Connecting pre-service teachers with contemporary mathematics practices:

Selecting and sequencing students' work samples

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Teacher educators can use students' work samples to introduce pre-service teachers to the selection and appropriate sequencing of tasks in a classroom. This not only allows PSTs to engage in an authentic act of teaching, but also reveals insights into their mathematical content knowledge.

One of the challenges facing teacher educators is providing our pre-service teachers (PSTs) with authentic experiences that cross the boundaries between Initial Teacher Education (ITE) and the classroom. An additional challenge facing the mathematics teacher educator, is addressing and deepening PSTs' mathematical content knowledge (MCK), which has been raised as a concern in the literature (e.g., Anthony, Cooke, & Muir, 2016). Practice-based strategies have been shown to be effective in bridging the perceived gap between theory and practice (Anthony, et al., 2016), integrating learning of knowledge for teaching (Clarke, Grevholm, & Millman, 2009) and deepening mathematical content knowledge.

This article describes how teacher educators used students' work samples to introduce PSTs to the Smith and Stein (2011) five practices; anticipating, monitoring, selecting, sequencing and connecting. For the purpose of this article we have chosen to focus on selecting and sequencing. In doing so we engaged the PSTs in an act of teaching, which also revealed insights into their own mathematical content knowledge. It is anticipated that our approach could be adapted for use by other teacher educators or as a stimulus in staff meetings by primary teachers and school mathematics leaders.

Work samples

Teachers regularly interpret students' work samples to assess their thinking and to provide evidence of understanding for formative and summative assessment purposes (Anthony, et al., 2016; Yeo, 2011). Representations of learning such as students' work samples, have the potential to support PSTs to learn not just about teaching, but how to use knowledge of teaching in action (Grossman, Hammerness, & McDonald, 2009). The work samples we used with the PSTs were generated from Year 2 students' responses to a multiplication task (see Figure 1).

Learning task

I had a full box of chocolates but someone ate some of the chocolates. The box now looks like this:

How can I work out the number of chocolates I started with?

Figure 1. Chocolate box array task.

The array task originated from the EPMC (Encouraging Persistence, Maintaining Challenge) Project (Sullivan, Walker, Borek, & Rennie, 2015), and was designed to encourage students to think about the properties of arrays. It was relevant for Year 2 students who are expected to recognise and represent multiplication as repeated addition, groups and arrays (Australian Curriculum Assessment and Reporting Authority, 2016).

Figures 2, 3, 4, 5, and 6 show the work samples that were generated by the students and selected by the teacher educators as a basis for the experiences described in this article. When choosing which samples to select from the 24 that the students generated, we were cognisant of the need to select examples that demonstrated particular aspects of multiplicative thinking and/or would provide insights into the PSTs' MCK and pedagogical content knowledge (PCK) (specific explanations of selection in relation to each of the samples is included further in this article).

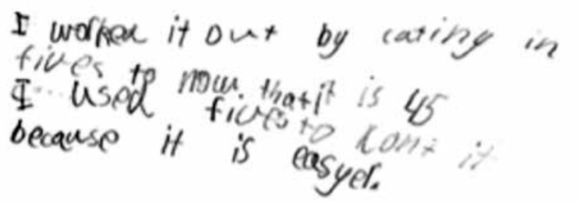

Figure 2. Sample A.

In Figure 2 the student recorded, "I worked it out by cating [counting] in fives to now [know] that it is 45. I used fives to cont [count] it because it is easyer [easier]."

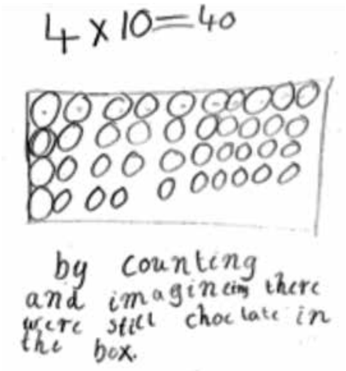

Figure 3: Sample B.

In Figure 3 the student recorded, "by counting and imagineing [imagining] there were still chocolate in the box".

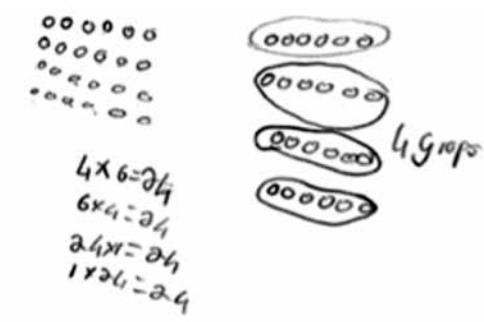

Figure 4. Sample C.

In Figure 4 the student recorded, "4 grops [groups]".

