pedagogy corner

The Year Puzzle

with
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The Year Puzzle challenge

To use the digits of any given year, in that order, with any legitimate mathematical operation to generate or create every number from 0 to 100. For example (using the year 1972):

$1 + 9 \times 7 - 2 = 62$

$\frac{(1+9)\times 7}{2} = 35$

$1 + 9 + 7 - 2 = 15$

$1 + 9 - 7 - 2 = 1$

$19 + 72 = 81$

$1 \times 97 + 2 = 99$

As readers will have heard me exclaim before, I'm a lesson collector; I love collecting 'interesting' lessons and teasing them apart to find out what makes them 'tick', particularly the pedagogy. What decisions did the teacher make that generated such an interesting and successful learning environment?

This lesson is based around a reasonably well-known number puzzle: The Year Puzzle.

I like the task and when I scrutinise it from various lenses or viewpoints such as, content, connections, problem solving, group work, differentiation and creativity, it seems very rich. There is no particular context (other than a puzzle) and no supportive technology. However, it has succeeded in many classrooms and one of the main reasons for this success is the pedagogy—those strategies used by the teachers which can make such a difference to the effectiveness or otherwise.

Before peering closely into the pedagogy of the lesson some general features of the task I like are:

- It covers—concurrently—in the one task just about every major number idea in the Years 4 to 7 curriculum. Basic operations, order of operations, brackets, powers and square roots, negative numbers and decimals. In addition, ideas from higher years such as factorials and repeated decimals can be introduced. This inter-connectedness is in direct contrast to the partitioned and separate manner in which this content is often treated
- **Differentiation**—multiple levels of success. The openness of the challenge allows it to be tackled (successfully) at various levels of ability. Some students will just use the four operations but will still be able to fill in many entries. Others with a wider range can use their understandings of, say, square roots or negative numbers. Teachers also report this is a good task which can be used anywhere from Year 3 to Year 12.
- **Problem solving, creativity and persistence**. The students are given a worksheet chart from 0 to 100 (see below). The early part of the challenge is quite productive and students fill in lots of entries on the chart. But after a time, gaps appear in the chart. Consider, for example, a student who has all the first 20 entries from 0 to 19, but has not yet created the number 16. The motivation and persistence to find this missing entry is usually very high and changes the focus from lots of combinations of calculations to problem solving. "How could I create 16?" This 'working backwards' strategy considerably raises the level of problem solving.
- **Group work**. Even though students may be working individually there is usually lots of informal discussion between students. "I just got the number 16!" "How did you do that?" says the student in the next desk, and then they exchange ideas, in effect learning from each other as well as explaining and justifying their 'creations'.
- **Change the rules.** For a younger class or struggling students we could loosen the rule about the

digits being in order and allow that "all four digits must be used but can occur in any order". There is little loss of creativity or skill practice.

Each of the above features can operate in a traditional classroom; students at their desks with a worksheet each, teacher in charge, wandering around and checking queries, encouraging sharing in small groups. However, this working atmosphere of the class can be significantly enhanced by some specific pedagogical decisions made by the teacher.

One class I watched, at about Year 5 or 6 level, was using the year 1972 (the teacher's birth year). The teacher deliberately used two of these pedagogical elements:

1. The large whole class recording sheet

The teacher gave each student a copy of the worksheet (see below), and after a few initial examples to clarify the task, students set about filling entries as they created various numbers.

However, the teacher went to the trouble of preparing a very large grid for class results. This made a powerful and significant difference to the classroom dynamic —the working atmosphere. Instead of the teacher-in-charge, the challenge now seemed to be owned by the whole class.

Students were invited to put their creations onto the class recording sheet. Cleverly, the teacher had made sure there was enough space for about three entries for each number from 0 to 100, which means about 300 possible entries. All students had to do was check with another person that their 'creation' was correct. The teacher was not the gatekeeper.

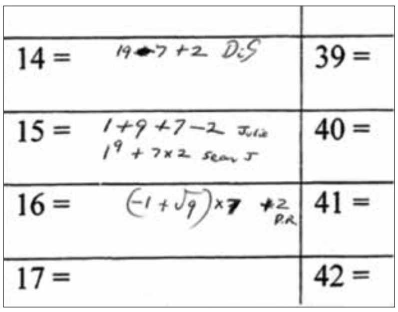

Figure 1.

The effect was immediate—lots of movement and lots of opportunity for each student to be represented, which gave a degree of ownership. The dynamic was that the students now owned the problem as a class; the teachers role was changed.

Another big benefit was that as students wrote their own creations on the grid it gave them a chance to peruse the work of fellow students and to notice interesting methods, to learn from them and often to go and converse.

