



Connective Intelligence for Childhood Mathematics Education

Inteligencia conectiva para la educación matemática infantil

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ABSTRACT

The construction of a connective brain begins at the earliest ages of human development. However, knowledge about individual and collective brains provided so far by research has been rarely incorporated into Maths in Early Childhood classrooms. In spite of that, it is obvious that it is at these ages when the learning of mathematics acts as a nuclear element for decision – making, problem – solving, data– processing and the understanding of the world. From that perspective, this research aims to analyse the mathematics teaching-learning process at early ages based on connectionism, with the specific objectives being, on the one hand, to determine the features of mathematics practices which promote connections and, on the other hand, to identify different types of mathematics connections to enhance connective intelligence. The research was carried out over two consecutive academic years under an interpretative paradigm with a methodological approach combining Action Research and Grounded Theory. The results obtained allow the characterization of a prototype of a didactic sequence that promotes three types of mathematics connections for the development of connective intelligence in young children: conceptual, giving rise to links between mathematics concepts; teaching, linking mathematics concepts through an active methodology, and practical ones connecting maths with the environment.

RESUMEN

La construcción de un cerebro conectivo comienza en las edades más tempranas del desarrollo humano. Sin embargo, el conocimiento que ya se tiene sobre los cerebros individual y colectivo apenas se ha incorporado en el desarrollo del pensamiento matemático en Educación Infantil, donde comienzan a gestarse elementos clave para tomar decisiones, resolver problemas de la vida cotidiana, tratar con datos y comprender el entorno. Desde esta perspectiva la presente investigación marca como objetivo general analizar el proceso de enseñanza-aprendizaje de las matemáticas en Educación Infantil a partir del conexionismo, considerando como objetivos específicos, por un lado, determinar las características de una práctica matemática que promueva las conexiones y, por otro lado, identificar los distintos tipos de conexiones matemáticas para fomentar la inteligencia conectiva. La investigación se lleva a cabo a lo largo de dos años consecutivos bajo un paradigma interpretativo con un enfoque metodológico basado en el uso combinado de Investigación-Acción y Teoría Fundamentada. Los resultados han permitido concretar un prototipo de actividad o conjunto de actividades que, en forma de secuencia didáctica, promueve tres tipos de conexiones matemáticas para desarrollar la inteligencia conectiva en Educación Infantil: conceptuales, que producen nexos entre contenidos matemáticos diversos; docentes, que vinculan diversos conceptos matemáticos a través de una metodología activa y de vivenciar las experiencias matemáticas con otras materias; y prácticas, que relacionan las matemáticas con el entorno.

KEYWORDS | PALABRAS CLAVE

Connectionism, connective brain, mathematics education, teaching-learning, didactic method, didactic strategies, didactic application, early childhood education.

Conexionismo, cerebro conectivo, educación matemática, enseñanza-aprendizaje, metodología didáctica, estrategias didácticas, aplicación didáctica, Educación Infantil.



1. Introduction

The classic definition of ecosystem establishes the importance of the harmonious combination of the environment and the community of living beings, as well as the relations between these beings and between them and their environment. From a social point of view, we are part of an enormous ecosystem whose balance is complex and highly dependent on the decisions made by those who live within it, which are influenced more and more by their capacity to access and interpret the vast quantity of information that is added to the realm of social communications on a daily basis. This information is catalysed by the intense barrage of information and communication technologies (ICT), whose capacity for evolution and metamorphosis is several degrees greater in magnitude than that which the human brain can accommodate. Given these conditions, theories are emerging from the field of neuroscience, focusing attention on collective intelligence and consciousness, which are also presented as connected (Pitt et al., 2013). Within this framework, connections not only help to maintain balance in an ecosystem exhausted by the dizzying social and informational changes mentioned above, but also act as drivers to change and transform the ecosystems themselves towards more sustainable and –why not?– more fair realities.

