Teachable moments

Anticipating and capitalising on mathematical opportunities


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When teachers provide students with the opportunity to engage in discussions there is potential for teachable moments to occur. As teachers it is important to recognise these opportunities and respond to them effectively.

One morning as I (the first author) was preparing my then 7-year-old daughter's lunch (which consisted of a sandwich with the crusts cut off), the following exchange occurred:

Beth: Mum, when you cut my sandwiches today, can you cut them into 4?
Mum: Yes I could, why?
Beth: Because I only had 2 yesterday and I ate those and was still hungry.

As a parent (especially one who might be a teacher of mathematics), do you capitalise on moments like this and use the opportunity to engage in a rich discussion about mathematics—for example, how the area remains the same, regardless of how many parts there are? Would you perhaps go further to convince her by cutting sandwiches into lots of different shapes and sections, and anything else you can find in the kitchen to cut up and demonstrate? As a teacher, do you recognise and embrace similar opportunities in your classroom when they occur, and if you do, how confident are you with capitalising on the mathematics these opportunities can address? This article provides examples of teachable moments that are based on mathematical teaching episodes that we have either taught or observed, followed by discussions on how we might respond to these moments. It is hoped that this article may act as a stimulus for collaborative professional conversations as to whether or not teachable moments can actually be anticipated, and, when they do occur, how they can be recognised and capitalised upon to enhance students' learning.

What are 'teachable moments'?

As teachers, we are constantly looking for ways to provide students with opportunities to engage in purposeful and authentic mathematical experiences. On a daily basis we need to select teaching content and approaches that will motivate and challenge our students and enable teachable moments to occur. We define a teachable moment as a teacher's instantaneous act in response to a student's answer, comment or suggestion; utilised either to address a misconception or to enhance conceptual understanding (Arafeh, Smerdon, & Snow, 2001; Askew, 2005; Clarke et al., 2002) We see teachable moments as being complementary to that which Rowland, Turner, Thwaites, and Huckstep (2009) refer to as 'contingency': in-the-moment actions and interactions in the classroom and the unpredictability of some of these actions.

To further illustrate this, consider the following exchange (adapted from Bishop, 2001).

Teacher: Can you name a fraction between $\frac{1}{2}$ and $\frac{3}{4}$?
Student: $\frac{2}{3}$
Teacher: How do you know?
Student: Because 2 is between 1 and the 3, and on the bottom, the 3 lies between the 2 and the 4.

We would consider this an example of a teachable moment and probably one that the teacher may not have anticipated. As a teacher, how would you respond? Would you follow up on the reasoning, possibly teasing out its limitations? Do you know yourself if it can be used in all cases? What examples could you provide? When there is a gap of two between the denominators and between the numerators, then the student's 'rule' will work:

$$\frac{1}{3} \text{ and } \frac{3}{5} \quad \left(\frac{2}{4}\right)$$

$$\frac{1}{5} \text{ and } \frac{3}{7} \quad \left(\frac{2}{6}\right)$$

In such situations, it is often useful to think of alternative examples in order to challenge students' assumptions, to determine how they might apply their reasoning and/or to demonstrate a lack of generalisability. It would seem reasonable, in this situation, to provide the following examples:

$$\frac{1}{2} \text{ and } \frac{2}{3}, \quad \frac{1}{4} \text{ and } \frac{2}{3}, \quad \frac{1}{2} \text{ and } \frac{3}{5}$$

It is important for the teacher to consider the choice of examples provided. The first two examples may lead to an unexpected response, for example:

$$\frac{1\frac{1}{2}}{2\frac{1}{2}}$$

equating to $\frac{3}{5}$ which is in between $\frac{1}{2}$ and $\frac{2}{3}$ and therefore would serve to support the student's argument. The last example is potentially more useful because if the student's rule is applied, it might result in $\frac{2}{3}$ or $\frac{2}{4}$, which would disprove the rule, as $\frac{2}{3}$ is greater than $\frac{3}{5}$ and $\frac{2}{4}$ is equivalent to $\frac{1}{2}$ rather than greater than $\frac{1}{2}$.

The above exchange illustrates that teaching mathematics is indeed complex and it is often challenging within the 'busyness' of the classroom to respond appropriately to situations such as these. Think about what you would do as a teacher: Would you ignore it and continue? Acknowledge the suggestion, but sideline it? Respond to the suggestion and incorporate it? (Rowland et al., 2009). Muir (2008) found that when faced with similar teachable moments, teachers either 'missed' the opportunity (i.e., no acknowledgement made), incorporated it into the lesson, or actively ignored it. These decisions tended to be influenced by the teacher's mathematical content knowledge, the context and intention of the lesson, and the students themselves, and often impacted upon students' understanding of the mathematics involved.

