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Equity-Directed Instructional Practices: Beyond the Dominant Perspective

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In this article, the author synthesizes four equity-directed instructional practices: standards-based mathematics instruction, complex instruction, culturally relevant pedagogy (CRP), and teaching mathematics for social justice (TMfSJ). The author organizes these practices according to the dominant and critical axes in Gutiérrez's (2007a) equity framework. Among 12 teachers from 11 schools in a large urban school district, the author presents case studies of 3 teachers who excelled with the aforementioned dominant equity-directed practices but struggled with the critical practices of connecting to students' experiences called for in CRP and critical mathematics called for in TMfSJ. The analysis explicitly explores the role of whiteness in these struggles. The author presents implications and recommendations for mathematics teacher education on how to better support teachers for equitable teaching that includes these critical equity-directed practices.

KEYWORDS: complex instruction, culturally relevant pedagogy, standards-based instruction, teaching mathematics for social justice, urban mathematics education

Most Black and Latinx[1] students in U.S. cities attend schools hyper-segregated[2] by race and socioeconomic class (Milner, 2013; Orfield, Kuscera, & Siegel-Hawley, 2012). School segregation—currently, accelerated by neoliberal processes of gentrification—is confluent with inequalities in teacher qualifications, experience, and turnover rates; advanced course offerings; money spent per student and condition of facilities; as well as deficit orientations to students and their families and communities (Anderson, 2014; Kitchen & Berk, 2016; Lipman, 2016; Martin & Larnell, 2013). Additionally, there is a mismatch between the students in urban schools and a teaching force that is largely White and middle-class (Chazan, Brantlinger, Clark, & Edwards, 2013; Martin, 2007). Despite these structural and systemic inequalities, if school achievement falls short compared to "better" resourced schools (often White, suburban schools), differences are then typically at-

[1] In some places in this article, I refer inclusively to these and other marginalized groups as "students of color."

[2] Frankenberg, Siegel-Hawley, and Wang (2010) define *hyper-segregated* schools as those with at least 90% of its students from "racial/ethnic minority groups," or at least 90% of its students White.

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tributed to the students and their families and communities for what is perceived as a lack of ability, effort, "grit," values, or parenting (Battey & Franke, 2015; Martin, 2009b; Milner, 2012; Nasir & de Royston, 2013; Pollak, 2013).

Hyper-segregated schools serving Black and/or Latinx students in U.S. cities are vulnerable to a cycle in which teachers' deficit views of students fuel low expectations (Martin, 2009b; Milner, 2012; Rousseau & Powell, 2005; Stinson, 2006). Low expectations are known to manifest in didactic pedagogy organized around remediation, rote drill and practice, and preparation for standardized tests (Anyon, 1997; Grant, Crompton, & Ford, 2015; Rist, 1970/2000; Thadani, Cook, Griffis, Wise, & Blakey, 2010), even though such pedagogy is less effective for students from marginalized, underserved groups (e.g., Diversity in Mathematics Education Center for Learning and Teaching, 2007; Franke, Kazemi, & Battey, 2007; Ladson-Billings, 1997; Silver, Smith, & Nelson, 1995; Spencer, 2009). Weak results on external assessments reinforce deficit views about students to reboot a cycle of low expectations for students and about teaching (Nasir, Cabana, Shreve, Woodbury, & Louie, 2014; Rubel & Chu, 2012). Supporting teachers in "urban" schools to acknowledge and modify their deficit views of students, increase expectations for students and for their own teaching, and develop or improve a robust set of equity-directed instructional practices disrupts this cycle (Aguirre et al., 2013; Rubel & Chu, 2012; Silver & Stein, 1996; Turner et al., 2012).

My goal is to present a research-based argument focused on teaching mathematics in hyper-segregated urban schools that moves away from a "failure-focused" master narrative (Martin & Larnell, 2013, p. 376). I begin with an overview of research about a set of four equity-directed instructional practices advocated for urban schools and synthesize these practices using Gutiérrez's (2007a) equity framework consisting of dominant and critical dimensions. Next, I present three cases of White teachers—those who demonstrated excellence with equity-directed practices that correspond to dominant dimensions of equity, on the one hand, but struggled with practices that correspond to critical dimensions, on the other. The goal is not to pin responsibility for systemic inequities on any individual teachers but rather I heed Dutro, Kazemi, Balf, and Lin's (2008) claim that "teachers' attempts—with all of their flaws and complexity—can provide rich texts for teachers to study collectively" (p. 295) and present this analysis accordingly.

[Facing] Race in (Urban) Mathematics Education

I foreground "race" in this literature review because of the significance of whiteness in the United States in reproducing subordination and widening society's opportunity gaps in and through mathematics education (Battey & Leyva, 2016; Martin, 2009a, 2009b, 2012; Nasir, Snyder, Shah, & Ross, 2012; Spencer, 2009; Stinson, 2006). There is an opportunity gap of inequitable access—to high quality mathematics curricula, teachers, instruction, textbooks, technology, and more—which contributes to a further widening of broader social inequities (Gutiérrez, 2008). As an example, consider how African American eighth-grade students are less likely to be recommended for algebra, even after controlling for performance in mathematics (Faulkner, Stiff, Marshall, Nietfeld, & Crossland, 2014). Because of the association between advanced mathematics coursework and wage earning (Battey, 2013a), hindrance away from algebra bears clear economic significance for African Americans (and other marginalized racial groups) and is part of the systematic maintenance of their subordination.

More generally, whiteness tacitly positions White people, their experiences, and their behaviors as superior (Battey & Leyva, 2016; Martin, 2009b), and it is supported by a set of corollary principles that function as "tools of Whiteness" (Picower, 2009, p. 204). For instance, the ideological principle of the United States as meritocracy is understood by many to be a central feature of American society, dictating that a combination of hard work and talent, or as cast in recent years, "grit" (Duckworth, Peterson, Matthews, & Kelly, 2007), yields success. Equivalently, the principle of meritocracy also dictates that lack of success is a result of a lack of effort or ability (e.g., Battey & Franke, 2015; Martin, 2009b). This principle functions as a tool of whiteness in how it ignores "systemic barriers and institutional structures that prevent opportunity and success" (Milner, 2012, p. 704) as well as institutional structures that facilitate opportunities and the distribution of rewards not according to merit but instead according to race and social background (Bowles & Gintis, 2002; McIntosh, 1988).

A second ideological principle that functions as a tool of whiteness is that of color-blindness (Bonilla-Silva, 2003; Martin, 2009b; Ulluci & Battey, 2011). Teachers who claim color-blindness—that is, they claim to not notice the race of their students—are, in effect, refusing to acknowledge the impact of enduring racial stratification on students and their families (Martin, 2008). As Ladson-Billings (1994) contends:

> Given the significance of race and color in American society, it is impossible to believe that a classroom teacher does not notice the race and ethnicity of the children she is teaching. Further, by claiming not to notice, the teacher is saying that she is dismissing one of the most salient features of the child's identity and that she does not account for it in her curricular planning and instruction. (p. 33)

Seemingly opposite to colorblindness is the "I can't relate" principle (Picower, 2009). Unlike the colorblind stance, through this relate principle, teachers self-identify as essentially different from their students. Yet their constructions of those differences are typically cast in terms of deficit constructions about students, their places, and their families. All of these ideological principles—the myth of the meritocracy, colorblindness, and "I can't relate"—function as tools of whiteness by absolving teachers from adopting new instructional practices that are proposed to further equity (e.g., Stinson, 2006), engaging in processes of reflection about equity (e.g., Rousseau & Tate, 2003), and confronting fears of people of color to learn about students and their communities (e.g., Aguirre, 2009).

Whiteness ideologies become amplified when an achievement lens is used to measure the quality of teaching in urban schools, yielding teacher caricatures in two forms (Martin, 2007). "Teacher as missionary" (Martin, 2007, p. 13), promoted in popular depictions of urban schools, refers to the notion of the transformation or taming of struggling Black and Latinx students as a result of the sacrifices and efforts of a White teacher. A related caricature is "teacher as cannibal" (Martin, 2007, p. 14), referring to a teacher who focuses solely on mathematics content, with little or no attention to relating to students or teaching mathematics that builds on or connects to students' social realities. Given this pair of undesirable caricatures, expanding the field's understanding of quality mathematics teaching in hyper-segregated, urban secondary schools remains of pressing concern. Next, I review four instructional practices that are advocated for mathematics teaching in (urban) schools.