Concerning our interpersonal connections, and beyond the obviousness of the global communication that the Internet and social networks facilitate, we know that we are indeed connected as suggested by the curious “six degrees of separation” theory, which states that, even if they do not know each other, any two people could send each other a personal message through a chain of contacts of no more than five links. This theory, proposed in 1929 by Frigyes Karinthy through a story called “Chains”, has also been subsequently considered and analysed by sociologists and mathematicians (Watts, 2004), who have aimed to demonstrate the theory and to endow it with logical rigour. That said, these kinds of connections only allow for the classic form of communication between transmitter and receiver through the establishing of suitable and reliable channels. The question lies more in the extent to which this apparent structure of online social connectivity can be exploited in order to make shared decisions which are good for the collective, and to drive forward and enrich a collective intelligence that manages this decision-making appropriately, that encourages the critical participation of citizens, and that is protected from the manipulations of small groups pursuing their own vested interests as opposed to the social good.

The construction of a connective brain starts in the early ages of human development, in which infants are still protected, to a large extent, from the barrage of messages sent by the media, with the exception, possibly, of television (Santonja, 2005), and the impact of ICT as an additional element of social communication. It is in this point that education plays a particularly relevant role, especially if we attend to the question of how the knowledge we already have about individual and collective brains may be incorporated into early childhood classrooms and, specifically, in the area of mathematical thinking, in which core elements start to emerge in relation to decision-making, problem-solving, data handling and understanding the environment. It was not in vain that Van-Overwalle (2011) highlighted the evidence indicating that many judgements and biases in social cognition can be understood from a connectionist perspective. Furthermore, he also pointed out that said judgements are underpinned by basic associative learning processes, often centred on an error minimisation algorithm. In relation to teaching, these ideas are already reflected in studies from the 90s, such as that of Askew, Brown, Rhodes, Wiliam and Johnson (1997), in which the connectionist teacher is characterised from the perspective of maths education.

1.1. Connectionism and learning

The learner and education are situated in live and dynamic contexts. Throughout history, different theories have emerged providing frameworks that aim to link research with educational reality. Some of the main theories of human learning started to be disseminated during an era in which technological resources were of little importance in people's lives. However, from the Second World War onwards, with the arrival of the technological revolution, psychological research resumed its interest in the human mind as an object of study and the computer began to acquire importance (Martorell & Prieto, 2002). As indicated by Caparrós (1980), dissatisfaction with the different versions of behaviourism led to an increase in new theoretical models that aimed to express human cognitive processes. Miller, Galanter and Pribram (1983) provide an overview of the works that constitute the beginning of the cognitive paradigm that attempts to explain how information is made available in the mind, through the elaboration of theoretical models that are subsequently validated through experimental techniques, computer simulations, or a combination of both, in order to describe knowledge. On the other hand, in constructivism, students are active since they organise their understanding by comprehending their experiences (Driscoll, 2005).

Given this dichotomy, the appearance of connectionism resulted in a revitalisation of cognitive psychology and,

as an educational approach, also generated considerable interest among researchers (Siemens, 2004; Downes, 2008; Bell, 2011), offering advancements which are potentially applicable in the field of mathematics education.

Since cognitivism is the most significant precursor to connectionism, it is worth highlighting the differences and similarities between both theories, which are summarised in Figure 1.

The architecture of the connectionist mind is based on artificial neural networks which are more or less complicated systems, made up of simple processing units. These units play a role analogous to neurones and relate to each other through connections of specific weights (different strengths) and generate complex systems of parallel computing (Crespo, 2007).

In these models, a minimum number of processing units allows different types of knowledge and the relations between them to be represented, whereby the loss of some units does not necessarily lead to the loss of information. Once trained in a particular task, these connectionist networks are resistant to contamination and enable the brain to acquire the learning of concepts, while also executing processes that, in line with McLeod, Plunkeett and Rolls (1998), tend to appear as mental processes in connectionist models. Namely:

- The combination of neural information is produced in parallel, even though the neurones are made up of different types. A large number of neurones are activated at the same time to complete the information by working together.
- The transmission of information is obtained through the relation between some neurones with others, in which the activation of processing units occurs as a result of different perceptions.
- The neurones are distributed in strata or independent cerebral layers and information is transmitted from one layer to the other or between different layers.
- Changes produced depend on the weight and strength of connection of the neurones, which are established through relations between responses or output units and transmitter or input units.
- Neurones constantly receive external stimuli which they process and modify. As a result, learning occurs

thanks to changes in the weights and strengths of the connections between such units, which are determined by perceptions.

According to Cobos (2005), the result of this is that the information received is coded through the neurones in a distributed manner, since various neurones are needed for us to be able to represent an object and, moreover, these neurones are an integral part of the representation of others.