Identifying teachable moments

The following vignettes contain examples of teachable moments. For each vignette, can you identify the teachable moment/s and think about how you would respond if you were the teacher? Would you sideline the opportunity? Would you incorporate it into the lesson? Would you actively ignore it? Could you have anticipated it occurring?

Vignette 1: Numbers between 4 and 5

Teacher: [Introducing task to class] Your challenge today is to make up a problem that is suitable for anyone in this classroom … you decide which sort of problem you want to do—whether it's a guess and check, whether it's a table combination, whether it's a tree diagram…you have to be able to work out the answer. OK. Then we'll come back in a group and we'll share the problem with everyone else.

Student: Can you make up a 'What am I' problem?

Teacher: Well yes, but you have to make it appropriate to Grade 5 and 6 people, OK, it's got to be age appropriate, so it can't be, "What am I? A number between 4 and 5."
[Some students laugh]

Another student: There's no numbers between 4 and 5 anyway.

In this example, we think that the teacher may have contributed to the student's statement that there are no numbers between 4 and 5 through her statement of "it can't be…a number between 4 and 5". It may be that she deliberately chose that example to see how students would react to address explicitly a commonly held misconception—that fractions and decimals are not numbers—or to develop an understanding that there are an infinite number of numbers between any two whole numbers. Unfortunately we do not know what the teacher's intention was, but we think that such a response should not be left unaddressed, and as teachers, we would definitely not ignore it, but rather incorporate it into the lesson. One way to address this might be to use a number line to show where $\frac{1}{2}$ is positioned, then extend this to show other fractions between 0 and 1, 1 and 2 and so on.

Vignette 2: Zero is not a number

The class is seated on the floor in front of the teacher who is conducting a lesson about square numbers. She asks for students to call out examples of square numbers and the following exchange occurs:

Susan: 16
Teacher: That's right, 16, why is 16 a square number?
Susan: Because 4 times 4 is 16.
Teacher: That's right—any others?
Bob: 4
Teacher: Yes, 4, because two times two is 4.
Jill: 9
Teacher: OK, think of another one. David?
David: Zero.
Teacher: Zero?
David: Zero times zero is zero.
Teacher: Zero? We've never actually had zero as a square number have we?
Sam: Zero isn't even a number.

In this situation the teacher is faced with a dilemma —the focus of the lesson is on square numbers, and examining what constitutes a number would require substantial deviation from the original intention of the lesson. Would this deviation be justified given the likelihood that not addressing the number issue (zero is not even a number) could lead to the development of misconceptions? The situation is further complicated by what definition of square numbers the teacher is using as a reference—technically it does meet the definition of a square number if you consider it to be 'number multiplied by itself', but it cannot pictorially be modelled as other square numbers can (e.g., 1 × 1, 2 × 2). In a follow-up conversation with the teacher she revealed that she avoided acknowledging this as she thought it would distract from the focus of the lesson and that it 'was not going to affect what they had to do'. Given the complex nature of this example, we would be inclined to briefly state that zero is actually a number, but then 'sideline' the opportunity to enable it to be addressed in a thoughtful and explicit teaching episode in the near future.

Vignette 3: How do we get to school?

The following vignette has been created from several interactions of the second author with children, to include a number of potential teachable moments and raise a new consideration—what do you do as a teacher when the discussion is not progressing in the way it was anticipated, and which teachable moment is the focus when faced with more than one?

The Foundation class has been discussing the bar graph (Figure 1) about how the children in another class come to school. They have, for example, volunteered

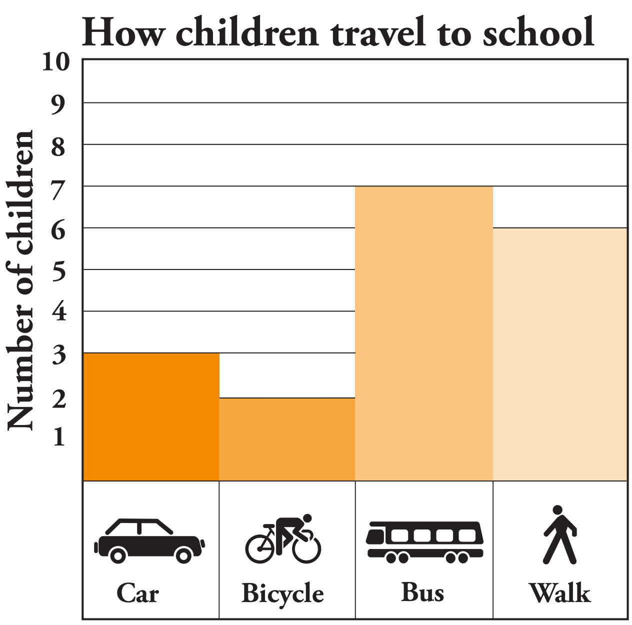

Figure 1. How would a new child come to school?

that more children come by bus than by car, six children walk, and the fewest ride a bicycle. The teacher is hence confident that the children understand the information shown in the graph. Aware that they are able to appreciate and describe the variation in the data, she decides to progress to exploring the expectations that might arise from the data (Watson, in press).