Equity-Directed Instructional Practices in Mathematics

I highlight equity-directed instructional practices from four models of progressive pedagogy that are typically recommended for the urban schools context: (a) standards-based mathematics instruction (SBMI; National Council of Teachers of Mathematics, 2000), (b) complex instruction (CI; Cohen, 1994; Cohen & Lotan, 1995; Lotan, 2006), (c) culturally relevant pedagogy (CRP; Ladson-Billings, 1994, 1995a, 1995b), and teaching mathematics for social justice (TMfSJ; Gutstein, 2003, 2006).

Here, and in the broader project through which the research was conducted, I identified one central instructional practice from each pedagogical model: (a) teaching for understanding from SBMI; (b) fostering multidimensional participation from CI; (c) connecting mathematics content to students' experiences from CRP; and (d) providing opportunities for using mathematics to read and write the world from TMfSJ. In brief, I selected these specific practices because of the way that they illuminate the nested relationships among these pedagogies. All four of these pedagogies rely on teaching for understanding as a basic cornerstone. Complex instruction then extends teaching for understanding with an articulated emphasis on participation. CRP further extends complex instruction by including a focus on the cultural context of teaching and learning. Finally, teaching mathematics for social justice accentuates CRP's engagement with cultural contexts but through a lens of power, using mathematics explicitly to analyze and respond to social injustices.

In the discussion that follows, I situate each practice in the respective literature in terms of its particular theory of learning and emphasize the obstacles identified in the literature especially salient for the context of hyper-segregated urban schools. Finally, I utilize Gutiérrez's (2007a) equity framework to synthesize the four practices into a pair of dominant and a pair of critical equity-directed practices.

SBMI – Teaching for understanding. Standards-based mathematics instruction (NCTM, 2000) conceptualizes learning as engagement with mathematics that results in conceptual understanding. This orientation to teaching mathematics favors the understanding of mathematical concepts over mere fluency with algorithms and facts (Carpenter & Lehrer, 1999; Hiebert & Carpenter, 1992). Mathematical understanding develops as part of social, discursive processes of conjecture, justification, and reasoning (González & DeJarnette, 2015; Selling, 2016; Sfard, 1998; Zahner, Velazquez, Moschkovich, Vahey, & Lara-Meloy, 2012). Because of its dialogic nature, teaching for understanding implies that a classroom's social culture is characterized by an emphasis on students' mathematical thinking, students' autonomy in choosing solution methods, the utilization of mistakes as learning opportunities, and intellectual authority residing in the mathematics itself (Hiebert et al., 1997). To organize a classroom around understanding, a teacher must develop social and socio-mathematical norms that support and sustain student participation in such a social culture (Bennett, 2014; Cobb, Yackel, & McClain, 2000).

The literature presents examples of teaching for understanding in mathematics in urban schools, characterized by high expectations for students (e.g., Bonner, 2014; Jamar & Pitts, 2005; Stinson, Jett, & Williams, 2013) and cognitively demanding mathematical tasks (e.g., Boaler & Sengupta-Irving, 2016; Kitchen, DePree, Celedón-Pattichis, & Brinkerhoff, 2007; Silver, Smith, & Nelson, 1995; Walker, 2012). Even though it could be considered less accessible to students whose first language is not English, as long as the focus is on underlying mathematical ideas and not accuracy or fluency of linguistic production, teaching for understanding is advocated as especially productive for all students, including English language learners (Moschkovich, 2013; Zahner et al., 2012).

A lesson's mathematical task is essential to teaching for understanding, given that tasks directly determine the kinds of mathematical work the students will engage with and how (Henningsen & Stein, 1997; Stein, Grover, & Henningsen, 1996). Yet across classrooms in the United States, students spend more time on low-demand mathematical tasks—that is, learning mathematics by practicing procedures (Boston & Wilhelm, 2015; Stigler & Hiebert, 2004), as part of traditional instruction comprised of lecturing and drill with practice (McKinney, Chappell, Berry, & Hickman, 2009). Even when teachers have access to standards-based curricula, they do not always opt to implement the high-level tasks (Boston & Smith, 2009). Especially relevant to the context of hyper-segregated urban schools, teachers' views about students' mathematical capabilities play a central role in their task selection and in the mathematical opportunities they provide (Battey & Franke, 2015; Cobb & Jackson, 2013; Jackson, Gibbons, & Dunlap, 2017; Wilhelm, Munter, & Jackson, 2017). Teaching for understanding demands that teachers view their students as possessing the prerequisite mathematical skills, literacy abilities, and problem-solving dispositions, a direct challenge to prevalent constructions of Black and Latinx youth.

CI – Multidimensional participation. A view of learning that emphasizes participation considers students as guided by the teacher as expert, knowledge as an aspect of practice, and knowing as inherently tied to participation (Lave & Wenger, 1991; Sfard, 1998). Such a perspective emphasizes that learners are always in a state of flux, that actions might be unsuccessful but are not tied to them as individuals or indicative of permanent, individual traits (Sfard, 1998). The range of mathematical opportunities provided by the teacher, however, determines opportunities for participation, quality of student participation, and perceptions of competence (Gresalfi, Martin, Hand, & Greeno, 2009; Wilhelm, Munter, & Jackson, 2017).

Instead of viewing diversity among students in terms of challenges it might present, CI reframes diversity as a resource, leveraging a construct known as "multidimensionality" (Cohen & Lotan, 1995). In a classroom with a range of mathematical opportunities, a wide array of mathematical practices is valued, such as asking good questions, employing different mathematical representations, explaining ideas, generalizing, justifying, or revising methods (Boaler & Greeno, 2000; Boaler & Staples, 2014). Multidimensionality offers greater breadth for mathematical competence (Gresalfi et al., 2009)—in terms of what students are accountable for and to whom (Dunleavy, 2015) as well as what kinds of agency they can exercise (González & DeJarnette, 2015). Essentially, multidimensionality supports engagement by broadening the ways that students can enact competence and creates pathways to success for more students (Langer-Osuna, 2016). A growing set of examples in the research literature ties mathematical achievement at urban schools to instruction that emphasizes multidimensionality (Boaler & Staples, 2008, 2014; Dunleavy, 2015; Horn, 2012; Nasir et al., 2014).

Teachers' views about Black and Latinx students as learners, their families, and communities impact not only the kinds of mathematical tasks they make available but also the kinds of mathematical participation they make available (Battey, 2013b; Battey & Franke, 2015; Cobb & Jackson, 2013; Grant et al., 2015; Jackson, 2009; Jackson et al., 2017; Wilhelm et al., 2017). In addition, teachers' beliefs about students and about learning underlie how they distinguish between mathematical and cultural activity, and how they determine which forms of participation will be considered suitable for their classroom (Hand, 2012). Teachers often view participation of marginalized students as off-task, unproductive, or distracting, even when it reflects students' membership of and competence in another social context, unbeknownst to the teacher (Hand, 2010).

The perception of order as a prerequisite to learning constrains individual teachers' views about classroom participation and leads to didactic teaching (Golan, 2015; Ladson-Billings, 1997; Nasir, Hand, & Taylor, 2008). School policies common to urban schools in the United States such as uniform dress requirements communicate a view of students as needing to be controlled (Battey & Leyva, 2016). "No excuse" policies popular in urban schools include intensive discipline systems and have been described as militaristic in the way that they dictate parameters for student behavior. Furthermore, overt messaging that announces to students that they are "smart" (Battey & Leyva, 2016) or the increasing branding of urban schools using names that include words like "success," "achievement," "aspire," "strive," or "ascend" communicate deficit views of students that likely impact teachers' choices about the range of options they provide for classroom participation in mathematics.

CRP – Connecting mathematics instruction to students' experiences. Cultural perspectives view learning as situated, as the "acquisition throughout the life course of diverse repertoires of overlapping, complementary, or even conflicting cultural practices" (Nasir, Rosebery, Warren, & Lee, 2014, p. 686). Youth encounter a range of cultural practices in their school day, through their activities outside of school, and across the course of experiences in their lives (Gutiérrez & Rogoff, 2003). Every cultural practice has its own stance and is governed by its own set of purposes, symbols, and discourses, which then demands negotiation (Rogoff, 2003). Teachers can recruit this negotiation process for school learning by building on or connecting to students' cultural practices, which, without interrogation, are typically organized according to White, middle-class cultural practices (Aguirre et al., 2013; Leonard, 2008; Matthews, 2003; Tate, 1995; Watson, 2012). CRP emanates from a cultural perspective on learning, guided by the principle that curriculum and instruction must draw on students' own cultural practices and not just the realities of others (Ladson-Billings, 1995a, 1997).

