

LEARNING TO THINK AS AN EFFECTIVE MATHEMATICS TEACHER: TEACHER EDUCATOR IMPACTS ON CURRICULUM KNOWLEDGE AND LEARNING TO TEACH

MARY F. HILL

LEXIE GRUDNOFF

AND FIONA ELL

THE UNIVERSITY OF AUCKLAND

ABSTRACT:

For improved student outcomes, teachers must integrate their knowledge about the curriculum, about how to teach it effectively and how to assess whether students have learnt it. Therefore, key tasks for initial teacher education are building curriculum knowledge, learning contextually appropriate ways to teach and how to use assessment for learning within each curriculum area. We interviewed four recent graduates who had demonstrated contrasting learning-to-teach profiles in an earlier quantitative study to understand how teacher education had assisted each of them to notice, recognise and respond to individual children's numeracy strategies. Their four stories indicate that teacher educators need to notice, recognise and respond to their student teachers with respect to their existing curriculum and pedagogical knowledge.

KEYWORDS:

teacher education; numeracy teaching; teacher professional learning; prior knowledge

In 2007, our teacher education research team began to collect information about our primary student teachers' numeracy knowledge and their ideas about teaching children mathematics. The teacher candidates (n=64) completed a curriculum-relevant assessment of their content knowledge of mathematics, and responded to three questions about an example of children's numeracy thinking (See Appendix A) (Ell, Aitken, Grudnoff, & Hill, 2007). The results of these assessments, at entry to and exit from the teacher education programme, yielded patterns suggestive of a link between higher levels of personal curriculum content knowledge and responses to children's work which were like those of experienced, effective teachers (Ell, Aitken, Grudnoff, Hill, & LeFevre, 2008). While the surveys showed that the teacher education programme (TEP) had improved the student teachers' ability to respond to children in mathematically appropriate ways, the summary nature of the tasks did not allow for more individual accounts of change as a result of teacher education. We therefore followed four of the participants from the 2007 study to investigate the sources of their knowledge, beliefs and practices in order to provide us with more in-depth information about how our student teachers experienced learning to teach mathematics. When interviewed they were halfway through their first year as beginning teachers and were purposively chosen as each was representative of patterns of responses within the survey data. We wanted to explore what may have been the particular influences on their learning before and during their teacher preparation programme. The key question was: How do individual student teachers perceive and attribute the changes that occurred during teacher preparation in their personal content knowledge and their ability to interpret children's work and suggest next steps for children's learning?

THEORETICAL PERSPECTIVES

Each student teacher enters a TEP with unique prior experiences, dispositions and learning histories. Research on children's learning strongly suggests that these are significant factors in determining classroom learning (Alton-Lee, 2003) and research on teacher education is beginning to explore the role that these factors might play in determining how teachers learn (Hammerness, Darling-Hammond, & Bransford, 2005; Levin & He, 2008; Richardson, 1996, 2003; Rosaen, Lundeberg, Cooper, Fritzen, & Terpstra, 2008)

Research into teachers' personal theories and beliefs, and the knowledge they derive from their experiences, indicates that such beliefs and theories can and do influence their practices and the opportunities their students have for learning (Clandinin, 1986; Elbaz, 1981; D. M. Levin, Hammer, & Coffey, 2009; Schwab, 1983). Furthermore, prospective teachers' beliefs and knowledge influence what is learnt during teacher preparation by acting as a filter through which student teachers acquire and interpret new knowledge (Levin & He, 2008; Richardson, 1996, 2003). The sources of these beliefs, however, have rarely been investigated (Levin & He, 2008). Where they have, they are not often linked to longitudinal inquiries into how holding such beliefs might influence learning to be a teacher. Levin and He (2008) reported a large qualitative study that investigated the sources of preservice teachers' personal practical theories and found that although their family background and own educational backgrounds were the source of about a third of the personal practical theories, 66% of their theories "had their foundation in either the explicit curriculum of their teacher education program or the(ir) ... field experiences" (p. 62). An important finding of Levin and He's (2008) was that relatively few of the personal practical theories of these preservice teachers seemed to have originated from children's views or ideas.

Learning to think and act in ways that promote the learning of students is a key intended outcome of teacher education. Student teachers should leave teacher education better able to understand, teach, assess and plan for the children in their care than they could when they arrived. We theorised that student teachers draw on their personal history as learners as they try to understand the teaching/ learning process and that their beliefs, knowledge and skills will be reshaped and reconsidered in the light of their new role as teacher. For example, in mathematics,

there is a difference between knowing how to add numbers and knowing how to teach someone else how to add numbers. As a teacher one needs to draw on one's own knowledge of adding, while at the same time coming to see adding in new ways—from the perspective of the difficulties a learner may face in learning to add numbers (Ball, Thames, & Phelps, 2008).

We agree with Edwards (2009), Hager (2004), Beach (1999) and others that becoming a teacher is not simply a matter of learning about curriculum and how to teach it, and applying this learning in classrooms. Rather, we view teaching as a process of informed decision making in complex situations and argue we need to prepare teachers who can read that complexity and respond to it resourcefully (Edwards, 2009). Building on Edwards' notion of resourceful practice, we theorise that in order to enhance outcomes for diverse learners, student teachers need to take mindful action in practice settings based on knowing what to teach, knowing how they might go about teaching, knowing why they might proceed in the way they have decided, and knowing who they are teaching and who might help them in the teaching process (see Figure 1).