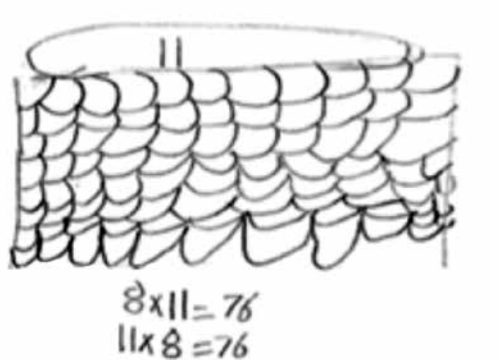

Figure 5. Sample D.

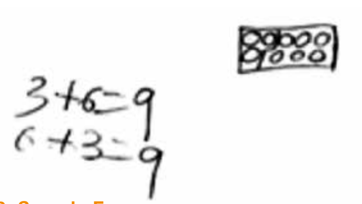

Figure 6. Sample E.

Selecting and sequencing

Anticipating, monitoring, selecting, sequencing and connecting form the five practices as described by Smith and Stein (2011). We see these as productive pedagogies and believe that they promote high-quality discussion and help students to communicate their ideas, enabling teachers to guide them in mathematically sound directions (Smith & Stein, 2011). We wanted in particular to focus on the sequencing and selecting practices and to encourage our PSTs to share and discuss how they might select student work samples, and consider purposeful choices for assisting students to extend their knowledge of arrays. Discussion included whether work samples might be ordered from least to more sophisticated in relation to knowledge and use of the array structure and accompanying symbolic notation. Similar conversations could take place with teachers and their colleagues during professional learning.

In the classroom, the selecting practice typically involves the teacher selecting particular students to share their work with the rest of the class. The selection is strategic, and guided by the mathematical goal for the lesson, along with the teacher's assessment of how each contribution will contribute to that goal. Once selected, the teacher then makes a decision about how to sequence the students' work. Again, this is strategic and depends on a number of factors. For example, the teacher may choose to begin with a student sharing a strategy that the majority of students used in order to make the discussion accessible to as many students as possible.

The teacher may choose, however, to begin with a more concrete representation, such as a drawing or diagram, and then move to more abstract strategies. For example, if there was a common misconception or an error, then the teacher may begin with that work sample before moving to a correct solution. Alternatively, the teacher may wish to have related or contrasting strategies presented one after the other in order to facilitate easier comparison of responses.

Sequencing decisions are not trivial and require considerable mathematical knowledge for teaching, as the key is "to order the work in such a way as to make the mathematics accessible to all students and to build a mathematically coherent story line" (Smith & Stein, 2011, p. 44). It is not simply a matter of beginning with an incorrect response or a naïve response and moving towards a more sophisticated response (although in the classroom this will often occur). It was anticipated that PSTs would have had little exposure to selecting and sequencing in their limited field placements, hence our decision to devote tutorial time to explore these two practices.

Implementing the task with the PSTs

We acknowledge when teaching, selecting and sequencing requires considerable mathematical and pedagogical knowledge from the teacher. As teacher educators, we were interested in how our PSTs would respond if they were responsible for selecting and sequencing three of the five students' responses to the array task. Before reading further, you might like to pause and carefully examine the five student work samples (Figures 2, 3, 4, 5, and 6) and consider:

> If you had to select three students to share their solutions, which work samples would you select and why, and in what order?

During a tutorial class, two different cohorts of third year Bachelor of Education PSTs (N=39) were asked to engage with the five practices, with a focus on the selecting and sequencing. The cohorts had limited experience of assessment related to analysing students' work samples and were not familiar with the five practices. We firstly asked the PSTs to attempt the array task themselves, as we wanted them to think about the mathematics underpinning the task and the types of responses that might be generated. The PSTs engaged with the chocolate box array task and discussed how students in Year 2 may respond. For example, they discussed how students may have visualised the problem; how many chocolates could be in the first row; how students decided on the number of rows in the box of chocolates; and if students used strategies such as skip counting to find the total.

Other suggestions included: using materials to model the situation; drawing an array for the number of rows and number of chocolates in each row.

The PSTs were then shown the Year 2 student responses labelled as work Samples A, B, C, D and E (Figures 2, 3, 4, 5, and 6) and asked, "If you had to select three students to share their solutions, which work samples would you select, why, and in what order?"

How did the PSTs respond to the task?

As expected, there was much variation in terms of first, second and third choices. Table 1 shows the frequency of responses for each. Sample E was the most popular first choice, which was perhaps not surprising as it was arguably the weakest response and we had anticipated that PSTs would be inclined to sequence from weakest to strongest responses. The work sample shows that the student attempted to fill the box by adding six chocolates to the three chocolates shown in the problem and recorded a corresponding number sentence. The response indicates limited understanding of an array or the meaning of multiplication. We included this sample as it shows knowledge of the array structure and the concept of multiplication and were interested to see if the PSTs would notice and articulate this. PSTs' reasons for selecting this first included:

> By explaining this first, you can initially eliminate any misconceptions about the box being that size. Students will then be able to (if not already) recognise that there is empty space and therefore more chocolate at the bottom of the box.