Emdin (2016) emphasizes the importance of integrating students' out-of-school experience into curriculum—an integration that he terms building "bridges to learning" by integrating students' "contexts" with "content" using symbolic or tangible artifacts from places from which youth come as "anchors of instruction" (p. 291). As Ladson-Billings (1994) explains:

> Culturally relevant teaching is a pedagogy that empowers students intellectually, socially, emotionally, and politically by using cultural referents to impart knowledge, skills, and attitudes. These cultural referents are not merely vehicles for bridging or explaining the dominant culture; they are aspects of the curriculum in their own right. (pp. 17–18)

Thus, a teacher might plan a lesson to focus centrally on specific cultural practices as objects of mathematical study (Civil, 2002; Kisker et al., 2012; Mukhophadhyay, Powell, & Frankenstein, 2009; Turner et al., 2012) or include representations that draw upon students' cultural practices. For example, the Algebra Project curriculum (Moses & Cobb, 2001) builds on students' experiences with public transit toward developing understanding of integers, but the lessons were not organized to thematically investigate the local transit system.

Contextualizing mathematics in students' cultural practices can lend a sense of practical utility to mathematics, often interests students, mediates between students' formal and informal knowledge, and supports mathematics identity development (e.g., Birky, Chazan, & Farlow Morris, 2013; Brenner, 1998; Hubert, 2014; Kisker et al., 2012; Martin, 2000; Vomvoridi-Ivanović, 2012; Walkington, Petrosino, & Sherman, 2013). However, superficial understandings of students' cultural practices can backfire as an equity strategy when teachers do not anticipate how students will take up realistic aspects of contextualized mathematics (Brenner, 1998; Lubienski, 2000; Tate, 2005). To connect mathematics to students' experiences in productive ways, teachers need to develop ongoing practices around learning about their students—their students' interests, everyday activities, heritage, home languages, and more (e.g., Emdin, 2016; Gay & Kirkland, 2003; Milner, 2003; Vomvoridi-Ivanović, 2012)—all the while embracing the tension that "one's students can never be known" (Gutiérrez, 2009, p. 13).

Emdin (2016) explains, "Teaching more effectively requires embedding oneself into students' contexts and developing weak ties with the community that will organically impact the classroom" (p. 139). He articulates three steps for teachers, beginning with being in students' social spaces, engaging with those contexts, and then making connections between the out-of-school context and classroom teaching. Entering and spending time in students' spaces is known to be a central challenge for White teachers (Chu & Rubel, 2010), given that it requires negotiating commonly held fears of people of color and their spaces (Picower, 2009), to conduct what are effectively "border crossings" (Anzaldúa, 1987). This challenge explains the tendency of teachers to draw on their own cultural experiences when contextualizing mathematics in cultural experiences as opposed to their students' (Matthews, 2003; Watson, 2012).

The research literature describes interventions designed around supporting pre- and in-service teachers about how to connect mathematics instruction to students' experiences or everyday practices (Aguirre et al., 2013; Moll, Amanti, Neff, & Gonzalez, 1992; Rubel, 2012; Rubel & Chu, 2012; Taylor, 2012). Observing students in out-of-school settings is time intensive for teachers (Nasir et al., 2008). Moreover, even in the context of professional development or teacher education interventions in which teachers have been supported in observing students in out-of-school settings, their identification of embedded mathematical practices in those activities was rare, ostensibly because of the inherent complexity in doing so (Bright, 2015; Gainsburg, 2008; Nicol, 2002; Wager, 2012).

TMfSJ – Critical mathematics. Sociopolitical perspectives foreground learning as identity-work: "Learners are always positioning themselves with respect to the doing of mathematics, their peers, their sense of themselves and their communities, and their futures" (Gutiérrez, 2013a, p. 53). Key to viewing learning as identity-work is the recognition of how "knowledge, identity, and power are interwoven and arising from (and constituted within) social discourses," social discourses which "privilege some individuals and exclude others" (Gutiérrez, 2013a, p. 40). TMfSJ is a pedagogical model drawn from sociopolitical perspectives about teaching and learning mathematics, guided by the goal that students be "prepared through their mathematics education to investigate and critique injustice, and to challenge, in words and actions, oppressive structures and acts—that is, to "read and write the world" (Freire, 1970/1988) with mathematics" (Gutstein, 2006, p. 4).

Contextualizing mathematics in sociopolitical terms has been found to interest students more broadly in mathematics (e.g., Brantlinger, 2013; Hubert, 2014; Rubel, Lim, Hall-Wieckert, & Sullivan, 2016; Winter, 2007). Moreover, it has been shown to support students in learning to use mathematics to better understand the sociopolitical contexts of their own lives, so to be better able to effect change for themselves and for others (e.g., Frankenstein, 1995, 2009; Gutstein, 2003, 2006, 2016; Leonard, Brooks, Barnes-Johnson, & Berry, 2010; Rubel, Lim, Hall-Wieckert, & Katz, 2016; Rubel, Lim, Hall-Wieckert, & Sullivan, 2016; Turner, Gutiérrez, Simic-Muller, & Díez-Palomar, 2009). In so doing, students learn more mathematics: as Gutstein (2007) explains, "The two sets of goals—mathematical and social justice—dialectically interact with each other" (p. 4). The dialectic between mathematical and social justice goals relies on leveraging and "channel[ing] students' implicit critiques" of the social order that they (students) already have" (Goldenberg, 2014, p. 126). Research demonstrates the necessity of teachers' knowledge of sociopolitical contexts and solidarity with students and their communities for critical mathematics (Gutstein, 2003; Terry, 2011), with key recommendations that social justice issues be selected in collaboration with students, as "generative themes" (e.g., Gutstein, 2016).

Aside from the time demands posed by lesson planning for investigations that are local and context driven, mathematics teachers are typically inexperienced with teaching in this way (e.g., Bartell, 2013; Esmonde, 2014; Gonzalez, 2009). Furthermore, there is tension between such an approach and how contexts are typically employed in the teaching of mathematics. Typically, lesson planning in mathematics is driven by a predefined, interrelated set of mathematical concepts and skills, and real-world applications of those skills are introduced as examples. Planning a unit around a specific social justice issue is a different process, in which the issue and potential paths towards justice drive the mathematical content and not the other way around (Frankenstein, 2009).

The larger project in which this article is derived included activities to support teachers in studying their students' communities in various ways toward generating hypotheses about issues of social justice for their students and their families that could be integrated with their curricular plans. Analyses in the related literature attribute teacher reluctance about critical mathematics to perceived or experienced tensions with mathematical rigor (e.g., Brantlinger, 2013; Enyedy, Danish, & Fields, 2011; Garii & Rule, 2009; Gutiérrez, 2009), or around the necessary simplification of both the social phenomenon in question and the related mathematics (Dowling & Burke, 2012). Other analyses attribute teacher resistance to fears of overwhelming students (Gainsburg, 2008), causing those who are marginalized or those whose families benefit from inequitable power relations to feel uncomfortable (Aguirre, 2009; Simic-Miller, Fernandes, & Felton-Koestler, 2015); or to disinterest in or ambivalence about explicitly introducing social issues into mathematics teaching (Atweh, 2012; de Freitas, 2008; Simic-Miller et al., 2015).

Mapping Onto Gutiérrez's Equity Framework

Gutiérrez's (2007a) equity framework in mathematics spatializes equity according to four dimensions along two axes. The framework's dominant axis consists of access to and achievement in mathematics; dominant in its reflection of society's status quo. Access—to quality teaching, instructional resources, and a classroom environment that invites participation—is a precursor to mathematics achievement. Along the second, critical axis, are identity and power in mathematics; critical in addressing students' cultural identities and sociopolitical issues, from the perspective of marginalized groups (Gutiérrez, 2007a). Consideration of the role of identity in learning is a precursor to addressing issues of power as they relate to identity.

Teaching for understanding and multidimensionality are centrally and explicitly organized around increasing access and supporting achievement in mathematics and map onto the dominant axis in Gutiérrez's (2007a) framework. Teaching for understanding and multidimensionality can be seen as implicitly addressing identity and power along equity's critical axis as well. Borrowing a distinction made by Stinson and Wager (2012), instructional practices that encourage opportunities for equitable participation can themselves be an avenue toward social justice, and therefore, a way to teach *for* social justice. Instructional practices organized around increasing access often draw on processes that are endemic to students' identities in terms of out-of-school or cultural activities and experiences, such as the apprenticeship (e.g., Civil & Khan, 2001; Masingila, 1993) or assumptions of competence (e.g., Nasir, 2005). Likewise, an emphasis on understanding through multidimensional participation, relates implicitly to power in terms of questioning how smartness in mathematics gets constructed (Hatt, 2007), whether mathematical ability is seen as innate or learned (Dweck, 2006), and considering whose interests are being served when speed is privileged over communication; symbolic arguments are valued over visual ones; or a hierarchical, sequential view of mathematics is held over a connected, networked view (Horn, 2012; Martin, 2012). Teaching for understanding and multidimensionality create space for students to develop "practice-linked identities" as learners and doers of mathematics (Nasir & Hand, 2008), which can be especially significant for Black and Latinx students for whom such space can be new (e.g., Martin, Aguirre, & Mayfield-Ingram, 2013).