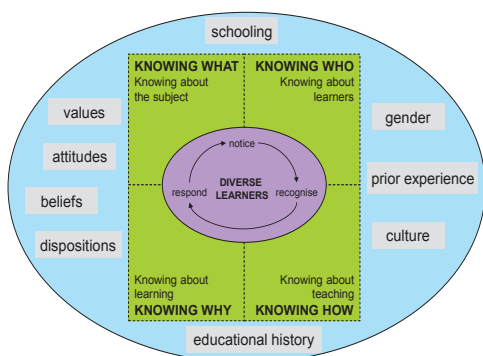


Figure 1: Thinking for Effective Teaching (TFET) model

Holding knowledge of what, who, how and why in a way that leads to effective teacher actions we termed “thinking for effective teaching” (TFET). Effective teaching is characterized as noticing what children are doing, recognising what it means in terms of their learning and responding with an appropriate action.

This idea arises from the literature on both mathematics education and on assessment for learning (Bell & Cowie, 1999; Fraivallig, Murphy, & Fuson, 1999). This characterisation avoids particular teaching actions being privileged as necessarily effective by making the three actions contextually defined. McNaughton and Lai (2009) argue similarly from their extensive research in reading comprehension teaching that teachers need to act as “adaptive experts”. Adaptive experts are highly skilled and deeply knowledgeable, having not only a base of relevant knowledge, but also strategic practices and metacognitive awareness of this practice.

The knowledge needed for adaptive expertise in teaching mathematics is termed “mathematics content knowledge for teaching” by Ball and colleagues (Ball et al., 2008). This knowledge underpins teachers’ ability to differentiate between children’s responses and to shape classroom dialogue towards mathematical argumentation (Walshaw & Anthony, 2008). In representing the teacher education process, the TFET model (above) posits that change in the types of knowing should be observed as the TEP proceeds. In addition, the types of knowledge should become more related as they are brought to bear on teaching situations with children. Using the example of addition, teacher candidates may hold knowledge about addition, and may have informally coached a child with homework, but may not have linked together the conceptual difficulties the child had with the mathematical nature of the addition task. Teacher education, according to the TFET model, should bring these experiences together in a way that enables teacher candidates to notice, recognise and respond to learners more effectively. In order to investigate how teacher education influences teacher learning, we decided to interview some of our

graduates who took part in the earlier 2007 study.

METHOD

Case study methodology was used to provide in-depth information that might cast light on the complexities of the process of becoming a teacher, through considering individual stories (Levin et al., 2009). The desire to understand, at an individual level, how the teacher education process affects teacher learning led us to use an interpretive approach to inquire about the participants in teacher education themselves, about their backgrounds and how they understood their practice (Borko, Whitcomb, & Byrnes, 2008).

PARTICIPANTS

The four participants (Cara, Natalie, Matt and Zoe) had completed their one-year, graduate-level teacher education programme and had been part of our initial research project (Ell et al., 2007, 2008). They all had degree-level qualifications which were completed prior to beginning teacher education and were invited purposively due to their results on the entry and exit tasks in the earlier survey study (see Table 1 below).

In the survey study (Ell et al., 2008), student teachers were rated as high (H), average (A) or low (L) in their personal mathematics content knowledge. They were also rated as being able to recognise (YES) or not able to recognise (NO) the features that indicated children’s numeracy thinking on the numeracy task (see Appendix A). The student teachers’ suggestions for taking the learning on were compared with those of expert/effective teachers. In this their answers were rated as expert-like (E) or not expert-like (NE). Table 1 summarises the performance of the four interview candidates on these measures.

DATA SOURCES

A semi-structured interview explicitly sought to use our TFET model (Figure 1)

Table 1: Results from the mathematics content assessment and the recognising and responding assessment for the four interview participants, at entry to and exit from the teacher education programme.

	Mathematics Content at entry	Mathematics Content at exit	Recognising at entry	Recognising at exit	Responding at entry	Responding at exit
Zoe	Low	Low	YES	YES	Non Expert	Non Expert
Natalie	Low	High	NO	YES	Non Expert	Expert
Cara	Low	Low	NO	YES	Non Expert	Expert
Matt	High	High	YES	YES	Non Expert	Expert

as a lens to view the participants' stories of learning to teach mathematics. The example of children's responses to the mathematics task from the earlier study (Appendix A) was used as a prompt for discussion of the key areas. At the time of the interview, all participants were employed in different schools as full-time classroom teachers with responsibility for their own classes. They had been teaching for six months. Each participant was interviewed at his/her school outside classroom hours. Two interviewers undertook two of the interviews. Each interviewer conducted one of the interviews alone. Interviews were recorded with the participants' consent, and subsequently transcribed. Each interview was approximately 45 minutes long.