Teacher: Now, can you predict how a new child would come to school?
Henry: Car.
Teacher: Why?
Henry: Because I come to school by car.
Teacher: Okay, Tony?
Tony: I agree, I think by car because he had never been to the school before.
Teacher: Supposing he had been to school for several months now, how do you think he will come to school now?
Tony: Bus.
Teacher: (hoping for reference to the bar chart): Why?
Tony: Because he has learnt where the bus stop is at the school and he has learnt where the bus stop is at his home and then he just goes in the bus down to school from his home.
Teacher: (disappointed, tries another student): What do you think Emma?
Emma: I reckon it would be bus…Because his mum probably goes to work and stuff … And his dad too probably.
Teacher: (tries again): Sophia?
Sophia: I think, maybe car.
Teacher: Why?
Sophia: Because if you went on a bike you might get hit and walking you might get hit, and those two are the safest [pointing to car and bus].
Teacher: (and again)Would anyone else like to say how the new child would get to school?
Susan: I catch the bus and I think the new kid catches the bus.

At this point the teacher is surprised and disappointed that none of the students have referred specifically to the bar chart in justifying their responses. She knows that they are capable of interpreting the message in the bar chart when asked to focus on it. Being a bit desperate but not willing to give up, she asks, "Is there any information from our chart that could help us suggest how the new student might come to school?"

Toby: Well, it would have to be one of the four ways, wouldn't it?

Teacher: Okay, remember we talked about how many students were represented coming each way? So, could we predict from that, which might be the least or most likely way the new student would come?

Sue: Well the most come by bus, so maybe bus. (The teacher heaves a big sigh of relief!)

Tania: But we *don't know* the student, so we can't be sure.

Teacher: Exactly, we can't be certain … There are many things in the world we don't know for certain, but we often use the information we have to make a prediction.

Tony: And if we knew he was coming, we could stand outside to see how he comes and if we were right!!

Teacher: Of course, sometimes we are able to check our predictions, but not always.

Taking into account the context of a Foundation class, the teacher should have anticipated that reasons given for transport to school were likely to be personal rather than data-driven in nature. Nevertheless, she redirected her questions to give opportunities for the students to consider the bar chart (e.g., asking students to hypothesise what might be different if he had been at school for several months) before referring back to it herself. Also, perhaps she could have questioned Sophia more about why she thought car and bus were safest, more subtly refocusing on the bar chart. The exchange shows there were a couple of mathematical opportunities presented and while not necessarily 'misconceptions', future teaching directions could explicitly target interpreting data and what it means to make a prediction and referring to this context as an example.

Final thoughts and conclusions

When teachers provide students with the opportunity to engage in discussions several potential teachable moments are likely to occur. To some extent, it is possible to anticipate and even orchestrate these moments, with task choices and examples often eliciting interesting responses from students. As teachers, we then need to consider how to respond appropriately to these situations; in-the-moment decisions are not often easy within the context of a busy classroom. It is hoped that this article will stimulate teachers to look for teachable moments in their mathematics lessons, respond appropriately, and regularly reflect on their actions. We suggest that teachers need to identify and consider if they ignored, sidelined, or incorporated such moments and ask themselves why they responded in the way they did, and if this was the most effective response.

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References

Arafeh, S., Smerdon, B., & Snow, S. (2001). *Learning from teachable moments: Methodological lessons from the secondary analysis of the TIMSS video study.* Paper presented at the annual American Educational Research Association Conference, April 1–15, 2001, Seattle, WA.

Askew, M. (2005). It ain't (just) what you do: Effective teachers of numeracy. In I. Thompson (Ed.), *Issues in teaching numeracy in primary schools* (pp. 91–102). Berkshire, UK: Open University Press.

Bishop, A. J. (2001). Educating student teachers about values in mathematics education. In F. L. Lin & T. J. Cooney (Eds.), *Making sense of mathematics teacher education* (pp. 233–246). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Clarke, D., Cheeseman, J., Gervasoni, A., Gronn, D., Horne, D., McDonough, A. (2002). *Early numeracy research project final report.* Melbourne: Australian Catholic University.

Muir, T. (2008). "Zero is not a number": Teachable moments and their role in effective teaching of numeracy. In M. Goos, R. Brown, & K. Makar (Eds.), *Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia* (pp. 361–367). Brisbane, Qld: MERGA.

Rowland, T., Turner, F., Thwaites, A., & Huckstep, P. (2009). *Developing primary mathematics teaching.* London: SAGE Publications Ltd.

Watson, J. (in press). Variation and expectation for six-year-olds. In A. Leavy, M. Meletiou-Mavrotheris, & E. Paparistodemou (Eds.), *Statistics in early childhood and primary education: Supporting early statistical and probabilistic thinking.* London: Springer.