There is a distinction, however, between how teaching for understanding and multidimensionality stop short at making matters of identity and power explicit in terms of *mathematics content*. Boaler and Staples (2008) amplify this distinction in clarifying that the mathematics content at Railside, an urban school whose success they analyze, was not "sensitive to issues of gender, culture, or class" (and presumably race). They argue, "There is more than one road to equity" aside from "culturally sensitive materials" and that "such materials…. can be uncomfortable for teachers if they require cultural knowledge that they do not possess or if their classrooms are extremely diverse" (p. 640). Instead, Boaler and Staples (2014) advocate for equity in terms of participation:

> Railside students learned to appreciate the different ways that students saw mathematics problems and learned to value the contribution of different methods. ... As the classrooms became more multidimensional, students learned to appreciate and value the insights of a wider group of students from different cultures and circumstances. (p. 32)

This "more than one road to equity" perspective contrasts with Style's (1988, p. 1) "windows" and "mirrors" metaphor that explains how education needs to include "window frames in order to see the realities of others" and "mirrors in order to see his/her own reality reflected." As Style (1988) claims:

> Knowledge of both types of framing is basic to a balanced education which is committed to affirming the essential dialectic between the self and the world. ... School curriculum is unbalanced if a black student sits in school, year after year, forced to look through the window upon the (validated) experiences of white others while seldom, if ever, having the central mirror held up to the particularities of her or his own experience. (pp. 1–4)

Gutiérrez (2007a) elaborates: "The goal is not to replace traditional mathematics with a pre-defined 'culturally relevant mathematics,' but rather to strike a balance between the number of windows and mirrors provided to any given student in his/her math career" (p. 3). Without such a balance, "when someone with the authority of a teacher, say, describes the world and you are not in it, there is a moment of psychic disequilibrium, as if you looked in the mirror and saw nothing (Rich, 1986)" (p. 3). This kind of "psychic disequilibrium" can explain the potential conflict between "identities that students are invited to construct in mathematics class and the kinds of persons they view themselves to be" (Cobb & Hodge, 2002, p. 279), especially salient to dis-identification with classroom mathematical activity (Cobb, Gresalfi, & Hodge, 2009; Nasir, 2002; Tate, 1995). Moreover, without addressing identity and power in mathematics itself, mathematics is constructed as neutral and as universal, a positioning that can be seen as parallel to colorblindness (Battey & Leyva, 2016).

Not only is the teaching of mathematics political (e.g., Fasheh, 1982; Felton-Koestler & Koestler, 2017; Gutiérrez, 2013b), but also learning is "always tied to culturally rooted perception of the learning settings" and to "one's cultural relation to the content" (Nasir et al., 2012). Seen in this way, the critical dimensions in Gutiérrez' (2007a) classification are necessary, and *demand* connecting mathematics content to students' experiences, their communities, and possibilities for social transformation through TMfSJ's critical mathematics (Cobb & Jackson, 2013). Gutiérrez (2007a) explains:

> We must keep in mind all four dimensions, even if that means that at times one or two dimensions temporarily shift to the background. A natural tension exists between mastering the dominant frame while learning to vary or challenge that frame. As such, access, achievement, identity, and power are not going to be equally or fully present in any given situation ... the goal is to attend to and measure all four dimensions over time. (pp. 4–5)

Even the Critical Practices Often Not Critical Enough?

Despite Ladson-Billings' (1995a) underscoring of the importance of teachers engaging in "cultural critique" of the political underpinnings of students' social circumstances, the practice of connecting mathematics to students' experiences from CRP is largely taken up in terms of contextualizing mathematics in general, everyday experiences. Typically, mathematics teachers select "real-world" contexts related to sports; middle-class leisure activities; or adult experiences related to home remodeling, shopping, banking, or budgeting (Bright, 2015; Gainsburg, 2008; Matthews, 2003; Wager, 2012; Watson, 2012), or initiate situated classroom settings related to classroom experiments or field trips (Wager, 2012). Ladson-Billings (2014) has observed:

> As I continued to visit classrooms, I could see teachers who had good intentions toward the students. ... They expressed strong beliefs in the academic efficacy of their students. They searched for cultural examples and analogues as they taught prescribed curricula. However, they rarely pushed students to consider critical perspectives on policies and practices that may have direct impact on their lives and communities. (p. 77)

Sealey and Noyes (2010) underscore this distinction in their case study of three schools, in which mathematics was constructed in terms of its practical relevance, the transferability of its processes, or its professional exchange value, but absent was a sense of the *political* relevance of mathematics and how it relates to power.

The ever-expanding role of state testing, prescribed curriculum, and increasing demands related to accountability undoubtedly create obstacles for teachers. But there is a "natural tension" between "mastering the dominant frame" by striving toward equity through the practices on the dominant axis ("playing the game") and "challenging the dominant frame" through the practices on the critical axis ("changing the game") (Gutiérrez, 2007a, p. 4). As Gutiérrez (2013a) exacts, "Without an explicit focus on issues of identity and power, we are unlikely to do more than tinker with the arrangements in school that contribute to the production of inequities in the lived experiences of learners and educators" (p. 62).

Here, I explore this tension by studying the difficulties for teachers in adopting the critical equity-directed practices of connecting mathematics instruction to students' experiences from CRP and critical mathematics from TMfSJ. Different from the collection of rich case studies in the literature that profile experienced mathematics teachers with a full set of well-developed, equity-directed practices (see, e.g., Birky et al., 2013; Bonner, 2014; Bonner & Adams, 2012; Clark, Badertscher, & Napp, 2013; Ladson-Billings, 1995b, 1997; Leonard, Napp, & Adeleke, 2009), and different from literature that documents failures of teachers in urban schools, I present these cases to serve as texts that illuminate "teachers' attempts—with all their flaws and complexity" (Dutro et al., 2008, p. 295).

Methods and Context

I draw on data gathered as part of a larger project aimed to design and study professional growth for secondary mathematics teachers around the four aforementioned equity-directed practices. Participants consisted of 12 teachers from 11 secondary schools (ten in grades 9–12 schools and two in grades 6–12 schools), who were recruited through a local teacher organization, geographically dispersed across a large city in the United States. Teacher demographic data are summarized in Table 1. The participating teachers taught a range of secondary mathematics, in grades 6–12, at schools with higher percentages of low-income, Black and/or Latinx students than the aggregated city percentages. I identify as an Ashkenazic Jew, pass as a White woman, and was the project director and lead facilitator of the associated professional development (PD).

The PD was initiated with an 8-day summer institute in the summer of 2012 focusing on the four equity-directed practices and continued with monthly 2-hour group meetings. In all sessions, teachers regularly engaged in collaborative mathematical problem solving, experiences which I used to then draw out overlapping themes of conceptual understanding, challenging mathematics, participation, and critical thinking about mathematics. Ilana Horn visited the summer institute and introduced teachers to the theme of multidimensional participation; teachers were provided with Horn's (2012) book *Strength in Numbers: Collaborative Learning in Secondary Mathematics* (see Appendix A for focal topics of PD sessions).

Table 1
Demographic Data of Participating Teachers

Variable	*N*		*N*
Gender		*Certification*	
Women	10	Alternative	7
Men	2	Traditional	5
Racial/ethnic self-identification		*Years teaching*	
White	9	Three or fewer	3
Afro Caribbean	2	Four to six	4
Mexican American	1	Seven to nine	4
		Ten or more	1

Over the course of the project, teachers participated in a sequence of activities (see Rubel, 2012) designed to facilitate and support them to enter students' spaces and to learn to identify students' funds of knowledge (i.e., learn to make connections to their everyday experiences and prior knowledge). Teachers conducted a series of various forms of community walks to learn about their students' lived worlds and experiences. For example, during the fall semester, teachers were asked to complete the following assignment: to choose a place near their school and spend time observing the space, people, and activities. Teachers could opt to stand in one place to observe, for example, in front of a building or inside a corner-store; or to traverse a neighborhood's streets, perhaps pursuing a specific theme of interest, such as locations of play-spaces for youth. Teachers shared their experiences and reflected together at one of the group meetings. Across the PD, teachers and I shared an array of examples of resources around opportunities to think critically with mathematics, including using the activities and lesson ideas in Gutstein and Peterson's (2005) edited volume *Rethinking Mathematics: Teaching Mathematics for Social Justice.*

Across the school year, the research team (consisting of me and a graduate research assistant) observed five lessons per teacher, with the same groups of students, in evenly spaced rounds across the school year, for a total of 58 observed lessons.[3] The research assistant observed and took field notes during all 58 lessons; I co-observed a subset of 28 of the 58 lessons. After each observation, we classified the lesson's main mathematical task, as implemented by the teacher and enacted by the students, using Stein and colleagues' (1996) levels of cognitive demand. We

[3] Two observations were cancelled because of a major weather event and the emergency relocation of a school. Both involved extensive disruptions at respective school sites, which would have led to unrepresentative observations.

resolved any discrepancies in ratings through discussion and through coordination of the task's narrative description with Stein and Smith's (1998) rubric.