ANALYSIS PROCEDURES

The interview transcripts were read independently by the first two authors. Then the researchers compared their coding, and categories were identified to describe the content of the interviews. Although some of the categories were strongly related to the TFET model, due to the fact that the interview questions were based on the model, further categories were also identified. The first author then closely coded all of the transcripts and sent them to the second author for recoding. Where there were discrepancies these were compared and discussed. Reliability in coding was achieved in this negotiated sense.

FINDINGS

The interview helped us to understand how the four "knowings" in the TEFT model had (or had not) changed and the extent to which these participants drew on their prior experiences and teacher education to make sense of their learning. We were also interested to understand how prior experiences and learning to teach had impacted the capability to notice, recognise and respond to individual learners differentially and appropriately. The findings are presented as four stories of change and then discussed in light of the model and the key factors that participants identified as contributing to their content knowledge and ability to recognise needs and respond to learners in mathematics.

MATT'S STORY

Matt had high personal mathematics content knowledge and could recognise the significant features children's work at entry to teacher education. His ability to respond like an expert teacher improved during the teacher education mathematics courses. Matt shows the overall cohort pattern of a link between high personal content knowledge and recognising and responding like an expert teacher at exit from the programme.

Matt thought that he had learnt to be more of a facilitator and questioner from his TEP. He had learnt to ask children to explain their ideas. Before entering teacher education he had believed that as a teacher, "you just teach".

He attributed this change from a transmission to a facilitative view to his university and practicum experiences. Learning about assessment for learning and the importance of understanding learners' ideas in math education had also changed Matt's approach to teaching. Matt thought that the TEP was focused on teaching facilitative approaches.

When asked what he had learnt in terms of mathematics content from teacher education, Matt commented that he didn't think that the courses covered the curriculum strands, and that he thought the content coverage was "a bit random". He explained that he had already gained his mathematics content knowledge before coming into teacher education and with this background he could work things out for himself.

Matt regarded children's personal freedom to learn as important. He believed that children all have different ways of understanding mathematics. He felt this view had changed as a result of teacher education. Before teacher education he thought he would have seen children's potential as fixed, but he had shifted this position to one where potential is unlimited for every child.

[Before learning to teach] I might not have considered that every child had full potential. Being a teacher ... I don't notice the cap on the potential of students ... they can keep going and going and going and there is no stopping them.

Matt explained that as a beginning teacher he was still transforming his thinking.

I guess my experiences this year overwhelm what I learnt from last year.... it takes it over... Like if I were to explain it, I would draw on my experience of teaching the kids rather than what I was taught last year.

When we asked again about the children's responses in the numeracy task (Appendix A), Matt was clear that it certainly does matter that they solved the equation in different ways.

Matt had clearly learnt from both teacher education and his first-year teaching experiences. However, remnants of his earlier beliefs are still evident in his ideas about teaching. For example, while Matt states that the TEP led him to change his views about children's potential to learn, and assessment, his views about the need for direct instruction and teaching basic facts remained. In fact, he saw some of the approaches taught at university to be what he termed a "crusade". Although Matt could talk about inquiry approaches, facilitative methods and the numeracy strategies, he still believed that traditional teaching methods (drill and repetition, text book exercises, for example) are also valuable.

ZOE'S STORY

As noted in Table 1, Zoe had a low personal mathematics content score both before and at the conclusion of the TEP, indicating that her mathematical content knowledge in the areas assessed had not been influenced by the teacher education process. She recognised children's needs in the numeracy exemplar task before she began teacher education, but both before and after the programme was not able to make expert-like suggestions about "where to next" for the children. The teacher education mathematics courses appear to have had little effect for Zoe.

When interviewed, Zoe was teaching 6-year-old students at a high-decile school. When asked how her mathematics courses in her TEP had affected her mathematics content knowledge, Zoe responded, "very little". She didn't feel very strong in mathematics and recalled learning about the numeracy strategy and "maybe some teaching ideas". In fact, Zoe was quite critical of her mathematics teacher education experiences.

I had two very different lecturers [in] math. I only really learnt early numeracy strategy—maybe some teaching ideas—there were huge gaps. I thought having only one assignment worth 100% was unfair. My grandma passed away, I had 3 jobs, so—I just scraped through.

Zoe stated that she was "very scared" to be teaching children mathematics as a beginning teacher. Six months into the year she still didn't feel she was giving her best to the children. Zoe thought that she had probably learnt about how children learn mathematics since being a full-time teacher. She stated that she had learnt

as she'd gone along, reflecting on changes in her daily work and thinking, "how can I make it better?" She also explained that as a beginning teacher, goals were set regularly with her supervising teacher to check on her own learning and teaching.

Zoe didn't think that she had changed her thinking about the children's responses to the numeracy task (Appendix A) during the teacher education programme. When asked to comment on the children's responses during the interview, Zoe noticed that all three children were at different stages. She stated that it didn't matter that they were all at different stages of learning because "hopefully they would be put into groups". She said:

The first one is counting all, so I'd start to imagine with them by hiding objects. The middle one, I'd be pushing them to basic facts and looking for other/easier ways to do it. The last, I'd be bumping problems up, up to 100 and perhaps even addition and subtraction or giving them problems in a different context.