This focus allows us to consider connectionism as a new bridge between cognitive science and neuroscience (Caño & Luque, 1995), and invites us to analyse its repercussion on learning.

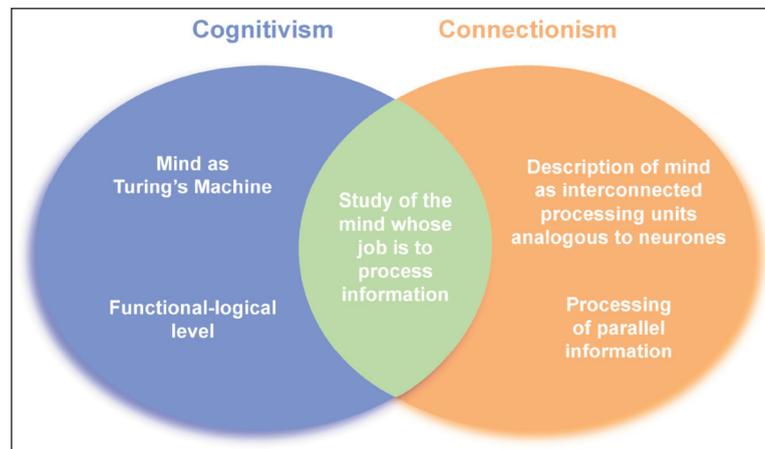


Figure 1. Similarities and differences between cognitivism and connectionism.

1.2. Connectionism and early childhood mathematics education

According to Merzenich and Syka (2005), one of the most relevant factors in the achievement of effective learning and the development of memory is attention, understood as “the main process involved in the control and execution of action” (Llorente, Oca, & Solana, 2012: 47). Accordingly, this is the faculty of choosing notifications out of the different senses that people perceive in the successive moments of their lives and of driving cerebral processes (López, Ortiz, & López, 1999). Thus, in order to be able to process information, children must be attentive, but it is also important that the processes used to develop learning are suitable.

In this sense, connectionism is adopted as a teaching model and as a model for the analysis of mathematics learning in Early Childhood Education, taking into account that the capacity to connect, associate and recreate are the identifying traits of this theory (Siemens, 2004).

Returning to the questions related to mental processes that tend to appear in connectionist models, the following analogies are established to work on mathematics at early ages:

- Neurones are related to each other in parallel to develop information: mathematics activities should not be presented in a linear manner given that different factors intervene when concepts are being evoked. As a result, it is vital to touch and see different material, but not in isolation.
- Neural information reaches the brain through perceptions: the visual, auditory, tactile, and olfactory stimuli that come from the external world are vital in attracting the child's attention and interpretations of these stimuli play a very important role in learning.
- In the same way that the information of cerebral layers is transmitted from one layer to another, mathematics concepts are built on each other, progressing gradually from the simplest to the most complicated one (Skemp, 1980).
- The more connections, the greater the evocative capacity, and as a result, concepts are fixed more strongly in the memory, are remembered with greater clarity, and at the same time, conceptual relations are recuperated better since different connections participate in our memory footprints, with each of these supporting numerous different footprints (Rumelhart & McClelland, 1992).

From this perspective, connectionism advocates a holistic form of education in which the development of contents following a temporal sequence is replaced by global development. According to this view, concepts are presented at the same time so that, on invoking some of them, not only are specific storage units activated, but also the units that save the mental images of related concepts, thus improving the evocation conditions. Specifically, early childhood mathematics education should be a coherent system which prioritises the mental construction aspect of the elaboration of an internal framework, in which different concepts are developed at the same time, leading to the subsequent creation of new concepts and mathematical processes.

This approach has already been discussed to some degree by different organisations and authors. From the approach of Realistic Mathematics Education (RME), Freudenthal (1991) proposes the beginnings of interconnection, according to which the blocks of maths content should be connected to each other. The National Council of Teachers of Mathematics of the United States (NCTM, 2000) considers connections as one of the five fundamental mathematical processes that should be worked on at all ages: Teaching programmes at all ages should equip students to: recognise and use connections between mathematical ideas, understand how mathematical ideas are interconnected and built on top of each other to produce a coherent whole, and recognise and apply mathematics in non-mathematical contexts (NCTM, 2000: 68).

