COMMENTARY

Calling for Research Collaborations and the Use of Dis/ability Studies in Mathematics Education

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Despite discussions of “mathematics for all,” opportunities that support the development of mathematical reasoning and understanding of mathematics as a human endeavor often do not exist for mathematics learners identified in schools as having dis/abilities. Indeed, mathematics for all is consistently used to motivate the allocation of resources and attention to mathematics education in legislation, policy documents, and organizations’ vision and position statements. Mathematics education researchers have served as advocates for marginalized students pointing out limitations in the mathematics for all rhetoric (Martin, 2003). Yet, students with dis/abilities are often left out of discussions regarding mathematics for all and equity research that has worked to contextualize and operationalize “achieving equity” and the process of “eliminating inequity” (Tate, as cited in Martin, 2003, p. 14).

Mathematics education researchers and organizations representing them use the term “equity” to refer to access and opportunities for all students. For example, the National Council of Teachers of Mathematics’ (NCTM) recent Principles to Actions: Ensuring Mathematical Success for All called for systemic improvement in mathematics education for all (NCTM, 2014). “Access and equity” is identified as one of six guiding principles for school mathematics (p. 5). Yet, the NCTM’s (2014) position statement on access and equity leaves dis/ability out of the subgroups to which these goals apply. Access and equity are identified as applying to racial, ethnic, linguistic, gender, and socioeconomic groups. Furthermore, access and equity in standards-based mathematics education remains elusive for those stu-

1 We use the term dis/ability to forefront power imbalances inherent in constructing and identifying dis/ability and the consequences of such imbalances in and out of school. The concept of dis/ability as socially constructed offers an entryway to reconstructing what mathematics education researchers mean when they use the term “disability” and to addressing inequities for individuals labeled with this educational and societal construct. The word “disability” is used when directly referencing works by other authors as they applied the term unless their work similarly use the term “dis/ability” (e.g., de Freitas, 2015).

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studies with dis/abilities (Browder, Spooner, Ahlgrim-Delzell, Harris, & Wakeman, 2008; Tan, 2014). In public schools, 13 disability categories are sanctioned for special education services. Generally, only students in a few dis/ability categories, such as learning disability or speech/language impairments, spend a majority of their school day in the general education classroom. Students identified as having “moderate” to “severe” disabilities (e.g., intellectual disabilities, autism, multiple disabilities) spend most of their school day in segregated special education classrooms or schools (U.S. Department of Education, 2013). Jackson and Neel (2006) compared mathematics instruction practiced in and outside general education classrooms. Based on their classroom observations, the researchers coded time that teachers in four schools spent on conceptual instruction, algorithm instruction, and instruction or tasks unrelated to mathematics. Jackson and Neel operationalized algorithm instruction as “focused on imparting factual knowledge and mathematical procedures” (p. 597). A major finding was that, on average, approximately 30% of the time in general education classes was devoted to algorithm instruction compared to 75% of the time students in special education classes were subjected to this form of instruction.

An additional concern is that accessibility for students with dis/abilities in general education mathematics classrooms remains a challenge. Insufficient classroom supports lead to learning obstacles (Baxter, Woodward, & Olson, 2001; Bottge, Heinrichs, Mehta, & Hung, 2002) and privileging narrow forms of mathematics expressions and sensory capacities (de Freitas, 2015). As a result, access to the general education mathematics classroom and curriculum depends on the degree to which students with dis/abilities resemble conventional ways of operating such as communicating mathematical thinking via speech and writing. Hence students with dis/abilities, even in general education classrooms, lack opportunities to build upon their existing ways of thinking mathematically.