By her own admission, mathematics is not Zoe's strength and she had not found her teacher education experiences helpful in mathematics. Although it appears that Zoe is able to notice that students' responses to the numeracy task are different and important, her teaching suggestions made during the interview were still brief. Despite her reflections on her teaching in mathematics, Zoe appears to not be thinking deeply about the students' needs.

CARA'S STORY

Cara entered and left the programme with low levels of mathematics content knowledge. However, like a group of her peers, she changed in her ability to recognise and respond to children's answers to the numeracy task over the course of the programme. This was contrary to the overall cohort pattern where higher personal content knowledge is related to expert recognising and responding.

When interviewed Cara was teaching 6-year-olds at a high-decile school. Cara believed that her international experiences, volunteering and work teaching English had strongly influenced her to become a teacher and had impacted her teacher education experiences as well. She put it like this: "... being born and raised in an international environment and then also doing anthropology, I suppose I realise that there is not just one way of being".

Although Cara thought the TEP had not challenged her perceptions, looking back she realised that she had changed her thinking from a transmission view of teaching to one more focused on attending to students and their learning. Cara thought that a variety of experiences in teacher education had contributed to this change of view, although she still agreed with her mother that there is "some stuff that the kids probably need to know". Cara believed that she had not learnt new content in mathematics from her teacher education courses. She did think, however, that the teacher education mathematics courses had introduced her to how to teach mathematics:

It was an introduction to the numeracy strategy and it was "here are the pink books, this is how you use them. Here's the test, this is how you use it". It was more of a "how to navigate your way through the pink books and on the website".

Since she had been teaching Cara had been involved in a significant amount of professional development in numeracy. She thought this had been a strong influence on her teaching, particularly the use of manipulative materials and imaging. But she also commented that teaching brings you down to reality in many ways.

Like you go into [the lecturer's] lesson and you come out thinking "oh my gosh that sounds so amazing and we will totally do that" ... and then you get to the classroom and you think "no that's just not going to happen".

In terms of mathematics Cara believed that with every topic her maths content knowledge and how to teach it was increasing. She felt that she was "staying safe" by using the numeracy books and systems she had learnt in teacher education.

When asked about the children's responses to the numeracy task (Appendix A), Cara explained that it does matter for the teacher that they are at different levels.

No, it doesn't matter that they did it in different ways but it does show those different levels—mathematical strategies. So you have your "count all" [children] and then your "count on" [children] and then you have your "part/whole" [children]. And well it would also mean that you can't teach them in the same way.

Cara went on to explain the teaching strategies she might use with each group, demonstrating that she understood that knowing how the children were solving the task was important for differentiating the teaching.

Cara's story demonstrates how the teacher's level of mathematics understanding affects teaching competency. The numeracy task that was used to gauge teacher candidates' ability to notice, recognise and respond to children's learning was deliberately set at a very simple level; children tend to be learning about these ideas in their first three years at school. Although Cara's content knowledge was still rated as low by the exit test, she did have a part/whole understanding of the number system and thus was able to notice, recognise and respond differentially in this context.

Cara's understandings of diversity and the need to differentiate for each learner existed before entering the teacher education programme but have been strengthened as a result of it. She had changed her thinking from a strictly transmission approach to a more facilitative one (as she termed it) where she sought to find out what individual learners know and can do as a basis for her teaching.

NATALIE'S STORY

Natalie began teacher education with low personal mathematics content knowledge, an inability to recognise the key features of children's work in the numeracy task and non-expert suggestions for teaching (see Table 1). At the end of the TEP, however, Natalie had high personal content knowledge on the exit test, and recognised and responded to the children's answers like an expert teacher. In contrast to the other participants, Natalie's results suggest a great deal of learning during teacher education, particularly as a result of the mathematics courses.

When interviewed, Natalie was teaching 6-year-olds in a mid-decile school. Natalie was keen to talk about how her mathematics understanding had changed during the course of her TEP. It is not exaggerating to say that she believed coming to understand the part/whole nature of numbers during the TEP had been an extremely dramatic change for her.

So there was this big exciting thing and I came home and it was like "did you know you could do all this with numbers?" And everyone is like "yah". But no one ever told me this; this is so exciting! But it was, it was really exciting for me, the day I discovered that you could do all this other stuff with numbers like breaking them into pieces

because I always thought numbers had to be these solid-like units that you just move, like did stuff to.

Natalie has an MSc in genetics, but she was still learning about how the number system works and about the various numeration strategies children need in order to learn and use mathematics efficiently.

Well I never realised, the entire way through doing two science degrees and all the way through school and everything, I never realised that you could do that with numbers ... It was just like this huge revelation. It was so exciting.

Explaining how she had managed to successfully complete two degrees that required extensive use of statistics, Natalie admitted that she had found the mathematics really hard. She had resorted to getting her flatmate to explain it to her. Throughout her schooling and tertiary study, getting "the right answer" had been Natalie's main strategy. She attributed her new learning to one of her teacher education lecturers in particular. Natalie believed that having a lecturer with mathematical content knowledge was important.