On the basis of these ideas, Alsina (2012) outlines different types of connections and numerous Early Childhood Education mathematics teaching-learning contexts. Specifically, he presents two main types of connections: a) connections between the different blocks of mathematics content and between mathematical contents and processes (intradisciplinary connections), b) mathematical connections with other areas of knowledge and with the environment (interdisciplinary connections).

Some preliminary studies have provided evidence concerning the positive effects of working on different concepts at the same time (Ortega & Ortiz, 2003; Vicario-Solorzano, Gómez, & Olivares-Ceja, 2014). But as far as the authors are aware, no research has been carried out in early childhood mathematics education which are based on connectionism. To advance in this direction, this study analyses the process of teaching-learning mathematics in Early Childhood Education on the bases of connectionism, with the specific objectives being, on the one hand, to determine the characteristics of a mathematical practice which promotes connections, and, on the other hand, to identify the different types of mathematics connections required to foster connective intelligence.

2. Material and method

The study presented here has been carried out under an interpretative paradigm. From the perspective of research into mathematics education, it is understood that this paradigm focuses on describing the personal significance of facts, the analysis of relations between people and their environment, as well as the cognitive and attitudinal aspects of the participants (Godino, 1993). From this research perspective, and in line with the objectives proposed, a qualitative methodology has been applied to obtain data (Pérez, 1994). Specifically, two methods have been used: a) the Action-Research method (AR) (Kemmis & McTaggart, 1992), comprising six cycles which will be specified below, and b) Grounded Theory (Strauss & Corbin, 1998), to analyse the data obtained in each of the AR cycles and to obtain categories.

2.1. Participants

The study has been carried out in the Early Childhood and Primary School known as "Federico García Lorca" in Valladolid (Spain), considering the mathematical activities of two consecutive years carried out with 271 children of the different levels of Early Childhood Education (3-6 years old), having previously obtained the necessary informed consent of their parents. The study has been conducted with the participation of six teachers with considerable professional experience, who are also active in ongoing educational innovation processes, which they provided information on for each six-month period in which the study was carried out. In addition, the study also counted on the participation of an external agent, a support teacher from the school with thirty years' teaching experience in Early Childhood education, specialised in mathematics education at this level.

2.2. Design and procedure

In the first instance, different meetings were planned with the researchers and the Early Childhood Education teachers. In the first meeting, connectionism was presented as well as its teaching potentiality. Subsequent meetings discussed ways in which mathematical activities could be carried out from this perspective and debates were set up aimed at clarifying how to collect the data.

The six participating teachers elaborated documents in which they reflected on all the activities carried out on a daily basis and during the different periods of the study, outlining the concepts worked on in a connected way, as well as their observations and the results of the activities. In addition to these reports, other six were obtained in which the external observer commented on her reflections of the teaching during each experimental period and the degree of satisfaction of the teachers.

The periods in which the reports were elaborated corresponded with the months of November, February and May of each academic year, resulting in the six cycles of AR. In a complementary manner, video recordings were

carried out during maths dictations (blank sheets of paper with maths instructions duly sequenced and adapted to their level) in the first year of the study and in the second year to test the children's progress in the development of logical mathematical reasoning.

In summary, each AR cycle takes into account the following aspects:

- Reports of the six participating teachers: they

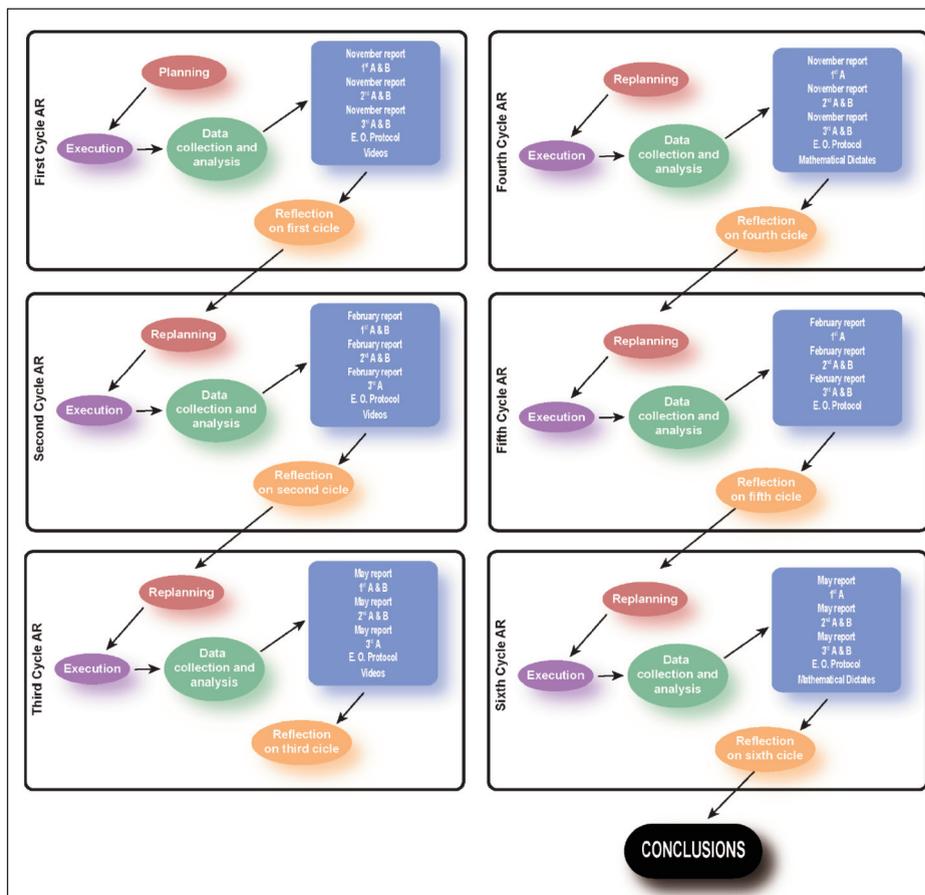


Figure 2. The Action-Research cycles.

record the content worked on during the term corresponding to each educational level in a coded format that records the day, month, academic year, educational level, group and connection of concepts. For example, code 2N11B (2, 3) corresponds to the activity carried out on November 2nd of the first year of the study, in class Year 1 B, with the concepts worked on at the same time indicated in parenthesis. The organisation of the data is structured in tables with the following headings: code of the activity, observations and possible categories.

- External observer's report: reports on the most noteworthy aspects of the teaching with absolute freedom, without any influence from the research team. Specifically, the protocol followed comprises seven items with different sub-sections: 1) the child and aspects of logic, 2) the child and quantity, 3) the child and geometrical and topological aspects, 4) the child and aspects of measurement, 5) the child and stories, games and problematic situations, 6) teacher's level of satisfaction, 7) other observations of interest.

- Video recordings: video recordings are made to be able to observe the children in action. Recording times oscillate between approximately 10 and 30 minutes.

- Evaluation of the activities: a count of the positive responses made by the child in relation to the connected activities they have carried out in each working day.

- Establishing of categories: the data collected in the reports, the video analyses and the tables are compared in order to establish emerging categories.

The flow diagram shown in Figure 2 summarises the followed procedure.

- The "Constant Comparison Method" of Grounded Theory (Strauss & Corbin, 1998) has been used to obtain the categories. The following levels of analysis have been considered: First level of analysis: the first steps consisted in reading and re-reading the information obtained through the different research instruments, in order to become familiar with the content and to develop a first impression. Following this, the information was segmented into fragments according to the ideas contained, identifying those expressing similar or related ideas through a common denominator. At this first level, the information received is organised by fragmenting or segmenting it into units: as the information is read, the different mathematical connections detected are highlighted and noted. In other words, the "raw data" (original material) starts to be transformed into "useful data" through initial coding and classification.

- Second level of analysis: on the basis of this first coding and classifying of the information obtained through the different instruments, group categories are established, such as "connections between different mathematical content" or "connections between mathematics and other disciplines", among others. In this sense, the coding and categorising process involves the triangulation of comparing, ordering and structuring to establish categories that enable the data to be compared (Gibbs, 2012).

- Third level of analysis: the categories are renamed, using the "Constant Comparison Method" described by Strauss and Corbin (1998), which includes the comparisons carried out in relation to the similarities, differences and connections of and between the data. The units capture and encapsulate meanings and actions. Thus, as relations are created and units compared in order to forge a preliminary analysis of the ideas, the names and content of the units also change, highlighting new relations and possible interpretations between categories. In this way, units are renamed, eliminated, compared, etc. and attention is focused on discovering them.