2. The clinics

Some students in the class were confident in using square roots, while some others were not. For example, one student entered the following on the class chart:

$$16 = \left(-1 + \sqrt{(9)}\right) \times 7 + 2$$

Others noticed and said "What's that 'funny' symbol mean?" Now the teacher was faced with a common dilemma: "Do I stop the whole class and give an explicit teaching piece on square roots—even though I know that is unnecessary for many in the class and I might be inhibiting their progress?"

The solution, which contributed greatly to the working atmosphere, was to invite just those who wanted to be there, to come to a corner of the board and have a mini-lesson on the meaning of square roots. This empowered students to go back to their desks knowing that:

$$\sqrt{(9)} = 3 \quad \text{and also} \quad \sqrt{(9+7)} = 4$$

The effect on the students was immediate and positive. They had acquired a new tool that empowered them to fill in more of their chart.

> The clinic has a common sporting parallel in sports teams such as netball or football, often just those who want or need goal shooting practice are invited to develop their skills at a clinic.

Similar clinic opportunities arose when students wrote on the class chart:

Powers: $1 = 1^{972}$ or $81 = 1^9 + 72$

Decimals: $5 = (.1 + .9) \times 7 - 2$

Factorials: $3! = 3 \times 2 \times 1 = 6$,
hence $\sqrt{(9)}\,! = 6$
$4! = 4 \times 3 \times 2 \times 1 = 24$,
hence $\sqrt{9+7}\,! = 24$

These two pedagogy decisions on one level may seem small but on another they convert a teacher-driven lesson into a more open-ended investigative lesson, and in this way confers much more ownership to the students and to the class as a whole. It can hence be a much more enjoyable working atmosphere for all.

I am indebted to Clarice Lisle, a teacher at Ballarat Grammar, for contributing toward the above observations and commentary. Clarice has also added the following reflective comments from her experiences with the task:

> This activity promotes a growth mindset culture. Students start off discovering the obvious answers such as 1 + 9 + 6 + 4 = (20) etc., eventually reaching their 'skill level threshold'. This is a critical point, where teachers can make a difference with how students perceive themselves as mathematicians; gaining valuable insight into the relationship that the students have with their learning.
>
> Some seek to discover other ways to meet the challenge, whilst others are reluctant to take risks and are undermined by their low levels of confidence and anxieties. This is where I actively promote the idea of 'shared knowledge', ensuring that our learning environment is safe. Everyone has something to offer. Students have the opportunity to be an expert in a particular skill and run a short master class explaining their thinking.
>
> I treat the task with curiosity myself, modelling mathematical inquiry by actively undertaking my own investigation and sharing 'my discoveries'. Somehow, we (the class) can whip up an enthusiasm for the task with each student in pursuit of the elusive 100 equations. (Whilst I don't promote competition, a competitive edge is a source of motivation, especially for some of the high achieving boys).
>
> The critical point is also where you can draw the students' attention to enhancing their outlook towards mathematics, encouraging them to own their learning and monitor the inner dialogue they have with mathematics. "I am getting better at Mathematics" "I don't get this yet, but will work on it." 'Can I see how that works again?" "I can't".
>
> Let students walk alongside the struggle rather than resist it, and acknowledge it as the key to new insights. This is a great activity that could be ongoing throughout the year so that students can reflect on their learning and attitudes.
>
> We look towards being able to explain our thinking with each equation. I do allow the odd one that can't be explained, but remains a mystery.

Note

A copy of the worksheet chart is printed on the following page.

The Year Puzzle

Year =

0 =	25 =	50 =	75 =
1 =	26 =	51 =	76 =
2 =	27 =	52 =	77 =
3 =	28 =	53 =	78 =
4 =	29 =	54 =	79 =
5 =	30 =	55 =	80 =
6 =	31 =	56 =	81 =
7 =	32 =	57 =	82 =
8 =	33 =	58 =	83 =
9 =	34 =	59 =	84 =
10 =	35 =	60 =	85 =
11 =	36 =	61 =	86 =
12 =	37 =	62 =	87 =
13 =	38 =	63 =	88 =
14 =	39 =	64 =	89 =
15 =	40 =	65 =	90 =
16 =	41 =	66 =	91 =
17 =	42 =	67 =	92 =
18 =	43 =	68 =	93 =
19 =	44 =	69 =	94 =
20 =	45 =	70 =	95 =
21 =	46 =	71 =	96 =
22 =	47 =	72 =	97 =
23 =	48 =	73 =	98 =
24 =	49 =	74 =	99 =

100 =