Research represents a crucial component in the process of recognizing and addressing these inequities. Ongoing commitment to equity and equity research in the mathematics education community has resulted in an increase in the number of equity focused research manuscripts submitted to journals (Stinson, 2013). In the last 10 years (2006–2016), the Journal for Research in Mathematics Education (JRME) has included four reports of empirical studies focused on exploring teaching, learning, and curriculum of students with disabilities. The focus of the studies were impressions (Lynch & Star, 2014) and reasoning (Lewis, 2014; Xin, 2008) of students with learning disabilities. Similarly, Lewis and Fisher’s (2016) review spanning 40 years of mathematics education research exclusively focused on students with mathematics learning disability. These articles begin to point the way

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2 Lewis and Fisher (2016) operationalize mathematics learning disability (MLD) in line with a dyscalculia diagnosis, which they described as a “biologically based difference in the brain” (p. 338). For the purposes of this commentary, we situate MLD within the broader category of learn-
toward research that illuminates mathematics reasoning and learning of students with disabilities. However, there is more to do to address dis/ability beyond those identified with learning disabilities to include other dis/abilities related to communication, emotion, behavior, attention, body, and health. Indeed, outside of learning disabilities, no empirical study published in JRME has involved individuals with dis/abilities as doers and thinkers of mathematics. Given the narrow focus of and few empirical studies on dis/abilities in mathematics education research, Karp (2013) has termed this group the invisible 10%.

We also find that dis/ability is underrepresented in mathematics education in the larger research community. Although Lubienski and Bowen (2000) identified disability as a subgroup within the discussion of equity and mathematics for all, scholars have noted limited numbers of available educational research studies focused on mathematics education involving students with disabilities (Karp, 2013; Lambert & Tan, 2016). Lambert and Tan (2016) analyzed 408 peer-reviewed journal articles with a mathematics education focus. They found that of the 42 articles explicitly including students with disabilities, two were published in mathematics education journals. The remaining 40 articles were published in special education or psychology journals whose fields are heavily influenced by behaviorist-based learning theories such as direct instruction (Woodward, 2004). To be sure, Lambert and Tan’s (2016) research found that learners with disabilities and those without are conceptualized differently in mathematics educational research. Whereas approximately 40% of studies subjected students with disabilities to learning traditions grounded in fixing or remediating deficits (e.g., medical or behaviorist models), only about 6% did so for students without disabilities. When it came to constructivists and social-constructivist traditions, 7% of the reviewed studies involved students with disabilities compared to 40% involving students without disabilities. Similarly, Lewis and Fisher’s (2016) review of mathematics education research on students with mathematics learning disability found that most of the 164 studies “involved topics aligned with the kindergarten through third-grade standards and focused almost exclusively on basic arithmetic calculation” (p. 357). Hence, the limited knowledge base in mathematics education research focused on students with disabilities points to alarming inequities. We attribute the construction of dis/ability as central to these inequities.

**Broken Minds as the Origin of Marginalization**

The construction of and response to dis/ability located within an individual is strongly ingrained and operated upon in society at large and consequently in
schools. The field of special education emerged as a “rational” way for schools to organize and maintain order by viewing disability as a pathological condition (Skrtic, 1991). Skrtic argued that society and public education are grounded in theories of organizational rationality and human pathology. Accordingly, school failure can be attributed to inefficient organizations and defective students. This rationality/pathology resulted in the emergence of special education “as a means to remove and contain the most recalcitrant students in the interest of maintaining order in the rationalized school plant” (p. 152).