3. Analysis and results

In the first place, an example of an activity carried out with 23 three-year-old students at the end of the third term of the first year of the study is shown, and, in the second place, the qualitative analysis of the activity is presented to establish a prototype of connectionist mathematical practice and the system of categories obtained from it.

3.1. Description of connectionist mathematical practice

The main objective of the activity presented is that the three-year-old children understand some basic aspects of the relativity of mathematical concepts.

Before starting the activity, and in order to contextualise the situation, the children had worked on the red and blue colours of the long and short rods as an introduction to measuring, and had played freely with the material used previously. When the activity in question starts, the children are seated in a circle and the teacher places different materials in the centre (Figure 3), such as a box of different sized ropes, some Cuisenaire rods and a worksheet.

With regard to the sequencing of the activity, it is presented in four different parts: 1) the comparison of the length of different ropes from the box is worked on through observation, 2) the children are asked to measure two ropes laid out along the floor with the number 2 rod and the number 9 one, and they are asked which rope they

put more rods next to (the difference must be significant for them to say that there are a lot next to one and only a few next to the other), 3) the rods used to measure each rope are put into different piles to be able to see the difference more clearly and, one by one, the children are asked to turn around and estimate which is short and which is long through touching them, 4) individual work is carried out on a worksheet, colouring two hose-pipes –one short and one long– following the code indicated by the mascot on the worksheet, who points to a short red rod and to another long blue one.

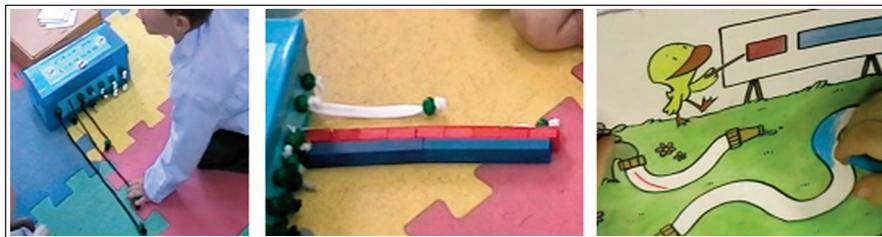


Figure 3. Comparing lengths, starting to measure with rods and individual work.

Throughout the didactic sequence, the teacher asks different questions to guide the students in the process of discovering the reason for the relativity of some concepts. An extract of the transcription of the activity is provided below by way of illustration:

– Teacher: “Today we’ve brought in a very special box”.

– Maria 1, smiling: “What’s this box called?”

– Everyone: “The box of rope”.

One of the girls remembers the special name they gave to the box.

– Maria 2: “The box of mice”.

– Teacher: “The box of mice because we have mice tails inside. And if we pull the mouse’s tail and it says ‘ouch’, it’s because there’s a mouse inside. Is there a mouse?”

– Everyone: “Yes! It’s inside”.

The teacher takes the “mouse box”.

– Teacher: “Let’s see if there’s a mouse inside. Let’s see, let’s see... let’s see if it comes out”.

The teacher shakes the box and pretends to play with the mice, putting her hand inside.

– Teacher: “Let’s see. Keep still little mouse. Will you keep still, please?! It can’t wait to get out! My oh my. Let’s close the box or it will escape”.

The teacher shows the ropes (which represent the mice tails) as she takes them out of the box.

– Teacher: “Ok. Let’s see if there are a lot of mice or just a few. How many do you think there are? A lot? Or just a few?”

– Everyone: “Loooooots”.

Table 1. Connections between the different blocks of mathematical content

	Numbering and calculating	Logic	Geometry
Measuring	Discriminating quantities: a few/many.	Recognising the properties of objects: colour	Identifying positions: inside and outside.
	Identification of the number 2 and associating it with a quantity.	Relating elements with the same characteristics.	
		Discriminating the differences between objects.	
		Differentiating sizes through the senses (sight and touch): long and short.	
	Interpreting codes		

3.2. Results extracted from the qualitative analysis

On the one hand, the detailed analysis of the teaching activities of the six AR cycles has led to the establishment of a basic mathematical practice prototype

for teaching-learning mathematics in Early Childhood Education and, on the other hand, to the establishment of a system of categories.