The research assistant converted field notes into a detailed, time-indexed, narrative memo that included a description of the lesson's tasks and materials, as well as a synopsis of the lesson's structure, teacher moves, and students' observable participation. I reviewed the narrative memo within a day or two of the observed lesson, and clarified any missing details. I interviewed teachers individually two to six times during the school year, using a face-to-face, semi-structured, traditional question-and-answer format (Hollway & Jefferson, 2000). Finally, teachers wrote reflective statements that year and in the following school year, and we archived these statements as well as any additional email correspondence.

We used the lesson narratives and field notes to divide each lesson into a set of time-indexed units of activities. The unit of activity was determined by a change in what the students were asked to do by the teacher or by a change in whether the students were directed to focus individually, in groups, or as a whole class (Stapleton, LeFloch, Bacevich, & Ketchie, 2004). We coded the participation structure of each activity according to what the students were asked to do by the teacher, similar to Stodolsky (1988), but using top-level categories (as well as further refined subcategories) of listening; investigating or problem solving; discussing; reading, writing, or reflecting; using technology; or practicing skills (adapted from Weiss, Pasley, Smith, Banilower, & Heck, 2003). Students being asked to listen to lecture presentations, copy notes from the board, or practice skills on worksheets were categorized as passive. Activities were categorized as active when students were, for example, asked to participate in discussions, investigate with technology, write about a solution strategy, reflect about a concept or process, explore mathematical concepts with manipulatives, prove a mathematical conjecture, or listen to a classmate's presentation.

Next, for each lesson, we calculated the ratio of the difference between active and passive minutes to the total instructional time, denoted as "Difference in Participation Proportion" (DPP; Rubel & Stachelek, in press). DPP ranges from –1 to 1, from a lesson whose participation structures are entirely passive to a lesson whose participation structures are entirely active. Exactly three teachers (B Mary, C Tracy, and D Molly) had high DPP measures and four or five observed lessons with tasks of high cognitive demand (as shown in Figure 1). I used case study methodology (Merriam, 1998) to create cases of these three teachers.

Data sources for the case studies include five narratives of classroom observations group meetings, and audio of four to six interviews per teacher. Data were converted to text and uploaded to Dedoose (research software), organized chronologically and by teacher. I used a grounded analysis approach (Strauss & Corbin, 1998) with the narratives of the classroom observations to iteratively categorize aspects of the case teachers' instruction using a priori codes related to connecting to students' experiences and critical mathematics as well as a code for views about their students as learners of mathematics. In subsequent iterations, I created and used sub-codes as shown in Table 2. This coding process enabled the development of descriptive and interpretive, written case summaries. Next, I triangulated these findings using interview data and teacher written reflections, looking for confirming or disconfirming evidence. Finally, I conducted member checking (Lincoln & Guba, 1985) with the case teachers to validate interpretations.

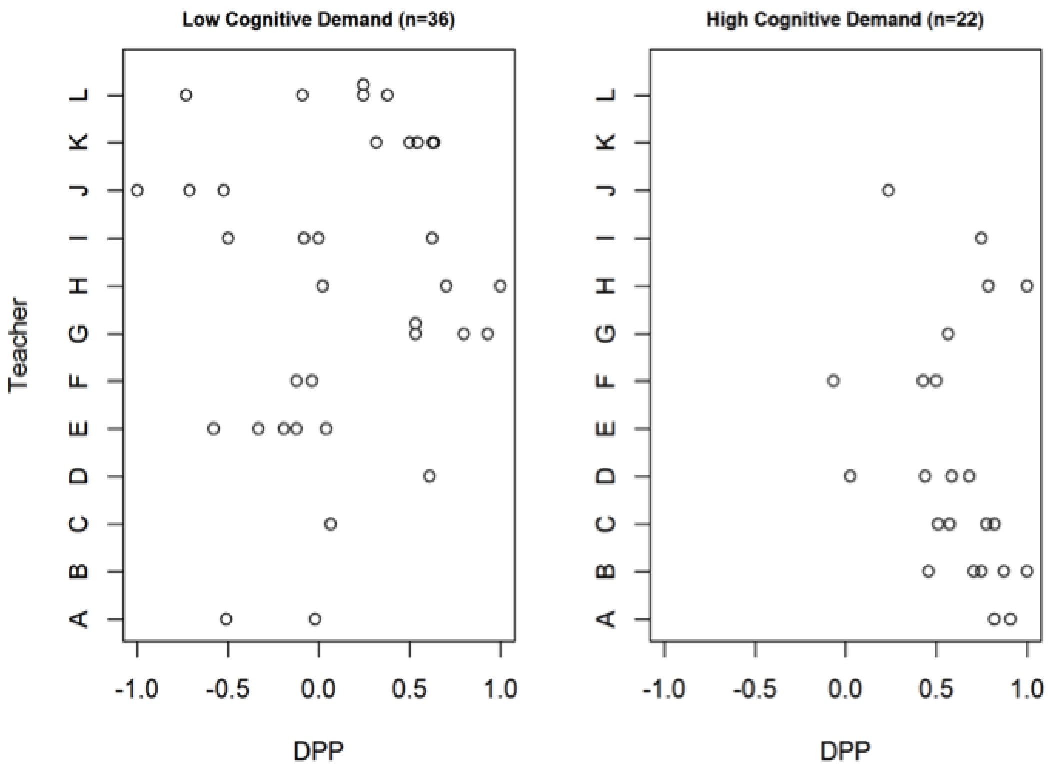

Figure 1. DDP and cognitive demand by teacher.

All three case teachers identified as White women and had started teaching careers after having moved to the city as young adults. Mary and Molly had entered teaching through an alternative certification pathway in response to a local teacher shortage (Boyd et al., 2012), and Tracy, with an undergraduate degree in mathematics, had entered teaching through a traditional pathway, with teacher certification from another state. In the academic year of data collection (2012-13), Mary taught 10th grade geometry; Molly taught 6th grade mathematics; and Tracy taught an 11th and 12th grade Algebra II class in the fall semester, and a 9th grade Algebra I class in the spring.

Table 2
Sub-codes Connecting to Students' Experiences and Critical Mathematics

Parent Code	Sub-code
Connecting to students' experiences	• Nature of context • Role of context in lesson • Connections between mathematical and everyday meanings • Input from students to launch task
Critical mathematics	• Actualized connections of content to social justice • Potential connections of content to social justice
Views about students as learners of mathematics	• Positioning students as successful learners of mathematics • Positioning students as mathematical resources for other students • Normalizing struggle in mathematics

Findings

I present an analysis of the difficulties for the case teachers, who demonstrated success with the study's articulated dominant equity-directed practices, in adopting the critical equity-directed practices of connecting mathematics instruction to students' experiences. These three teachers selected and implemented high demand tasks across all (or nearly all) of their observed lessons, in contrast with other teachers, who more often built their lessons around low-demand tasks (as shown in Figure 1). All of Molly's, Mary's, and Tracy's observed lessons had high DPP values, meaning that they included significant durations of opportunities for active participation. In what follows, first, I provide more details about the three case teachers, in terms of their success with the dominant equity practices. Then, I shift the results to focus on an analysis of their struggles with the critical equity-directed practices.

Success with Dominant Equity-Directed Practices

Mary: 10th Grade Geometry. Mary was 30 years old and in her sixth year of teaching, all at one school. Mary's school was grades 9–12, with most students identified as Hispanic or Latino (69%) and the remainder as Black or African American (29%), and most qualifying for free or reduced-price (FREP) school meals (86%). Because of a history and perception of violence, students at this school were required to take off their shoes and belts and pass through a metal detector, which was staffed and supervised by police, to enter the building. Her 10th grade classroom was overflowing with colorful, hand-made posters highlighting geometric definitions and relationships, photographs of students, and reminders about classroom rules and expectations. The classroom contained rectangular tables arranged for assigned groups of four students. Mary typically welcomed students enthusiastically as they entered the classroom and encouraged them to sit down quickly and get started with a task displayed on the Smartboard. Mary moved energetically around the classroom and spoke at a rapid pace, conveying enthusiasm for mathematics and for the lesson.