Meanwhile, special education philosophy and research have historical roots in psychology and medicine guided by positivist orientations. These orientations encourage interventions for students with disabilities emphasizing performance (Baroody, 1999; Paul, French, & Cranston-Gingras, 2001). Within the medical model, the concept of disability reflects organic deficiencies, “broken” bodies or minds as “something to fix, cure, accommodate, or perhaps endure” (Andrews et al., 2000, p. 259). Recently, for example, the study of developmental dyscalculia, a mathematics learning “disorder” included in the Diagnostic and Statistical Manual of Mental Disorders (DSM-5; American Psychiatric Association, 2013), has sought to identify biomarkers of dis/ability within individuals by attempting to locate the cause of mathematical “disorders” in the brain (de Freitas & Sinclair, 2016). To identify these biomarkers, tests are used to determine the ability to closely estimate and compare quantities without counting. The outcomes of these tests are normed constituting “intact” or “deviant” forms of “number sense.” Although mathematics education scholars have critiqued these tests (e.g., de Freitas & Sinclair, 2016) for over-emphasizing narrow dimensions of number sense, the results are taken at face value reinforcing the location of disability within an individual. Indeed, scholars (e.g., Paul, French, & Cranston-Gingras, 2001) have noted that despite advancements in social science perspectives, special education researchers have maintained a strong commitment to a positivist epistemology. As previously discussed, special education researchers conduct the majority of studies in mathematics education involving students with disabilities. Hence the view of students with dis/abilities as mathematics doers and thinkers has remained narrow. At the same time, we should be careful not to cast all special education researchers as grounded in these historical traditions; there are important exceptions (e.g., Cawley & Parmar, 1992; Lambert, 2015).

Taken together, the construction of and response to dis/ability becomes problematic for mathematics education researchers. Similar to Skrtic’s (1991) assertion that school organization delegates responsibility of the education of students with dis/abilities to special educators, we deduce that research practices fall along categorized lines of special education researchers and mathematics education researchers. The absence of dis/ability in mathematics education equity debates perhaps reflects how research involving students with dis/abilities falls within the purview of
special education researchers. We invite mathematics education researchers to challenge such research fragmentation while questioning positivist methods emphasizing narrow forms of mathematics performance (Browder et al., 2008; Tan, 2016). Attending to these contradictions represents a constructive path toward building upon what we know about students as mathematics thinkers and doers. Yet, Martin (2003) asserts that the process of eliminating inequity will move beyond “rendering students eligible for opportunities that we assume and hope will exist for them” (p. 14) toward empowerment to use mathematics to “alter the power relations and structural barriers that continually work against their progress in life” (p. 15).

**Measureable Costs of Inequity**

Outcomes for individuals with dis/abilities in and out of school illustrate Martin’s (2003) point. Students with disabilities continue to be suppressed in segregated and self-contained environments where “life skills” curriculum fails to prepare them for real life (Frattura & Topinka, 2006; Tomlinson, 2012). Students with disabilities have the lowest high school graduation rates among all students, almost 20% lower than the national average (U.S. Department of Education, 2015). Those who graduate from high school are less likely to enroll in and complete postsecondary educational programs than their counterpart students without disabilities (Newman et al., 2011). Moreover, restricted work opportunities after graduation result in large employment disparities between those with and without disabilities; 21% of working-age individuals with disabilities are employed either full- or part-time compared to 59% of those without disabilities (Harris Interactive, 2010). Limited employment opportunities result in 28% of people with disabilities aged 18–64 living in poverty, as compared with 12.5% in the general population (DeNavas-Walt, Proctor, & Smith, 2012). In addition, individuals with disabilities are less likely to report that they are very satisfied with life than those without disabilities—34% versus 61%, respectively (Harris Interactive, 2010).

Outside school, individuals with disabilities have taken leadership roles in self-advocating for civil rights despite having negative school experiences such as being bullied, labeled, and marginalized (Caldwell, 2011). Keith Jones is one such advocate. Mr. Jones reflected on the low expectations that he experienced in and out of school as well as the work of gaining access to meaningful learning in mathematics:

Because of my being...I have to be secluded, stashed away...talked about in a way that there’s no expectation of me doing anything. And then you want me to succeed? You want me to be a product of this society...produce...put into this economy? But from the time I’m born to the time I die I’m being told I ain’t shit! ...Teachers didn’t really push...it was more of ‘Okay, here’s some manila papers and crayons, color’.
Mr. Jones’s reflection addresses Martin’s (2003) assertion that achieving equity in mathematics is not bound by time in school. Indeed, mathematics education researchers are positioned to examine and address inequities outside of school and beyond; consequently, historical, social, and economical analyses are all crucial in this work (King Thorius & Tan, 2015). Unfortunately, Mr. Jones’ development as a self-advocate is an uncommon outcome for most individuals with dis/abilities.