In addition, one PST indicated that she would share this sample first, then "Ask the class if they think this is correct and why it wouldn't be?"

Other reasons included the following:

- By explaining this first, you can initially eliminate the box being that size and the misconception that the rows do not have to be equal.
- There is little evidence of the student's thinking process, so their explanation as to why they came to the conclusion of three plus six is nine would be insightful.
- To help students to understand what an array actually is.
- I just need to know what the student was thinking.

Table 1. Pre-service teachers' number of choices for students' responses (Samples A–E) (n = 39).

Participants	A	B	C	D	E	No choice
First choice	5	11	6	4	13	0
Second choice	5	12	14	4	4	0
Third choice	3	12	16	4	3	1

Sample B (Figure 3) was selected as the first choice by 11 PSTs because it was described as the "strongest response". We included it because it shows the connection between a diagram, equation and explanation of thinking, along with evidence of understanding the row and column structure of an array. One PST noticed the detail, and justified her choice by stating that it "showed three components, a diagram, equation and an explanation of thinking". Other reasons were: "the student could imagine where the other chocolates could be… and drew all the imagined counters," and, "This work sample showed their working out as well as how they came to their answer."

Another PST indicated that:

> I would choose this because it is a good demonstration of how the students could use different strategies to work this out, where this student drew a picture and counted the chocolates he imagined in the box.

Interestingly, the term array was not generally used by the PSTs when justifying their choice for this sample.

Sample C (Figure 4) was selected by six PSTs. The sample shows some understanding of commutativity and symbolic notation of multiplication, and correct number facts for 24. We included this sample because it showed both the array structure and equal group structure representations for multiplication. When the PSTs were interpreting Sample C, at least three PSTs indicated the student was using, "grouping thinking to calculate and show different arrays," rather than understanding that the 'groups-of' idea is different to the array model.

Sample A (Figure 2) shows how the student recorded his thinking without reference to a drawing or an array, suggesting he was visualising how many chocolates might fill the box. We included this sample because it illustrates the transition from concrete to mental imagery. This sample was chosen by five PSTs as their first choice. Examples of PSTs' justifications included, "because it provided evidence of how and why the student came to the solution of 45 for the number of chocolates". One PST suggested that a possible discussion point would be to highlight an inclusion of a drawing of the student's thinking (an array) to enrich the response.

Sample D (Figure 5) was selected by four PSTs as their first choice but was the least common first, second or third choice. This sample shows that the student was able to record an attempt to draw 8 rows of 11 chocolates but she recorded a multiplication number sentence that did not match her array. We included this response because the student made a reasonable attempt at drawing an array but did not attend to the importance of having equal columns and equal rows. In addition, the response did not reflect a realistic solution and we were interested in seeing how the PSTs would react to an incorrect response. A reason for one PST selecting Sample D first, was that she perceived it as being the "weakest response" and:

> … the student is displaying array knowledge and I think it would clue the struggling students into looking at the array.

Another PST chose this sample as the second choice and explained:

> To remind the students of the 'gap' or empty space between chocolates, encouraging everyone to think about how the circles relate to the imaginary chocolates…the need to replicate the way the chocolates would fit into the box.

Reference to an incorrect response was made by one PST who chose Sample B and then Sample C (second choice) but did not select a third choice, and wrote, "The rest are so wrong I would not show them to the class."

Overall, we found that when justifying their choices, the PSTs tended to focus on students' strategies, and some mathematical ideas related to multiplicative thinking and student misconceptions. This was encouraging in that it showed that PSTs were noticing aspects that we considered important and that the work samples provided a stimulus for drawing out these ideas. Their justifications also indicated some limitations in their mathematical content knowledge (e.g., eight of the PSTs' justifications for choosing samples D or E did not identify the student's error or miscalculation)—limitations that we may not have been aware of if we did not engage them in this experience. Similarly, it also revealed insights into their beliefs about particular teaching strategies and approaches that could and should be addressed through their teacher education (e.g., the importance of discussing misconceptions and errors, rather than focusing only on correct responses).

Conclusions and implications

This article has provided an example of how teacher educators can link theory and practice through engaging PSTs in activities that relate directly to how students learn mathematics. The experience showed that PSTs were able to clarify and justify their choices when analysing and sequencing students' work samples. Their justifications provided us as teacher educators with insights into their beliefs and MCK which we could then address in future workshops. We believe this was facilitated through the careful selection of the five work samples and would recommend that if planning to conduct a similar lesson, other teacher educators carefully consider the samples to be shared. The PSTs also reacted positively to the experience, with feedback indicating that it was a worthwhile task in that "it allows us [PSTs] to have a better understanding of what students are learning and what to look for". It is hoped that the ideas in this article will provide other teacher educators with a strategy that can be used with their own PSTs to further their understanding, and gain insights into their MCK across a range of mathematical topics. We also think that the experience could be conducted just as successfully with primary teachers and/or school mathematics leaders as part of a staff meeting or professional learning experience.

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