Mary supported the development of students' mathematical understanding with various tools, notably using her classroom's Smartboard to produce colorful, visual, dynamic, and interactive mathematical representations. For example, in a lesson on vertical angles, Mary used the Smartboard to measure and compare two congruent angles, but one with significantly longer rays than the other, in anticipation of a common misconception that angles with longer rays have a greater measure (Fischbein, 1987). The dynamic functionality of the Smartboard allowed Mary to rotate, translate, and compare angles with the students. Students were regularly invited to operate the Smartboard, to explore conjectures, draw pictures to express an idea, or represent mathematical thinking.

Mary often incorporated physical materials as tools to represent and model mathematical relationships. For example, in a lesson about triangle similarity, Mary built a model to use measures of individual students' heights to indirectly compute other heights. Students worked with physical measurement tools, a mirror and measuring tape, and used a model involving similar triangles to calculate the unknown (and theoretically not directly measurable) height of the classroom. This task afforded students the opportunity to discover that the room's height could be measured indirectly and to explore how the model adapts when a different person's height is used.

Even though Mary's student population was majority Latinx, with 19% of these students classified as Limited English Proficient (the official federal classification, not mine), Mary's lessons included significant amounts of mathematical discussions (25% of lessons, on average). She supported student participation in mathematical discussions by launching lessons (see Jackson, Shahan, Gibbons, & Cobb, 2012) with activities that directed students to individually write down and then share their prior knowledge of mathematical meanings. For example, in a lesson about vertical angles, Mary began the lesson with an individual, written prompt: "I think congruent means _____." Mary then wove together responses from several students in a whole class discussion to arrive at a shared understanding of congruent figures or angles, which would be crucial to the rest of the lesson (Lesson, 10/24/12). It was commonplace for students, at the end of class, to express gratitude to Mary for her teaching; at the end of one observed lesson, as she gathered up her things to leave the room, one student shared her feelings of gratefulness, "I appreciate all the teaching you gave us" (Lesson, 3/13/13).

Molly: 6th Grade Mathematics. Molly, age 25, was in her fourth year of teaching. Her school was grades 6–12 school in the same neighborhood as Mary's school. Nearly all of the students were identified as Black or African American (66%) or Hispanic or Latino (30%), and most qualifying for FREP school meals (86%). Entering students had standardized test scores slightly above the city average, and students were required to wear uniforms. Students were required by the school to line up in the hallway before they could enter the classroom, a hallway with posters reminding students that "You are a mathematician" and "You are a problem solver." Inside the classroom were colorful bulletin boards showcasing students' work. Molly typically greeted her students at the start of the period, at the door, shaking hands with some students and welcoming many in by name. Students sat in assigned groups of three at trapezoidal tables. Molly demonstrated a friendly, if stern, playfulness. In general, many students enthusiastically volunteered responses to her questions by actively raising their hands. If students seemed distracted, Molly would lead the class in coordinated clapping rhythms. In two of the observed lessons, Molly initiated brief sets of calisthenics exercises that incorporated review of mathematical number facts as a way to refocus the group.

Molly's lessons emphasized sense-making and connections between students' solutions and ideas. She organized her lessons around recruiting multiple methods of solving problems, which she typically then shared and compared in whole-class discussions. For example, in a lesson about area, Molly began with a gridded rectangle and a gridded nonstandard polygon, and asked students—first working individually, and later working in a group—to find at least two methods to find each area. Molly organized a whole class discussion in which students explained and compared two methods, and Molly highlighted a contrast between decomposition and composition approaches. The lesson continued by extending students' understanding of area and of methods of finding area to derive with students a method of finding the area of any triangle by building it into a rectangle of known dimensions, toggling between whole class discussion and individual mathematical investigations.

Tracy: High School Algebra. Tracy was 27 years old and in her third year of teaching. Tracy's school was grades 9–12, with a central location that attracted students from diverse geographies. Most of the students at Tracy's school were identified as Hispanic or Latino (73%), a smaller number of students as Black or African American (14%), with 75% qualifying for FREP school meals. Incoming 9th graders' test scores were higher than the city average, and students were required to wear uniforms. Tracy taught Algebra II to 11th and 12th graders in the fall semester, and Algebra I to 9th graders in the spring. Her classroom was mostly bare-walled, with few posters or visual displays and without Smartboard technology. Students were seated in individual desks and chairs that were placed side-by-side to create pairs. At times, Tracy asked students to form larger groups, at which point the students would push their desks together into groups of four. Tracy spoke to students in her lessons in what seemed likely a deliberately slow pace, peppered with pauses that provided both the space for students to share their own thoughts, questions, or concerns. She primarily stood at the front of the classroom, except when circulating around during group work. In addition to her outwardly reserved manner, she regularly communicated a dry sense of humor, using gesture and tone, which seemed to appeal to the students. In her classroom, students appeared comfortable asking questions, commenting on their own confusion, or venturing to provide potential solutions to a given task.

Tracy's lessons were typically organized around whole-class discussions that were layered around various types of individual or group-focused investigations, involving rigorous mathematical activity around high demand tasks and with supporting tools. One paradigmatic observed lesson began with a short, individual activity in which students were asked to write about their ideas as to the difference between volume and surface area. The subsequent discussion was organized around students' sharing their ideas in reference to various cardboard models of three-dimensional figures. Once there were some agreed upon definitions and units of perimeter, surface area, and volume, Tracy gave each pair of students several physical models of nonstandard three-dimensional figures, as well as plastic unit cubes, and students were asked to determine strategies for finding volumes. The class was later reconvened as a whole group to share their strategies, with Tracy guiding them toward generalization. Students were given a handout containing a set of word problems of varying difficulty to practice applying an understanding of volume and were directed to work on these in groups. The lesson concluded with a whole class discussion that featured the sharing of student solutions to three pre-designated problems.

As stated previously, the focus here is on the difficulties for teachers in adopting the critical equity-directed practices of connecting mathematics instruction to students' experiences called for in CRP and critical mathematics called for in TMfSJ. Alternate analyses could probe the factors and micro-practices supporting or challenging aspects of these teachers' successes with the teaching for understanding and multidimensionality. In the context of their success with the dominant equity-directed practices, the cases of Mary, Molly, and Tracy are ideal terrain in which to analyze their difficulties with the critical practices.

Challenges in Connecting to Students' Experiences

The three teachers made efforts to connect mathematics to out-of-school contexts, but these were limited to experiences assumed to be general or teacher-initiated classroom settings. Molly and Tracy contextualized story problems in terms of general out-of-school contexts related to sports, business, or personal finance but not with specific connections to students' experiences. For example, Tracy's lessons included contextualized problems around basketball shots for the topic of binomial probabilities (Lesson, 9/28/12), a business plan for a clothing company for the topics of normal distribution (Lesson, 11/9/12), or compound interest to explore exponential functions (Lesson, 11/30/12). Similarly, Molly made connections in her lessons to general experiences, such as drawing an analogy to the general experience of getting her height measured at the doctor's office as a way to explain how the altitude or height of a polygon has perpendicularity as part of its definition (Lesson, 3/8/13).

In Mary's class, geometry, she oriented her lessons largely around teacher-initiated, classroom-situated geometric settings and not on geometry specifically connected to students' physical spaces outside of school. For example, in a lesson about triangle similarity, Mary's lesson built on similarity relationships to enable students to use their own heights to indirectly measure the height of the classroom. Students worked with physical measurement tools, a mirror and measuring tape, and used a model involving similar triangles to calculate the unknown (and theoretically not directly measurable) height of the classroom.

Lacking knowledge and fearing people of color and their spaces. One interpretation of the difficulty for the case teachers in connecting to or building on students' experiences is a lack of knowledge about students' experiences because of fear of entering the spaces in which their students live, shop, play, or worship (Picower, 2009). Indeed, Molly noted and reflected a sense of inadequacy in connecting to her students' experiences that she explained in terms of feeling like an outsider relative to her students and their communities: "As a young, White teacher, who doesn't live in my school's neighborhood or in the neighborhood where my students live, and who didn't grow up in a city, what position am I in to connect to my students' experiences?" (Reflection, 5/2014) The case teachers were reticent to embed themselves in students' spaces to learn about their students. Molly's question as to "What position am I in to connect to my students' experiences?" is reminiscent of the "I can't relate" tool of whiteness presented by Picower (2009), and seemed to function as a kind of release for Molly from the need to cross boundaries or confront fears of students' communities to begin the never-ending process of learning about students.