Including individuals with dis/abilities in the discussions of mathematics for all is a starting place, but it is crucial to move to actions addressing the potential dreams and desires of such students. Mathematics education researchers are central to this move. Specifically, mathematics education researchers can utilize diverse research tools and frameworks for understanding mathematical thinking, knowing, and ways of being. Mathematics education researchers also have diverse ways of talking and thinking about mathematics. Collectively, such powerful research tools and knowledge of mathematics education are needed to counter blatant forms of dis/ability-based discrimination (i.e., ableism) in and out of schools. For example, ableism may manifest in discussions surrounding the “appropriateness” of engaging students with dis/abilities in constructivist and reform-oriented pedagogies. Woodward and Montague (2002) point out that special education scholars have resisted such pedagogies for students with disabilities. This resistance has resulted in the endorsement of teacher-directed practices for students with disabilities focused on explicit forms of instruction (National Mathematics Advisory Panel, 2008) constituting practices that qualify as evidence-based (Gersten et al., 2009). These practices are systematic procedures targeting measurable responses while providing reinforcement and error correction feedback (Spooner, Knight, Browder, & Smith, 2011). Translating these systematic procedures to mathematics education practices, Saunders, Bethune, Spooner, and Browder (2013) provided the following description:

The teacher identifies a skill to teach the student (e.g., how to identify obtuse, acute, or right angles) and finds an appropriate prompt to help the student get the right answer. This prompt is anything the student needs to get the right answer, and stays the same throughout the time it takes the student to learn the skill. It could be a verbal model (teacher presents an obtuse triangle and says “obtuse”), a gesture to the correct answer, or even a physical prompt by moving the student’s hand to the correct answer. First, the teacher presents the problem (e.g., “What kind of triangle is this?”) and immediately uses the prompt to help the student get the right answer. After doing this for a number of trials (sessions), the teacher fades that prompt by simply delaying it: The teacher presents the problem (e.g., “What kind of triangle is this?”) and waits 4-6 seconds before delivering the prompt. This gives the student time to answer independently, but also provides support. (p. 29)
How should mathematics education researchers respond knowing that students are being treated this way? We invite mathematics education researchers to rally together using their research expertise and voice to speak against blatant forms of ableism such as illustrated above. It is worth reiterating that these procedures are representative and widely supported by special education mathematics researchers hailing systematic instruction as one of the most significant advances in the field (Spooner & Browder, 2015).

To support individuals with dis/abilities to achieve their goals as mathematicians and humans, they must be seen as both. Opportunities to awaken the mathematicians within students with dis/abilities go beyond having access to “evidence-based” direct and systematic instruction. Instead, a mix of pedagogies affording these mathematicians with opportunities to explore and create communicates the intent of mathematics for all and of all (Tan, 2017). In doing so, mathematics education researchers work to address inequities and rise to the challenge of Paulo Freire’s critical question: “How many critical intelligences, how much curiosity, how many enquirers, how many capacities that were abstract in order to become concrete, have we lost?” (Freire, D’Ambrosio, & Mendonça, 1997, p. 8).

Evidence of Ability within Students with Dis/abilities

Mathematics education researchers have begun to support individuals with dis/abilities to achieve their goals as mathematicians and humans by building a body of knowledge identifying mathematics ability. Some scholars maintain that the mathematics learning of students with dis/abilities, including those constructed as having a more “severe” disability, is similar to their non-dis/abled peers (Baroody, 1999; Tan, 2014; Tan & Alant, 2016; Van den Heuvel-Panhuizen, 1996). These scholars posit that students with disabilities can benefit from reform-based practices (Baroody, Bajwa, & Eiland, 2009). Elaborating on such an assertion, we illustrate ways students with dis/abilities engage in, make meaning from, and express mathematics.

