A "true" story about mathematical reasoning made easy


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Use of the "Is it true?" task is a simple, yet effective way in which to encourage students to reason mathematically and to move towards sophisticated reasoning capabilities.

Reasoning underpins mathematical understanding

Mathematical reasoning is one of the four proficiencies in the *Australian Curriculum: Mathematics* (AC:M) where it is described as: "[the] capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising' (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2017, p. 5); and more recently is spearheading efforts to reform mathematics teaching (Herbert & Bragg, 2017). Reasoning is essential for students to make sense of mathematics (Kilpatrick, Swafford, & Findell, 2001) and described as "the glue that holds everything together, the lodestar that guides learning. One uses it to navigate through the many facts, procedures, concepts, and solution methods and to see that they all fit together in some way, that they make sense" (p. 129). However, previous research has shown that many teachers emphasise explaining, and yet neglect other reasoning actions such as justifying and generalising (Clarke, Clarke, & Sullivan, 2012). Similarly, Loong, Vale, Herbert, Bragg, & Widjaja (2017) reported that many of the primary teachers engaged in their Mathematical Reasoning Professional Learning Research Program possessed a limited understanding of mathematical reasoning and its teaching, prior to observing and discussing demonstration lessons that focussed explicitly on reasoning. Despite the emphasis on reasoning in the AC:M, little is known about teachers' choice of tasks intended to extend the reasoning capabilities of their students.

Mathematical reasoning can be challenging to teach and assess. The first challenge is to choose a task that furthers primary students' capacity to reason. This is not easy. Clarke, Clarke, and Sullivan (2012) asserted that "teachers need support in identifying tasks that prompt reasoning" (p. 32). In this article, we describe how to incorporate reasoning prompts to explore students' misconceptions, then more specifically present a task with a "truth prompt" that has the potential to afford students with the opportunity to reason mathematically. We will explain the rationale for the choice of this task and how it was developed into a lesson with potential to assess reasoning. In addition, examples of children's reasoning will be presented and discussed.

Truth in your classroom

As we will outline below, common misconceptions the students were experiencing in one class have been taken to generate a rich, thought-provoking task that elicits mathematical reasoning simply by utilising an incorrect statement and adding a reasoning prompt. The truth prompts below (see Figure 1) can be applied to any correct or incorrect statements from your students' work samples, or an amalgamation representative of their common misconceptions, to create a rich task.

> Is it just sometimes true, or is it always true?
> When is it true? How do you know?
> How could you demonstrate or show
> or prove that it is true?

Figure 1. Truth prompts (AAMT, 2011).

These prompts can be applied to a wide variety of domains. For example, when exploring geometry with students, the following questions might be posed: "A rhombus is a square. Is it just sometimes true, or is it always true?" or conversely, "A square is a rhombus. Is it just sometimes true, or is it always true?". Similarly, statistics and probability questions might resemble: "When you flip a fair coin ten times in a row, it will land on heads 5 times and tails five times. How could you demonstrate or show or prove that it is true?". In measurement a question might be: "The area of a rectangle does not increase when the perimeter increases. When is it true?"

Is it true?

The "Is it true?" prompt was utilised by a Grade 3 Canadian teacher because of her concern about the misconceptions her children held in regards to place value, particularly when adding two digit numbers that required regrouping. During a collaborative planning meeting, the example given by the teacher was that many students in her class, when presented with the sum 27 + 34 = _____, in written and horizontal format, would record the solution as 511. The children were adding the 7 and 4 together and writing 11, then adding 2 and 3 together and writing 5 in front of the 11: thus resulting in an answer of 511, rather than correctly regrouping to obtain a total of 61. The teacher had tried various, mostly unsuccessful, approaches to overcoming this place-value misconception, and was flummoxed as to how to proceed. Undeterred, an alternative approach was explored which utilised mathematical reasoning prompts to overcome this misconception.

The two goals of this task (presented in Figure 2) are to overcome the students' place value misconceptions and prompt mathematical reasoning through encouraging children to justify their stance. The mathematical misconceptions include grouping, regrouping, and renaming for two-digit addition. The particular reasoning foci are explaining and justifying true statements, exploring and analysing relationships between numerical structures, and forming conjectures and generalisations.

The "Is it true?" task commences with an inaccurate calculation presented to the whole class, namely, 27 + 34 = 511.

> 27 + 34 = 511
> Is it true?
> Why or why not?

Figure 2. Example of "Is it true?" task.

After allowing five to ten minutes of individual time for the students to consider the problem, the teacher invites the class to: "Convince your partner why this statement is true or not". This invitation from the teacher to explain, justify, and convince, encourages oral communication of the students' reasoning. The reasoning language encountered might include; "because", "true", and "so". The oral interaction between peers encourages a consolidation of their thinking and offers an environment to articulate their thoughts succinctly. This shared reasoning time is when peers are more likely to challenge the thinking of their partners, thus facilitating the opportunity to justify, create new knowledge and understandings. This paired time is followed by a whole class discussion about all students' reasoning so that a debate can occur. The final aspect of the task includes students reflecting upon and consolidating their reasoning. Students are invited to articulate any follow-up questions for further exploration in subsequent lessons (Bragg, Loong, Widjaja, Vale, & Herbert, 2015).