Discovering that numbers could be manipulated in these ways also changed Natalie's beliefs and attitudes about children's learning and her approach to teaching. She explained that she had realised that children learn in different ways. Due to her dawning understanding about the number system, Natalie now believed that a variety of teaching approaches are necessary to ensure all children learn effectively. For example,

Some children can do it without [materials] because they can just understand the theoretical structures and frameworks. But then other children really need that. Like the visual learners really need that visual representation.

She also described how courses about pedagogy and assessment had changed her views about differentiating her teaching. She gave examples of catering for individual learners by building on their strengths and interests. She had also learnt about motivation, or as she explained it, "hooks": "They need to have hooks. Every child needs a hook to get connected to you and to connect to whatever you are teaching them."

However, Natalie still didn't feel confident about teaching mathematics at the end of her TEP. She was also sceptical about what she had learnt on her practicum placements.

I learnt a little bit of it on practicum I guess... To some extent but in others ways I don't think that I'm entirely convinced that practicum was really all that useful.... it was more about how to

get the kind of ideas that I had out in "kid" language and that's really what practicum was for me.

Natalie reported that she was learning on the job through trial and error, reflecting on her practice, attending professional development courses and watching other teachers. Most of all Natalie believed that any teaching approach had to fit in with the way that she, the teacher, thinks.

When asked about how she would respond now to each of the students in the numeracy task (Appendix A), Natalie explained that she would value each of their responses and then work with them differentially according to the strategies and understandings they still need to develop. She then talked the interviewer through a range of possible responses for each level of solving the task and explained how she would take the learner on from there. She related this to children in her own class now and explained how she was taking them forward in their numeracy learning.

Natalie explained in the interview that it is important to understand the learning processes children use in mathematics. She credited her "paradigm" shift in understanding the part/whole nature of numbers as a major reason why she was now able to teach these ideas and strategies effectively. She definitely saw this as coming from the TEP but found it difficult to know exactly where other elements of her knowledge and practice had come from.

Natalie's story demonstrates how powerful teacher education courses can be when they address the mathematics content and strategy knowledge of the student teacher. Natalie came into teacher education with some critical gaps in her mathematics knowledge. Luckily, one mathematics teacher educator focused on the very aspects she needed to understand to teach early numeracy as an expert. Natalie had a rich background in science and mathematics which helped her to recognise the potential of the numeracy and algebra ideas that she was introduced to in her mathematics courses, which appear to have directly influenced the changes we see in her entry and exit results. Although Natalie says she is not able to state explicitly what she learnt from the TEP or from beginning teaching, she gave many examples in her interview of how the TEP had, in fact, caused her to rethink both her knowledge and her beliefs.

DISCUSSION

As these four cases demonstrate, learning to be a teacher is intimately connected with the multiple factors noted in the outside ring in the Thinking for Effective Teaching (TFET) model (Figure 1). This finding is consistent with other evidence (Edwards, 2009; Grudnoff, 2008; Levin & He, 2008). All four interviewees explicitly linked their educational history, prior experiences, family background, beliefs and dispositions to their current views about learners, learning and teaching. Because these views about learners, learning and teaching ultimately guide their actions in the classroom (Chant, 2002; Chant, Heafner, & Bennett, 2004; Clandinin, 1986; Levin & He, 2008) it is argued that teacher educators need to access and address student teachers' knowledge and views in order to build the knowledge and beliefs novice teachers need for effective teaching.

In contrast with Zoe, Natalie described how her mathematics lecturer had noticed, recognised and responded to her lack of understanding of the part/whole nature of the number system. Natalie attributed the shift in her ability to respond to children's ideas about mathematics to her mathematics lecturer addressing her existing theories. Although Natalie's was the most dramatic shift, she, Matt and Cara all reported that the specific focus on their existing ideas about teaching mathematics had impacted upon their ability to respond to children's explanations of their problem-solving strategies.

The TFET model theorises that learning about knowing what; knowing who; knowing how; and, knowing why is critical to developing each student teachers' ability to notice, recognise and respond appropriately to learners. Consistent with other evidence (Ell et al., 2007, 2008; Levin & He, 2008), the interview findings supported this theory.

KNOWING WHAT

Although "mere content knowledge is likely to be as useless pedagogically as content-free skill" (Shulman, 1987), content-specific knowledge and understanding is essential for teaching (Ball et al., 2008). In Ball et al.'s terms common content knowledge encompasses the kinds of mathematical knowledge and understanding that people need to live successfully in the various spheres of private, social

and civil life (Walshaw & Anthony, 2008). With this knowledge, teachers are able to plan, assess and carry out instruction in classroom settings. But as well as common content knowledge, Ball et al. (2008) argue that a second domain, specialized content knowledge, which is unique to teaching, is required. Ball and colleagues demonstrate that this special knowledge is implemented when, for example, teachers look for patterns in student errors or size up whether a non-standard teaching approach would work in a particular circumstance. However, to understand and use such specialised content knowledge, common content knowledge is still essential.