Van den Heuvel-Panhuizen’s (1996) study involving 61 fifth- and sixth-grade students diagnosed with an intellectual disability found that participants utilized particular and, in some cases, sophisticated ratio problem-solving strategies (e.g., multiplicative reasoning, drawing concrete and abstract models) despite not being formally taught about ratios. Similar findings have pointed to sophisticated problem-solving strategies in other mathematics domains such as subtraction (Peltenburg, Van den Heuvel-Panhuizen, & Robitzsch, 2011) and combinatorics (Peltenburg, Van den Heuvel-Panhuizen, & Robitzsch, 2013). In fact, Baroody’s (1999) research synthesis determined that students with disabilities, including those with severe disabilities, do engage in mathematical practices such as inductive and deductive reasoning, and adapting/devising mathematical strategies. Baroody con-
cluded that because these students have the cognitive building blocks necessary to develop meaningful mathematics learning they can benefit from purposeful, meaningful, and inquiry-based approaches to mathematics learning.

Conviction about the learning potential of students with disabilities coupled with concerns about curricula lacking academic rigor were the impetus for development of a conceptually based mathematics curricula for these students (Göransson, Hellblom-Thibblin, & Axdorph, 2016). The authors examined mathematics lessons that teachers constructed in six classes involving students with disabilities. They found that teachers effectively created learning environments where students inquired, held mathematics conversations, shared their insights, and became interdependent.

These studies confirm a core belief of mathematics education researchers—that all humans are mathematics thinkers and doers. Yet, narrow notions of what constitutes mathematics thinking and doing constrains this position. As such, these studies support the notion that dis/ability lies not within individuals but rather resides in limited opportunities and in rigid mathematics educational practices (e.g., narrow forms of assessment).

**Dis/ability Studies in Mathematics Education**

We join scholars who have challenged mathematics education researchers to build a process for exploring inequities central to our field (e.g., Gutiérrez, 2013; Karp, 2013; Martin, 2003), inviting researchers to include students with dis/abilities in these efforts. Our intention is for mathematics education researchers to partner with special education and dis/ability studies in education researchers to bring more diverse theoretical perspectives to research involving students with dis/abilities. In addition, we suggest the consideration of dis/ability studies in mathematics education (DSME) as a complimentary theoretical framework for research and advocacy. DSME synthesize elements of dis/ability studies in education (DSE) and embodied mathematics.

**Dis/ability Studies in Education**

DSE is an emerging field that examines dis/ability as a social construction resulting in social exclusion and oppression (Gabel, 2005). DSE departs from the field of special education and groundings in positivist traditions (Valle & Connor, 2011; Ware, 2005) depicting dis/abilities as deficits located within individuals. Rather, DSE aims to “fix” traditional practices leading to a disabling view of individuals. Thus, in drawing from elements of DSE, DSME focuses on reimagining mathematics education practices that enable and empower every student in ways that approach characteristics of students with dis/abilities (and all students) as repre-
sentative of human diversity. Rather than drawing lines of exclusion, DSME innovates for inclusive practices, expanding our capacity to support all learners in robust mathematics learning communities.

Another crucial element of DSE is privileging individual voice and lived experiences. Self-advocates, such as Keith Jones, provide critical insights to guide mathematics education researchers in their efforts. Such insights foster understanding of dis/ability experiences, which offers considerations for directions of inquiry (Bagliero, Valle, Connor, & Gallagher, 2011). Moreover, Gutiérrez (2013) urged mathematics education research to draw upon lived experiences of marginalized individuals and/or to apply and (re)write “theories and frameworks that give voice to others” (p. 57). As such, researchers can recognize “the embodied/aesthetic experiences of people whose lives/selves are made meaningful as disabled, as well as troubles the school and societal discourses that position such experiences as ‘othered’ to an assumed normate” (American Educational Research Association, 2016).