"Is it true?" Enabling and extending prompts

Enabling and extending prompts allow teachers to differentiate students' experience of a task. For those students who require assistance in accessing the initial "Is it true?" task, the enabling prompts simplify the complexity of the task so that the student may respond successfully to these prompts and subsequently the original task (Sullivan, Mousley, & Jorgensen, 2009). An example of reducing the complexity is via simplifying the numbers in the problem to: "27 + 4 = 211. Is this true? Why or why not?". Adding a single-digit number, 4, to 27 will illuminate the implausibility of the original statement. Alternatively, the teacher may reduce the cognitive demand of the task to help their students' understand how the numbers change and stay the same when grouping and regrouping through the following prompt:

- What does 27 + 30 = ? 27 + 31 = ? 27 + 32 = ? 27 + 33 = ? 27 + 34 = ?
- What pattern do you notice?

An analysis of the pattern increasing by one, (57, 58, 59, 60, 61) will highlight the inconsistency of the total of 27 + 34 resulting in 511.

Manipulatives are recommended for students who continue to struggle with understanding the incorrect assertion in the statement. If, from the previous pattern-noticing prompt, a student answers 57, 58, 59, 510, 511 to the pattern, then number lines or hundreds chart are suitable support materials to assist the child in observing the pattern of the base-ten number system. The pattern-noticing prompt would be stated again, "What patterns do you notice on the hundreds chart? How is this different to the previous pattern?" Alternatively, the teacher may ask students to model both 27 and 4 with materials such as MAB, unifix blocks, etc., and count the total of the two merged sets of blocks. In the case of 27 + 4 = 211, the visual representation of the 31 blocks merged blocks will assist students in understanding that the sum could not reasonably be 211. The teacher would encourage the students to explain their thinking.

Extending prompts are used when students require more challenge from this task. The following extending prompts are suitable for eliciting reasoning at a deeper level:

- Explain why someone might mistakenly think this problem is true.
- How might you model this problem to help a friend who thinks this is true?
- Explain how estimation might help you with this problem using a real-life example.

Alternatively, extending questions which include the reasoning prompt and continue to explore the misconceptions of grouping and regrouping in place value at a more complex level are:

- How might you alter one of the numbers to make this addition problem equal 511?
- 270 + 340 = 511
  Is it true? Why or why not?
- 97 + 34 = 1211
  Is it true? Why or why not?

The rationale for the choice of these numbers was to explore the complexity in regrouping with tens.

Exploring place value in conjunction with subtraction is an alternative route to aiding reasoning. For example:

- 511 – 34 = 27
  Is it true? Why or why not?
- 34 – 27 = 13
  Is it true? Why or why not?

For students who can perform the calculation correctly but reason incorrectly, the following suitable reasoning prompts might be:

- Convince me your solution is correct;
- Explain how that process works.

When upper primary students are connecting decimal numbers with the metric system and undertaking calculations using the four operations (ACARA, 2017), these following equations combined with "Is it true? Why or why not?" are beneficial in exploring these concepts whilst supporting reasoning:

- 2.7 + 3.4 = 5.11
  Is it true? Why or why not?
- 5.11 – 3.4 = 2.7
  Is it true? Why or why not?

Similarly, these question prompts could be reversed to create subtraction examples:

1.29 + 3.36 = 4.515
3.26 + 6.82 = 9.108

Assessing reasoning

The next challenge facing teachers is how to assess reasoning. In assessing the task "Is it true?", teachers would look for attempts by the students to justify their thinking and verify that the statement is incorrect by drawing on their prior knowledge of place value. More sophisticated reasoning is apparent in students who use a correct logical argument and employ words such as "because", "so", and "therefore". For example, a student might respond, "No, it is not true because 20 + 30 = 50, 7 + 4 = 11, so 50 + 11 = 61". Students who display a broad knowledge of place value, may respond with, "We need to regroup the numbers in any column, if they add to more than nine. Therefore, 7 plus 4 equals 11 ones, which is 1 ten and 1 one. So we now add the 1 ten to the 2 tens and 3 tens to make 6 tens." A student responding in this way is noticing and exploring relationships within numerical structures of patterns. We have considered a particular task which provides opportunities to encourage students' reasoning and its assessment. This approach, however, could be utilised for many different tasks as illustrated above in the section 'Truth in your classroom'.

Conclusion

In this paper, we introduced a simple and effective way of incorporating reasoning prompts into the classroom to overcome mathematical misconceptions. The "Is it true?" task was a teacher-generated task developed to address both mathematical content and the reasoning proficiency, particularly the notion of truth. The opportunity to check the truth of a statement, and detect and rectify untrue conjectures, is part of the incremental steps to forming a logical argument, completing that argument, and, ultimately, providing a watertight argument. This growing sophistication in students' reasoning capability is achievable through teachers offering a systematic approach of support (Stacey, 2012). This task is one such way to offer effective support to students to grow their reasoning capabilities.

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