Natalie was the only graduate of the four interviewed whose level of mathematics content knowledge changed significantly during the teacher education programme. She was acutely aware of this and described how her lack of part/whole number knowledge had handicapped her throughout her education. Her story is especially revealing in terms of how addressing individual student teachers' mathematical knowledge and understanding makes a difference to the effectiveness of their teaching. Consistent with evidence about enhancing children's achievement (Alton-Lee, 2003; Bell & Cowie, 1999; Black & Wiliam, 1998), these findings indicate that when student teachers' personal mathematics content knowledge is noticed, recognised and responded to appropriately by teacher educators, mathematics knowledge is enhanced. And having strong mathematical content and pedagogical knowledge can have dramatic affects on how they themselves then teach. It is of concern that two of the four graduates interviewed left the programme with low mathematics content knowledge and three indicated that the mathematics courses had not affected their content knowledge at all.

KNOWING HOW

Our participants reported learning how to teach was a focus of the mathematics courses, the TEP and the field experiences. All gave examples of learning ways to teach in their mathematics courses. Cara described learning to use the numeracy books in some detail. Zoe, Cara and Natalie all thought that they had learnt teaching activities from their mathematics classes and from field experiences, and Matt learnt to be a facilitator.

In addition to becoming familiar with teaching guides, our interview evidence indicated that three of the four participants had learnt to attend to students' conceptions. Cara and Matt described this as learning about "facilitating" learning. Both had entered the programme with more of a transmission view of learning but felt that they had learnt about getting children to ask questions, listening to children as they explain their ideas and using an inquiry approach. Natalie, too, thought she had learnt about listening to children's ideas and explanations and using this information to make teaching decisions. Zoe, however, did not think she had learnt a great deal about how to teach from the mathematics courses.

Attending to students' thinking has been shown to make large positive impacts on student learning and achievement (Black & Wiliam, 1998). As Levin et al. (2009) put it, "a growing body of work points towards the strong influence that perceptive, ongoing diagnosis of student thinking has on a teacher's instructional moves and student learning" (p. 143). Walshaw and Anthony's (2008) review of the mathematics literature and Ball and associates' investigations (2008) make it clear that knowing what students know and think is essential for effective teaching. Our data indicates that teacher education can focus student teachers' attention to children's thinking in order to differentiate instruction. However, from our interviews it appears that the teacher education focus on student thinking in mathematics was neither systematic nor consistent. Using the numeracy project teachers' guides seems to have provided a coherent link between teacher education and teaching, but a more coherent and consistent approach to focusing on student thinking within mathematics teacher education is suggested.

KNOWING WHO

Knowing who the learners are, what they are likely to know in terms of mathematics, and what they might find confusing is also critical to effective teaching (Ball et al., 2008). All four interviewees described views about learners that they ascribed to their prior life and educational experiences as well as learning about students as learners in several of their teacher education courses. In contrast with stage-based views of teacher education that hold that novice teachers are unable to attend to children's thinking until they have begun to identify themselves as teachers and mastered classroom routines (Berliner, 1988; Kagan, 1992), all four participants talked about knowing and respecting children as people and as learners.

Cara, who had lived in several different countries as a member of a minority culture, talked about the importance of having respect for children's diversity. She believed that being born and raised in an international context, teaching English as a second language and completing an anthropology degree had all strengthened this personal outlook. Zoe felt she had learnt the importance of individualising attention from her own high school experiences and Matt also credited his secondary teachers with motivating his learning.

Natalie talked about providing "hooks" for getting children into learning. She credited the teacher education programme with her learning about students and catering for them as individuals. She could pinpoint actual lectures and materials used in the programme that changed her understandings about learning and learners.

When asked about the children's responses to the numeracy task, all of the participants recognised children's strategies and responded in ways that demonstrated that they could attend to individual learning. However, none of the participants talked particularly about their students as learners of mathematics. We would hope that, as a result of our mathematics courses, graduates would have developed knowledge about common learning progressions and student misconceptions that they bring to the teaching situation because teaching often requires knowledge "at the intersection of content and students" (Ball et al., 2008). This is an aspect for further inquiry in subsequent qualitative studies of our programmes.

KNOWING WHY

Apart from the evidence that Natalie had recognised that her new number understanding was critical in her ability to recognise and respond to learners appropriately, the interviews did not clearly demonstrate that these graduates had gained the special content knowledge (Ball et al., 2008) necessary to know why to use what approach when and with whom. Although both Cara and Matt demonstrated their ability to respond like expert teachers had changed during the TEP, neither appeared to recognise that they had shifted in their ability to do this. Zoe clearly had not gained this type of knowledge. However, with the help of her supervising teacher and through her participation in a professional development programme, she had become aware of the need to develop this expertise.

As alluded to above, knowing why is an essential content domain for teachers. Ball et al. (2008) explain in some depth how this knowledge builds upon, but is different

from, common or curriculum content knowledge. We argue that this domain is essential knowledge that enables teachers to recognise and respond appropriately to learners as shown at the centre of the TFET model, or as McNaughton and Lai (2009) term it, to act as adaptive experts. To investigate this aspect of these graduates' knowledge, we asked questions in the interview to find about why they had responded to the children's exemplar task in the way that each had.