3.2.1 Prototype of connectionist mathematical practice

A teaching sequence following a connectionist approach should follow the guidelines provided below:

- Organisation and group presentation of the different didactic material.
- Asking the students well-formulated questions one by one to help them start to discover the different mathematical content.
- Presentation of the content involved in the activity to help the children understand it.

- Repetition of the activity so that the children can practice all the content worked on.
- New period of dialogue through “mathematical conversations” about their experiences to increase the force of the connections made between content.
- Asking of new questions to help the children internalise the connections.
- The need to bring collective experiences to the level of personal experience by representing on paper to help memorise the contents worked on.

3.2.2 System of categories

As a result of the process of constantly comparing the data, three categories have been obtained which represent the baseline upon which the subsequent theory has been delimited:

- Conceptual connections: responsible for producing links between different mathematical content.
- Teaching connections: responsible for establishing links between different mathematical concepts through an active methodology and by working with mathematical experiences linked to other areas.
- Practical connections: establish relations between mathematics and the environment.

At the same time, these categories are interconnected, forming a neural network which has enabled us to establish which type of connections appear in each of them with more precision (Figure 4):

- Conceptual connections: the identification of sensorial qualities, of quantities of a numerical series, of forms of spatial situations, of aspects of measurement, of similarities and differences between scenes; grouped according to the following criteria; association of number and quantity; discrimination of quantities, of forms, of aspects of measurement; different relations, such as the pairing of the same objects, classifications, series, sorting, comparing objects; simple graphic representations; and starting to use mathematical language.

- Teaching

connections: active methodology, holistic teaching, evaluation and assessment.

- Practical connections: mathematics in the environment, as well as stories, games and didactic material.

It is important to highlight the relevance of the practical categories of connection in relation to the objectives of

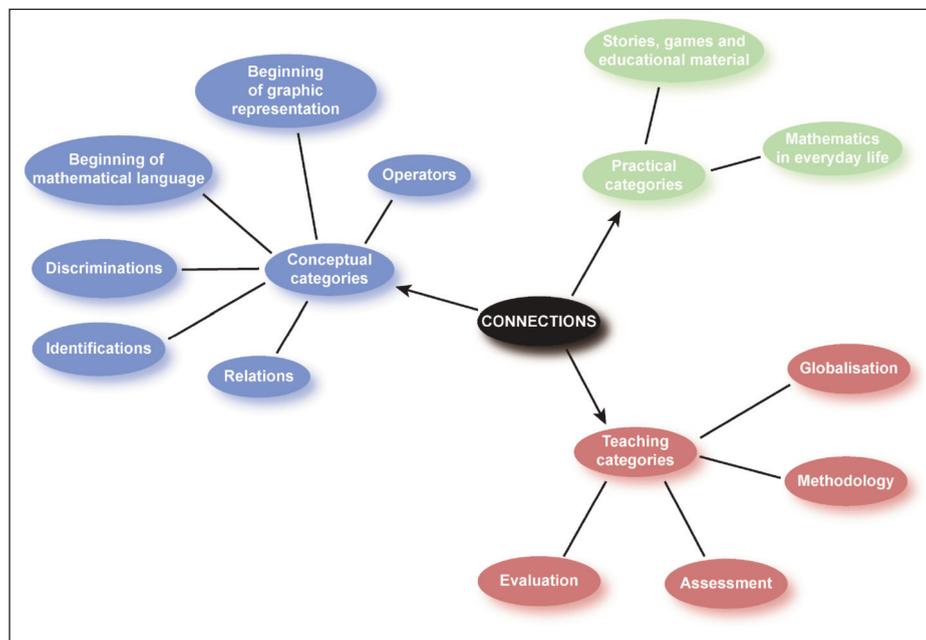


Figure 4. Structure of categories of emerging connections.

this study, since these connections are needed if children in schools are to carry out activities in which logic, numbers, information handling, geometry and measurements appear in a connected way, both in relation to their daily lives and with the use of different teaching-learning resources (stories, proverbs, poems, didactic material, etc.).

4. Discussion and conclusions

This article presents some advances concerning the role of mathematics education in the construction of connective intelligence in the early ages of human development, assuming that mathematical thinking plays an

important role in the individual's capacity to make decisions, solve problems, process data and understand their social environment.