Embodied Mathematics

Central to embodied mathematics is that making sense of and expressing mathematics involves the body in ways that are not fully understood. We draw heavily on the works of Elizabeth de Freitas and Nathalie Sinclair (see, e.g., 2014, 2016) to situate embodied mathematics and reframe dis/ability in mathematics education research. We find problematic that “specific ways in which mathematics is represented, communicated, and explained tacitly privilege certain sensory capacities” (de Freitas, 2015, p. 198). Such privilege fosters restricted views of what constitutes engaging in mathematics. Thus, de Freitas directs us to explore the possibilities of “radically different sensing bodies” (p. 189). Indeed, researchers have long known that by exploring other sensory capacities, students with disabilities often excel in creative productions (e.g., Carter, Richmond, & Bundschuh, 1973). In particular, students with dis/abilities may engage in mathematics through various modes of interactions such as swaying, rhythmic movement, gesturing, tapping, feeling, facial expressions, or gaze (Sinclair & Heyd-Metzuyanim, 2014). Unfortunately, such embodied modes are not privileged in the school environment, leading to a diagnosis of learning dis/ability and/or the dismissal of some ways of operating as non-mathematical. Thus, principles of embodied mathematics center on “the way that bodies are provisionally and temporarily enabled, directing our attention to the temporal contingency of dis/ability” (de Freitas, 2015, p. 189) to counter notions of definitive potentialities and capacities.

Embodied mathematics then recognizes the potential of the human body and where “bodies can be seen as differently abled and differently organized rather than disabled or distracted” (de Freitas & Sinclair, 2014, p. 145). In citing Eide and Eide (2011), de Freitas (2015) noted:
The curricular emphasis on alphanumeric aspects of mathematics, for instance, works against students with exceptional spatial skills. People diagnosed with dyslexia may struggle with procedural learning and rote memory tasks, but their memory of phenomenological details—details pertaining to physical aspects of an experience, such as tactile, motor, or spatial arrangements—exceeds that of non-dyslexics. (p. 199)

Other embodied arrangements such as music also provide access to mathematics engagement (Edelson & Johnson, 2003) through rhythmic movement. These movements possess spatial properties, sequencing, and patterning essential to mathematical concepts (Geist, Geist, & Kuznik, 2012). Students’ engagement in creating and moving to music provides opportunities for the development of insights about the structures of space and time in their creative activities. That is, embodied mathematics requires us to “simultaneously rethink the body in and of mathematics” (de Freitas & Sinclair, 2013, p. 454). Importantly, like DSE, embodied mathematics takes seriously the social-political dimensions and its “entanglements” in education (de Freitas & Sinclair, 2013). Thus, for mathematics education researchers, the implication of embodied mathematics, DSME, and other sophisticated epistemologies is to better understand “the minute sensations that contribute to our students’ learning and invention of mathematical concepts” (de Freitas, 2015, p. 192) situated within constructs of dis/abilities to address inequities in and out of schools.

Our role as mathematics educators is partnering with families, students, educators, and community members (NCTM, 2008) to support, create, and advocate by addressing inequities through responsive mathematics education research. To facilitate this work, dis/abilities, as both a collective group and individual experiences, must be explicitly included in mathematics equity research and advocacy alongside other marginalized groups. Importantly, this inclusion must cover the full-range of dis/abilities (e.g., autism, intellectual dis/abilities, emotional and behavioral “disorders”) not just mathematics learning disabilities. In doing so, we take on the responsibility of mathematics education research involving students with dis/abilities. We do this because we have long claimed that our work is about all students. We do this because we know the value of diversity and different perspectives in truly inclusive mathematics teaching and learning. We do this because we know that we must view every student beyond socially constructed labels and perceived limits. We do this because we are committed to honoring and understanding multiple ways of knowing, expressing, and engaging in mathematics.

References


Karp, K. (2013, January). *The invisible 10%: Preparing teachers to teach mathematics to students with special needs*. Judith Jacobs Lecturer at the Association of Mathematics Teacher Educators’ Annual Conference, Orlando, FL.