We were expecting that if the teacher candidates understood progressions in numeracy learning and the need to notice, recognise and respond to individual responses differentially, they would answer that, yes, it does matter that the children in the example responded with different strategies. However, even though at exit three of the interviewees had differentiated between the strategies, Natalie, Zoe and Cara all stated initially that it didn't matter. When probed, Cara and Natalie explained that what they meant by "it doesn't matter" was that although the children all gave different answers, their ideas should all be valued. Cara and Natalie both then went on to explain that for teaching, it did matter. From a mathematics teaching perspective, Matt, too, believed it did matter. Matt, Cara and Natalie then gave quite explicit and extensive descriptions of how they might support each of the students in the exemplar to extend his or her numeracy strategy development.

IMPLICATIONS

Seen in combination with studies such as Ball et al. (2008), Walshaw and Anthony (2008), Levin and He (2008) and Levin et al. (2009), our work indicates the need for a core curriculum for mathematics teacher education. Such a core curriculum should focus on student learning in mathematics as well as on student teachers' own subject content knowledge and thinking for effective teaching so that, as teachers, our graduates can notice, recognise and respond appropriately to students' explanations and performances in differing teaching contexts. Furthermore, we need some way of tracking this development in our teacher candidates. Based on research about children's learning (Alton-Lee, 2003; McNaughton & Lai, 2009; for example), it seems sensible to assume that teaching and assessment for learning approaches in school classrooms could be promising models for teacher education classrooms. Our findings suggest that teacher educators need to elicit what teacher candidates know and believe (in this case, about mathematics), interpret what this means in terms of learning to be a teacher, and act appropriately on this information based on an agreed set of learning outcomes for the mathematics

courses.

The findings of this small study also indicate a lack of coherence across our mathematics courses in terms of both the intended outcomes and the content taught. This suggests the need to look more closely at the intended learning outcomes of the courses offered, the consistency of the mathematics programme offered across different groups and the need to monitor the outcomes more closely. Hammerness (2006) reports success in designing and implementing a coherent programme at Stanford over the last decade, as does Darling-Hammond (2006). Using such work could guide us to increase our evaluative efforts (such as the ones reported in this paper) and build more coherent teacher education in order to improve student outcomes in our schools. We have research evidence about what works to increase outcomes for diverse students (Alton-Lee, 2003; Walshaw & Anthony, 2008), and now need to use the evidence from these best evidence syntheses to understand what teacher education practices lead to their consistent use by our graduates.

CONCLUSION

These stories provide insights about how our student teachers experienced and learnt in our teacher education programme and subsequent teaching experiences. The quantitative data (Ell et al., 2008) in the earlier study indicated that a majority of the teacher candidates learnt to notice, recognise and respond to learners as a result of these courses. The data from these interviews confirmed that this was the case, particularly for three of the participants. Natalie's story, in particular, indicates that when teacher educators notice, recognise and respond differentially to student teachers in their courses, they can have a dramatic effect on both the student teacher's knowledge and his or her future teaching. In contrast, Zoe's story underlines how difficult becoming an effective teacher can be without such targeted teaching. Furthermore, these participants' stories demonstrate that student teachers do not learn about teaching at university and transfer this knowledge to teaching in school. They learn to become responsive teachers of children through sociocultural processes of learning and by making consequential transitions (Beach, 1999) as they interweave their knowledge with context over time. As Edwards (2009) explains, they "mindfully navigate different settings and shape and are shaped by them" (p. 156). As a consequence of these findings, we intend to continue to investigate how student teachers learn and the ways in which teacher educators can act as adaptive experts and use evidence about teaching and learning to inform their own instructional strategies.

REFERENCES

- Alton-Lee, A. (2003). *Quality teaching: Best evidence synthesis*. Wellington, New Zealand: Ministry of Education.
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389–407.
- Beach, K. (1999). Consequential transitions: A sociocultural expedition beyond transfer in education. *Review of Research in Education*, 24, 101–139.
- Bell, B., & Cowie, B. (1999). A model of formative assessment in science education. *Assessment in Education: Principles, Policy & Practice*, 6(1), 101–116.
- Berliner, D. C. (1988). Implications of studies on expertise in pedagogy for teacher education and evaluation. *Educational Testing Service*. Symposium conducted at the meeting of the New Directions for Teacher Assessment: ETS invitational conference, Princeton, NJ.
- Black, P., & Wiliam, D. (1998). Assessment and classroom learning. *Assessment in Education: Principles, Policy & Practice*, 5(1), 7–74.
- Borko, H., Whitcomb, J., & Byrnes, K. (2008). Genres of research in teacher education. In M. Cochran-Smith, S. Feiman-Nemser, J. McIntyre, & K. Demers (Eds.), *Handbook of research on teacher education* (pp. 1017–1049). New York, NY: Routledge.
- Chant, R. H. (2002). The impact of personal theorizing on beginning teaching: Experiences of three social studies teachers. *Theory and Research in Social Education*, 30, 516–540.
- Chant, R. H., Heafner, T. L., & Bennett, K. R. (2004). Connecting personal theorizing and action research to preservice teacher development. *Teacher Education Quarterly*, 31(3), 25–42.
- Clandinin, D. J. (1986). *Classroom practice*. London, England: Falmer.