The case teachers' difficulties in connecting to students' experiences can be understood in the context of the group's engagement with activities in the project designed to facilitate their presence in students' spaces. At the monthly group meeting in which teachers were asked to share their experiences in visiting their students' spaces, they do so in a silent, snowball activity in which they recorded their responses to the following prompts on a large whiteboard: Where did you go? How did you observe? What did you notice? What did you learn? About half of the teachers in the group focused their observations around typical stereotypes of low-income urban spaces, commenting on noticing "heavy police presence," "vacant lots," "99 cent stores," "paucity of transportation access," and spaces that seemed "disused" with "creepy vibes." One teacher noted having observed in and around a corner store, "observing owner (Yemeni), workers (Dominican), and customers (dealers & thieves)." Another explained that she had traversed her selected area looking in particular at "housing types, conveniences, and cleanliness." There were few comments that did not reflect deficit views of low-income people of color, such as a presence of multi-generational families, murals with positive messaging, community resources like churches and daycares, and evidence of gentrification (Meeting, 10/2013).

Mary later reflected on the tension that she experienced while completing the activity, between her typical noticing of her students' places and what the assignment prompted her to notice. She wrote:

> I first took note of the heavy New York City Police presence, the run-down lots, the burnt cars and "We'll Buy Your House" signs. I thought of everything I "knew" (assumed) about the area. But then, as we walked through the subway station, I noticed beautiful stained-glass windows with sunlight streaming through. In the sunlight, a man stood at his card table selling incense and oils, two women laughed as they waited for a train, a toddler held his father's hand as they walked up the stairs. (Reflection, 5/2014)

Mary's reflection is confluent with other teachers' responses during the activity debrief and demonstrates how in this case, visits to students' spaces can re-inscribe prior deficit views about students and their communities. This result more broadly should generate cautions in terms of how these kinds of community walk activities organized for teachers might actually reproduce teachers' deficit views instead of challenging them (Philip, Way, Garcia, Schuler-Brown, & Navarro, 2013; Rubel, Hall-Wieckert, & Lim, 2016).

Learning about students subverted. Observations of the material conditions in students' communities can contribute to a subversion of teacher to "teacher as missionary," one of Martin's (2007) two teacher caricatures. In my own work with youth at a hyper-segregated urban school, for example, I organized a celebratory class trip to go out for ice cream. Instead of making efforts to learn about what local treats they might enjoy or teach me about, I presumed that they would want to go to sample expensive ice cream at a new outlet in a nearby, but gentrifying neighborhood, its very presence a consequence of this gentrification. While some might argue that this was an act of kindness or generosity, I understand now how this effort likely made the students uncomfortable in how it was a "White savior" maneuver.

Tracy's efforts to learn about her students led to a similar subversion. She described a "fun, kind of experiment" that she tried which she felt was "an amazing teaching moment." She invested personally in Red Cross training and in special insurance and founded a weekend cycling club for her students. She described how she had initially dreamed of riding with the students out of the city but had realized that "we're never going to get over the bridge, but it doesn't matter. ... It's not about that, it's about enjoying the scenery and being with my students" (Meeting, 5/2013). This example shows Tracy's willingness to spend time outside of school with her students, generosity with her resources, and her commitment to getting to know her students better. However, this particular effort was organized to engage students in one of her hobbies, without a parallel effort to observe or learn about students by engaging in one of their interests. Tracy's approach was to note, among her students, the absence of one of her own essential activities, and then make extensive efforts to bring it to her students. For Tracy, her appropriation of the cultural positioning of youth of color as living incomplete or unhealthy lives led her to connect to students through her own, White experiences.

Challenges to Critical Mathematics

Whiteness as blinding. Mary and Molly avoided addressing issues of power and social justice in the content of their mathematics lessons, even though their lessons included examples of real world contexts that could have easily lent themselves to critiques of power and social justice. For example, one of Mary's observed lessons, on the topic of geometric loci, was contextualized in terms of an unspecific investigation about home locations, relative to constraints like not living too close to a power plant or a busy road (Lesson, 2/11/13). The lesson did not mention the high asthma rates in the school's local neighborhood and relationships to types and rates of local environmental pollution that have been found to be worse in that low-income neighborhood.

Similarly, Molly organized a lesson around the context of the school being moved by the district to a new location the following school year (Lesson, 11/16/12). The lesson focused on using map scale to convert between inches on a map and actual distance. Students were tasked to create a walking route from the current location to the new location, and then use the map scale to determine the actual distance of the route. A range of questions could have been asked: Does the new location bring the school closer to you? Closer to most students? How does the planned move impact the student community in terms of length of commute or new walking routes? What categories of amenities are available at each location? Are students happy about the change? Molly's lesson was strictly limited to interpreting the map's scale in terms of actual distance.

Mary agreed, in hindsight, that this topic had the potential to provide opportunities for critical thinking with mathematics about placement and displacement, desirability and undesirability, and gentrification, topics that were pressing in her students' environments. When asked about why she had not connected this topic to sociopolitical circumstances impacting her students' lives, Mary explained that at the time, she had not noticed these potential connections, explaining that "because of my privileges, I have had blinders on to a way of seeing these things" (Interview, Summer 2014).

False neutrality. Different from Molly and Mary, Tracy developed unit projects for her Algebra II course on what she called "social issues" and described her approach to designing these units in terms of being "neutral about issues" and organizing students to examine a particular theme from "two opposing points of view" (Reflection, 6/21/13). Tracy presented the students with a topic and expected them to develop their own arguments but also arguments that support dominant, hegemonic perspectives about that issue using mathematics. For example, Tracy wanted students to learn how mathematics can be used to *defend* the stop and frisk tactic used by local police, a tactic that had serious implications for most of her students impacting their day-to-day mobility. Tracy's assumption was that learning to support both stances would support students in clarifying and strengthening their arguments and to practice using mathematics to support a range of arguments. Furthermore, by presenting an issue and arguing both sides could seemingly be a safeguard against fears about bringing political issues into the classroom.

Tracy's technique of framing the assignment around having students argue dominant and critical perspectives of an issue was intended to allow for students to develop their own opinions along a wider continuum and prepare them for advocating for their position by better understanding the opposing argument. But in framing the task for students to justify both positions, Tracy inadvertently positioned the two perspectives as equally viable, or as matters of opinion. For example, in the case of the analysis of stop-and-frisk data, Tracy seemed to be equating the position that Black and Latinx youth are stopped at greater rates than White people tactic and that this is inconvenient, degrading, unjust, and is part of a broader system of racialized policing practices with the position that Black and Latinx youth are justifiably stopped more often. In framing these positions as equally viable, Tracy undercut her students' sense of injustice about their own experiences and, therefore, failed to leverage their already existing critiques of the social order.

Tracy herself reported that framing the social justice tasks in this way, that she understood as neutral, indeed backfired with the students. However, she reverted to deficit views of her students in her interpretation. She said that she had "learned the hard way that not all students have the same experiences and you can't assume they know both sides to every social issue" (Reflection, Spring 2013) and not on how her anti-critical framing of the issue in the tasks for students might explain their lack of effectiveness.

Teaching for *social justice as tool of whiteness.* Stinson and Wager (2012) distinguish between teaching *for* social justice—that is, providing students from underserved, marginalized groups access to challenging mathematics (equity's dominant axis)—with the critical equity-directed practices, or teaching *about* social justice. In other words, dominant equity-directed practices do not challenge the status quo (Gutiérrez, 2007b). However, insofar as they are understood as teaching *for* social justice, addressing equity through the dominant equity-directed practices can seem sufficient, allowing schools or teachers to evaluate their programs, policies, or instruction as addressing equity. However, as I, and others, have argued, the critical equity-directed practices are necessary, since supporting students to "play the game" is not equivalent to "changing the game" (Gutiérrez, 2009, p. 11).

Molly's case illustrates how emphasizing the dominant equity-directed practices can obfuscate, or even co-opt, the critical. Her classroom featured a poster placed front and center above her whiteboard informing students that, "Without struggle there is no progress" (Douglass, 1857), a passage excerpted from Frederick Douglass's 1857 Emancipation speech, which he delivered in the context of the movement to abolish slavery in the United States. Nearly all students at her school identify as Black or African American, and Molly's choice to feature a poster quoting an African American thinker whose writing is tied to slavery and emancipation could be seen as setting the stage for critical mathematics, by positioning mathematics as a tool for civil rights. Douglass's words suggest that social progress requires struggle, and in the original text continue with:

> This struggle may be a moral one, or it may be a physical one, and it may be both moral and physical, but it must be a struggle. Power concedes nothing without a demand. It never did and it never will. Find out just what any people will quietly submit to and you have found out the exact measure of injustice and wrong which will be imposed upon them, and these will continue till they are resisted with either words or blows, or with both. (para. 8)

Displaying these words on the wall of a mathematics classroom could signal that mathematics could be useful in a struggle for justice.

