with the increase of missing rates, it was observed that the results of the average RMSEA values were increased.

For MCAR, the EM method performed better than the other methods in all missing rate conditions; whereas the RMSEA values obtained from the IN method was higher than the other methods. Average
RMSEA values which were obtained from the methods with different missing rates seem to be higher for MCAR than MAR.

Table 3. Average RMSEA of g Parameter for Simulation Conditions

<table>
<thead>
<tr>
<th></th>
<th>MCAR</th>
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<th></th>
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<th>MAR</th>
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<td>IN</td>
<td>PM</td>
<td>TW</td>
<td>EM</td>
<td>IN</td>
<td>PM</td>
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<tr>
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<tr>
<td>30</td>
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<td>0.0220</td>
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<td>0.0157</td>
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<td>0.0066</td>
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<tr>
<td>Missing Rate</td>
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<tr>
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<td>0.0064</td>
<td>0.0097</td>
<td>0.0070</td>
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</table>

When the interaction effect graph for the factors is examined, it was also seen that in both MCAR and MAR conditions, different sample sizes and numbers of items didn’t change the average RMSEA values which were obtained from missing handlings methods too much. However with the increase of missing rates, the average RMSEA values tended to increase.

![Figure 3. MCAR Average RMSEA Values of the Interaction Effect (g Parameter)](image-url)
3.3. Results for Classification Accuracy

The effects of different missing data mechanisms, sample sizes, number of items, and missing rates on the classification accuracy were given in Table 4 and the results of the interaction effects were given in Figure 5 and Figure 6. When Table 4 is examined, it was seen that average classification accuracy rates which were obtained from MCAR and MAR conditions were high. It was also seen that in MCAR conditions, the average classification accuracy rates which were obtained from missing handlings methods in different sample size didn’t change much. However, with the increase of number of items, the classification accuracy rates tended to increase. The classification accuracy rates decreased with the increase of missing rate. In MAR condition, the average classification accuracy rates which were obtained from missing handlings methods in different sample sizes and number of item didn’t change much. For MCAR and MAR with the increase of missing rates, it was observed that the results of classification accuracy rates were decreased. All methods except the IN method performed similarly under the missing rate conditions. It was seen that classification accuracy rates which were obtained from methods with different missing ratios were lower for MCAR than MAR.

Table 4. Average Classification Accuracy Rates of g Parameter for Simulation Conditions

<table>
<thead>
<tr>
<th></th>
<th>MCAR</th>
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<tr>
<td></td>
<td>EM</td>
<td>IN</td>
<td>PM</td>
<td>TW</td>
<td>EM</td>
<td>IN</td>
<td>PM</td>
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4. DISCUSSION

In this study, it was observed that, the increase of missing rate, average RMSEA values tended to increase and rates of classification accuracies were tended to decrease. It is consistent with the results of Dai [34]. Despite the low missing rates, average RMSEA values tended to increase with the increase of missing rates. Sample size manipulation was not affective for the parameter estimations and classification accuracies. Number of items were not affective for parameter estimation whereas it was affective for rate of classification accuracy. The increase of number of items, increased the rate of classification accuracy.
It was observed that average RMSEA values which were obtained for MAR were lower and rates of classification accuracies were higher than the average RMSEA values which were obtained for MCAR. These results are also consistent with the literature. In addition to that, for many conditions it was observed that, average RMSEA values which were obtained from missing data handling methods were low and rate of classification accuracies were high. Among the missing data handling methods, higher average RMSEA values and lower rate of classification accuracies were observed for IN method. The results of other methods seemed to be closer to each other. This result is also consistent with the literature [21],[23],[36].

5. CONCLUSION

As a result, missing rates affected s and g parameter estimations and classification accuracies for all methods. Also, the results of the study showed that performance of methods varied according to manipulation factors for different missing data mechanisms. In this study, it was aimed to investigate the impact of different missing data handling methods on DINA model parameter estimation and classification consistency. Further studies will be conducted for other cognitive diagnostic models investigations with different number of attributes, missing rates and missing data mechanisms.

REFERENCES


BIOGRAPHY OF AUTHOR

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