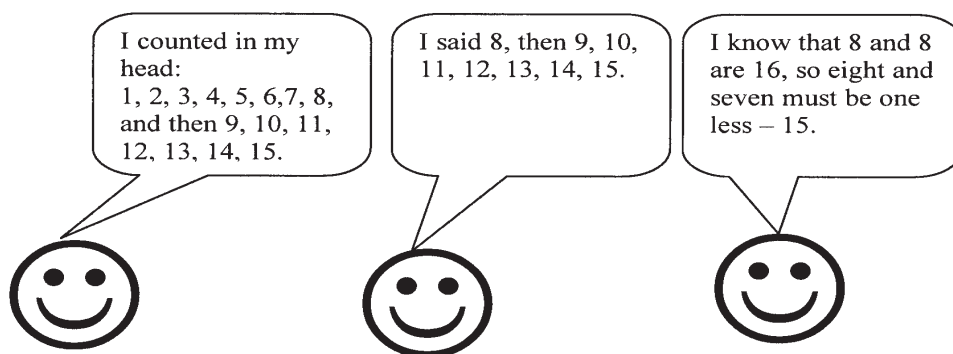
- Darling-Hammond, L. (2006). *Powerful teacher education: Lessons from exemplary programs*. San Francisco, CA: Jossey-Bass.
- Edwards, A. (2009) Becoming a Teacher, in H. Daniels, J. Porter and H. Lauder (Eds.) *Educational Theories, Cultures and Learning: vol 1*. London: Routledge, pp. 153-164.
- Ell, F. R., Aitken, G. A., Grudnoff, A. B., & Hill, M. F. (2007, April). *Developing baseline instruments to assess growth in teacher knowledge in an initial teacher education program*. Paper presented at the meeting of the American Educational Research Association, Chicago.
- Ell, F. R., Aitken, G. A., Grudnoff, A. B., Hill, M. F., & LeFevre, D. (2008, March). *Thinking for effective teaching in mathematics: How does it change in an initial teacher education course?* Paper presented at the meeting of the American Educational Research Association, New York.
- Elbaz, F. (1981). The teacher's "practical knowledge": Report of a case study. *Curriculum Inquiry*, 11, 43–71.
- Fraivallig, J., Murphy, L., & Fuson, K. (1999). Advancing children's mathematical thinking in everyday mathematics classrooms. *Journal for Research in Mathematics Education*, 30(2), 148–170.
- Grudnoff, A. (2008). (Ph D). University of Waikato, Hamilton, NZ.
- Hager, P. (2004). Conceptions of learning and understanding work. *Studies in Continuing Education*, 26(1), 3–17.
- Hammerness, K. (2006). From coherence in theory to coherence in practice. *Teachers College Record*, 108(7), 1241–1265.
- Hammerness, K., Darling-Hammond, L., & Bransford, J. (2005). How teachers learn and develop. In L. Darling-Hammond & J. Bransford (Eds.), *Preparing teachers for a changing world* (pp. 358–389). San Francisco, CA: Jossey-Bass.
- Kagan, D. M. (1992). Professional growth among preservice and beginning teachers. *Review of Educational Research*, 62, 129–169.
- Levin, B., & He, Y. (2008). Investigating the content and sources of teacher candidates' personal practical theories (PPTS). *Journal of Teacher Education*, 59(1), 55–68.
- Levin, D. M., Hammer, D., & Coffey, J. E. (2009). Novice teachers' attention to student thinking. *Journal of Teacher Education*, 60(2), 142–154.
- McNaughton, S., & Lai, M. (2009). A model of school change for culturally and linguistically diverse students in New Zealand: A summary and evidence from systematic replication. *Teaching Education*, 20(1), 1–21.
- Richardson, V. (1996). The role of attitudes and beliefs in learning to teach. In J. Sikula (Ed.), *Handbook of research on teacher education* (pp. 102–119). New York, NY: Simon & Schuster/Macmillan.
- Richardson, V. (2003). Preservice teachers' beliefs. In J. Raths & A. McAninch (Eds.), *Teacher beliefs and teacher education* (pp. 1–22). Greenwich, CT: Information Age Publishers.
- Rosaen, C. L., Lundeberg, M., Cooper, M., Fritzen, A., & Terpstra, M. (2008). Noticing noticing. How does investigation of video records change how teachers reflect on their experiences? *Journal of Teacher Education*, 59(4), 347–360.
- Schwab, J. J. (1983). The practical 4: Something for curriculum professors to do. *Curriculum Inquiry*, 13(3), 239–265.
- Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57, 1–22.
- Walshaw, M., & Anthony, G. (2008). The teacher's role in classroom discourse: A review of recent research into mathematics classrooms. *Review of Educational Research*, 78(3), 516–551.

APPENDIX A

Three children in a group that you are teaching are working out the answer to $8 + 7$.

They all get the right answer.

When you ask them how they did it, they say:



Does it matter that they did it in different ways if they all got the same answer?

Why/why not?

What would you do next with these children?