Molly's explanation, however, as to why she highlighted that particular quote, though, illustrates her alternate interpretation. She related her choice of Douglass's (1857) words to her desire that her students "have every possible opportunity and choice open to them." Molly explained that her selection of a passage around *struggle* refers to "the hard work, effort and confusion that I expect them to face during math class" and that *progress* refers to the promise of "advancement (that) does come from that work" (Interview, Spring 2014). Indeed, Molly's instruction generally reflected her stated belief that "math is not something you are innately good or bad at, but rather that everyone is expected to and can excel in math through hard work" (Reflection, Summer 2012).

Molly's interpretation of this Douglass (1857) quote is consistent with her interpretation of equity in dominant terms of access and achievement. She consistently and notably demonstrated high expectations for students by teaching for understanding and routinely provided her students with multidimensional opportunities for participation in mathematics. Through her consistency around these dominant equity-directed practices, Molly could be seen as an exemplar of teaching *for* social justice. At the same time, Molly's interpretation of struggle diverted from an understanding of struggle as resistance to injustice. Her framing of progress sidetracked an understanding of progress in terms of concession of power. Instead, her articulations of the importance of effort along with her affirmations around the belief that effort is always rewarded, more closely correspond to various tools of whiteness, like the myths of meritocracy and colorblindness.

Discussion

Aggregating instructional practices from four models of pedagogy in this study has highlighted nested relationships among these pedagogies and provides a mapping of a set of equity-directed practices from these pedagogical models onto Gutiérrez's (2007a) equity framework. Here, I used that mapping and offered an analysis of challenges for White teachers who demonstrate success with dominant equity-directed practices around critical equity-directed practices. The found imbalances could be an outcome of all four dimensions of equity not being equally present in this particular set of observed lessons. Indeed, the practices on the dominant axis are practices that lend themselves to daily instruction, whereas the critical practices arguably could be seen as practices that teachers engage with less regularity. Alternatively, the difficulty for the case teachers with the practices on the critical axis could be attributed to the professional development program, in its attempt to work with teachers on such a wide span of instructional practices from four pedagogical models across a single year. Perhaps teachers develop expertise with the practices on the dominant axis more readily than the practices on the critical axis.

The case teachers identified as White. Readers could reasonably infer that teachers of color enact or struggle with the critical equity-directed practices differently than White teachers. Examining this particular question in depth is beyond the scope of this article, but it is important to note aspects unique to the teaching of the two teachers of color in this group of teachers because they add significant nuance to implications of the cases presented here. Contexts of any kinds were absent in nearly all of the lessons of the two participating teachers of color (both taught first-year algebra). However, different from the case teachers, Harriet and Teresa found ways to connect their instruction to students through language. Teresa drew on strategically using Spanish (her native language) with students, to translate mathematical terms, to check in with students about their emotional state, and to redirect behavior. She regularly referred to her students as her "loves" and typically interacted with every individual student at some point in the lesson. Although her family had immigrated from a different part of the world than her students' families, her regular use of Spanish, especially as used to take care of the students, communicated a sense of familial care and connection.

Harriet's approach at connecting to her students occurred primarily through language as well. Her instruction was entirely in English but she often used youth language in her lessons, reminiscent of the Lee case described in Clark, Badertscher, and Napp (2013). For instance, Harriet would playfully ask the class, "Can I mess with you now?" before presenting more challenging exercises, or challenge that she would like "to mess with you, but not yet" (Lesson, 12/12/12). This language use created a playful atmosphere, and communicated both support and a sense of challenges. These examples suggest that the field expand our understandings of critical equity-directed practices, led through expertise of teachers of color. For example, the practice of creating hybridity between school mathematics and students' out-of-school practices called for in CRP can be expanded beyond the integration of real-world contexts from students' experiences. Fostering hybridity through language (e.g., Lee, 1997; Moschkovich, 2002; Rosebery, Warren, & Conant, 1992) or other kinds of "disruptions" of traditional learning environments (Ma, 2016) are avenues that need further exploration.

One contributing factor to the difficulty for teachers in engaging in the critical equity-directed practices likely resides in the limited emphases of teacher education on developing content knowledge and knowledge of mathematics for teaching among teacher candidates. As its modifier implies, the dominant notion of equity dominates any discussions or interventions around equity in mathematics education, while themes that engage the interplay of race, whiteness, and social justice in the context of the teaching and learning of mathematics are downplayed or sidestepped. Martin (2007) has set criteria for highly qualified teachers for African American children which highlight, identity and power:

> (a) developing deep understanding of the social realities experienced by these students, (b) taking seriously one's role in helping to shape the racial, academic, and mathematics identities of African American learners, (c) conceptualizing mathematics not just as a school subject but as a means to empower African American students to address their social realities, and (d) becoming agents of change who challenge research and policy perspectives that construct African American children as less than ideal learners and in need of being saved or rescued from their blackness. (p. 25)

The capacities outlined by Martin might be viewed as desirable but are not viewed widely as necessary. Martin, however, states unequivocally: "Teachers who are unable, or unwilling, to develop in these ways are not qualified to teach African American students no matter how much mathematics they know" (p. 25). This analysis of teachers who showed success with dominant equity-directed practice yet struggled with practices that relate to identity and power demonstrates that greater attention and focus is required around supporting mathematics teachers in developing critical equity-directed instructional practices. Mathematics teacher preparation needs to de-silence race (Martin, 2009b, 2013), address whiteness and its role in mathematics education (Battey & Leyva, 2016; Martin, 2013), and foster the development of political knowledge among teachers around issues related to schooling, education, identity, power, and mathematics, as well as cultivating their abilities to act on such knowledge (Gutiérrez, 2013b).

Rather than general education courses on race, culture, and diversity, this preparation could approach these areas of emphasis through the lens of mathematics. There is the potential to develop norms among teachers, in content and methods courses, for example, around the posing of critical questions about mathematics content: who created it, to answer what questions, to what ends or purposes, to whose benefit, and to whose demise (Brelias, 2015); whose experiences are reflected and valued in certain mathematics content or tasks and whose interests are ignored (Greer, Mukhopadhyay, Nelson-Barber, & Powell, 2009; Gutiérrez, 2002, 2007a); as well as the related question of why certain mathematics content is taught in schools (Gutiérrez, 2002, 2013a; Nolan, 2009). Content and methods courses could address the "formatting power" of mathematics, how it "colonizes part of reality and reorders it" (Skovsmose, 1994, p. 36), how mathematics gets privileged over other ways of knowing (Borba & Skovsmose, 1997; Gutiérrez, 2013a), and how it is used to intimidate others or to squelch debate (Ewing, 2011). Exploring the ways that mathematics interacts with identity and power through mathematics might be an effective way to further support teachers' development of critical equity-directed practices, rather than taking a general, non-discipline specific approach.

In closing, the findings I presented here add necessary nuances to an over-simplistic, master-narrative about a "pedagogy of poverty" in urban high schools (Haberman, 1991) to instead, identify and better understand challenges for teachers, especially White teachers in hyper-segregated urban schools. The cases of three teachers—competent early-career teachers, who demonstrated excellence at dominant equity practices while struggling with critical equity practices—communicate "teachers' attempts—with all of their flaws and complexity" (Dutro et al., 2008, p. 295). On one hand, we can reaffirm a commitment to an orientation of "perspectives and insights of possibility" (Milner, 2011, p. 88) around teachers and teaching. We can signal the notion of "Yet" from Dweck (2006) and Horn (2012) to choose a growth mindset about teachers and their teaching and interpret demonstrated excellence with the dominant equity-directed instructional practices with optimism regarding further growth. On the other hand, we must challenge any preconceptions that dominant notions of equity alone, through its compelling pillars of access and achievement, will be sufficient to challenge the status quo. Changing society guided by social justice demands that we find ways to better support mathematics teachers in developing and engaging critical notions of equity in and through their teaching.

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Appendix A

Contents and Resources of Professional Development Meetings

Month	Meeting Topic	Associated Resource
September	Levels of cognitive demand	Task sorting activity described in Smith, Stein, Arbaugh, Brown, & Mossgrove (2004)
October	Community walks in school neighborhoods and debrief	Protocol for studying community
November	Improving questioning	Boaler & Humphreys (2005)
December	Launching a mathematical task	Talk by K. Jackson, reading: Jackson et al. (2012)
January	Preparing for teacher presentation at national conference on learning about students' communities to inform teaching	
February	Racial microaggressions	Talk by Battey, about Battey & Leyva (2016)
March	Community based mathematics	Talk by Remillard and Lim, about Ebby et al. (2011)
April	Presentations by "veteran" teachers: 1. Classroom discourse 2. Critical about and with mathematics	Smith & Stein (2011)
May	Share out of classroom observation data and next steps	