While traditional channels of access to mathematics knowledge were based on the transmission of information in a sequential and linear manner, our study has explored the elements that should be taken into consideration in education in general, and in teaching in particular, to promote a new approach to the teaching-learning of mathematics which takes into account and fosters connections between different knowledge, as an essential element in developing citizens with the skills needed to manage decision-making tasks in a critical way. In this sense, in the field of mathematics education, over recent decades different organisations and authors have been advocating the importance of presenting mathematics knowledge in a connected way from early ages (Freudenthal, 1991; NCTM, 2000; Alsina, 2012). Despite this, research into early childhood mathematics education has not provided findings that offer specific guidelines for teachers to foster connective intelligence. In order to develop this specific line of enquiry, a study has been carried out over two consecutive years which has enabled us to establish a prototype of activity or set of activities, in the form of a didactic sequence that promotes connections between contents. Up to the time of this study, some authors

in the field of Early Childhood mathematics education have contributed data on learning trajectories in order to sequence (and be able to connect in a suitable way) mathematics contents of the same block (Clements & Sarama, 2009), or have explored the phases that should be taken into account in the design, management and evaluation of competency-based mathematical activities that include connections, among other processes (Alsina, 2016). However, no prior studies are available on the specific elements that should be

We present a prototype of an activity or set of activities that, in the form of a didactic sequence, promotes the connections between the contents in order to construct connective intelligence in the early ages of human development, assuming that mathematical thinking plays an important role in the individual's capacity for decision-making, problem-solving, data-processing and the understanding of the world.

considered in order to carry out mathematical practices from a connectionist perspective. The establishment of a prototype of connectionist activity thus represents an innovation in education, which is the result of specific research in this area, in the sense advocated by Llinares (2013).

Another important contribution of the study lies in the establishment of different kinds of connections. The interpretation of the results offers a body of central connections, with all the potential relations that exist between the different categories of each group, and which also depend on and interrelate with other groups, thus configuring a new way of working on the development of mathematical thinking in Early Childhood Education.

In order to promote connective intelligence in the classroom from early ages of human development, it seems more than apparent that the formats derived from education or, in other words, training models, should aim to provide teachers with in-depth knowledge of these different types of connections and the ways to develop them in their students, considering the features of a connectionist teacher as proposed by Askew and his colleagues (1997).

In summary, considering the objectives of our study, the main conclusions are as follows:

- Mathematics education can play an important role in the construction of a connective brain from early ages of human development, considering its role in relation to decision-making, problem-solving, data processing and understanding the social environment.
- The promotion of connectionism in early childhood mathematical practices requires a didactic itinerary characterised by a planning and management process including six phases: the use of didactic material for working in groups, asking the students' questions, group discussion, subsequent mathematical conversations, the asking of new questions and the individual representation on paper of the knowledge acquired.
- Connectionist practices present three main types of connections: conceptual, teaching and practical.

- Practical connections have particular relevance since they are responsible for connecting knowledge related to everyday life.
- Teachers should have in-depth knowledge of the different types of connections and the ways of developing them with their students.

In this regard, some of the main limitations of this study have been the fact that no prior analysis was carried out of the participating teachers' knowledge of connectionism, and the fact that no comparison was made of the students' mathematics learning in relation to other groups of students who have not learned mathematics in this way. Future studies will therefore be needed which use a specific model to explore mathematics teachers' knowledge, such as Mathematical Knowledge for Teaching (MKT) by Hill, Rowan and Ball (2005) and Hill, Ball and Schilling (2008), or the Didactic-Mathematical Knowledge Model (CDM) of Godino (2009), and Godino, Ake, Gonzato and Wilhelmi (2014), in order to conduct a more precise analysis of Early Childhood teachers' knowledge of connectionism and ways of implementing it in mathematics practice. Likewise, in order to validate the classroom application of this teaching model, whose goal is to foster connective intelligence, new quasi-experimental quantitative studies should be designed to compare the performance of students who learn with a connectionist approach with others learning with more traditional methods that do not take into account connections.

A more precise diagnosis of the question will thus help to establish a set of lines of practice which are much more appropriate in relation to both pre-service and in-service teacher training, given the increasing importance of fostering the connective intelligence of our students, considering the contributions of Neuroscience and other related sciences, which propose radical changes to the way in which individuals access knowledge.

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