Mathematics Dialogic Gatherings: A Way to Create New Possibilities to Learn Mathematics

Javier Díez-Palomar
University of Barcelona
<jdiezpalomar@ub.edu>

Abstract
This paper introduces the Mathematics Dialogic Gatherings (MDG) as a successful way to encourage adults’ learning of mathematics. We report on a group of adults who attended a MDG in an adult school placed in Barcelona. Participants in this group do not have an academic trajectory. They attend once a week a session in the adult school, where they read, share and discuss paragraphs from textbooks of mathematics. Popular gatherings are a historical way for adults to learn in Spain. MDG are based on the dialogic learning approach developed by Flecha and others. In this session I will provide evidence on adults’ discussions illustrating how they scaffold themselves through egalitarian dialogue to learn and understand the mathematical concepts included in the textbooks used within the MDG. Drawing on the data collected, I argue that adults learn as a result of a dialogue in which they negotiate the meaning of the mathematical objects discussed, using dialogic talk. I conclude that MDG have the potential to create further learning opportunities especially for those who have never attended formal school courses, or dropped out of their school.

Key words: Mathematics Dialogic Gatherings, classic readings in mathematics, dialogic learning

Mathematics Dialogic Gatherings (MDGs) were first implemented during the 1980s in La Verneda Adult School, in Barcelona. This school is well-known internationally because being the place were Dialogic Literary Gatherings (DLGs) started in late 1970s (Soler, 2015). At that time, Ramon Flecha, with other friends, lead a community-based movement demanding for a public adult school in a working-class neighborhood, in Barcelona. A group of people, most of them without academic experience (they never attended any school), occupied a famous building in that neighborhood turning it into a community center. They began to learn in that building, and they created the adult school. Some of them formed a group who decided to read classic texts. People, who barely had attended a school, started reading Ulysses by Joyce, Don Quijote de la Mancha by Cervantes, or Hamlet by Shakespeare, using dialogic learning (Flecha, 1997). In 1999 this school became the first Spanish educational experience published in the Harvard Educational Review (Sanchez, 1999). Now thousands of people have conducted DLG all around the World (Flecha, 2011; De Botton et al., 2014; Serrano & Mirceva, 2010).

Following the path opened by these people, a group of six women begun to met every week in an adult school, in Barcelona, reading the classics, but in the field of mathematics. This paper is the story of those women, the first MDG. First, I will provide the theoretical framework to understand the basis of MDGs. Then, I will describe the ways in which MDGs work. Going back to the founders of DLGs, I will offer a justification as to why is it important to use classic readings rather than any other type of reading. Finally, I will discuss adult mathematics learning drawing on a particular example coming out from the MDG meetings.
Theoretical Framework

Mathematics literacy involves a number of abstract cognitive skills, or what Vygotsky would call “high mental functions.” During the last century, Piaget’s ideas were celebrated and quoted to claim that cognition is a developmental process in which individuals construct knowing. He proposed the idea of cognitive “schemas” to understand what happens when someone learns a concept. In his Genetic Epistemology Piaget conceived the schemes as units of analysis to represent “units of learning.” In this sense, for instance, there is the scheme of number, defined as a mathematical concept involving quantity and order position within a series. The next number is always bigger than the previous one (1, 1+1, ..., n+1, being n+1>n). This is the “rule.” Then, Piaget claimed that learning happens when the individual needs to solve a cognitive conflict between his/her scheme and new information coming from the “environment.” In our example that conflict could arise when someone tell us that in between two numbers there is always a “new” one. Then, the “solution” for our “conflict” is rational numbers. We always look for equilibrium. When new information appears (threatening our original cognitive “equilibrium”), we tend to accommodate the new information into our scheme to get a new “equilibrium.” This effort of accommodating is what Piaget calls “learning.” According to him, children go through a series of stages, from simple reflexes (sensorimotor stage) towards abstract thought (formal operational stage). Learning is “determined” by age. The cognitive development is a linear process in which individuals move from concrete operations to formal (abstract) ones. In the realm of adult education Erikson went further proposing the stages of psychosocial development (Erikson, 1959). Later studies have fully rejected Piaget’s assumption that learning depends on age (Mehler & Bever, 1967).

The core idea of Genetic Epistemology about schemes and the process of “assimilation-accommodation-equilibrium” has been accepted by the international scientific community. But learning is not just an individual process; it is a social one. According to Vygotsky (and his followers), learning emerges as a result of social interactions within individuals with different ability levels. When there are two or more people, it is always possible to create what Vygotsky (1978) called “zone of proximal development;” every individual within the group can achieve his/her “potential” mathematics ability with the help of someone else that already can do it. Later on, David Wood, Jerome S. Bruner and Gail Ross (1976) developed the idea of “scaffolding” in trying to understand how this process works (as a learning process). According to them, the teacher supports the students’ thinking giving them “hints” thus students can build their understanding over them.

This approach has been also used with success within the adult learning mathematics field. Catherine A. Hansman (2001) claims that adult learning occurs in context between adult learners’ interactions among them. Talking about parent involvement, González, Andrade, Civil and Moll (2001) also used this approach to characterize how adults use their previous knowledge to create “zones of practices in mathematics,” resulting learning as a consequence. However, although all these studies seem to confirm that learning is a social process in which people participation in heterogeneous groups (or pairs) participate in mutual interactions to support each other, they do not explain how does it work this social process.

Neil Mercer (1995), who has dedicated his professional life to investigate the role of language and the development of children’s thinking, published a taxonomy to differentiate between “disputational, cumulative, and exploratory talk.” These three categories help us to understand how individuals (students, adult learners, etc.) use language to learn. Mercer explores the relationships between quality of dialogue, reasoning, and academic results. In doing so, he ends up with the idea of “exploratory talk,” which is this kind of talk that individuals use to share relevant information, engaging with others’ ideas. According to Mercer,
Exploratory talk, by incorporating both conflict and the open sharing of ideas, represents the more ‘visible’ pursuit of rational consensus through conversation. More than the other two types, it is like the kind of talk which has been found to be most effective for solving problems through collaborative activity.

(MERCER, 1995, P. 105)

Drawing on this idea, it seems that “learning” is somehow connected to dialogue and reasoning. In recent years, Díez-Palomar and his colleagues (Díez-Palomar & Cabré, 2015; Garcia-Carrión & Díez-Palomar, 2015) proposed the idea of dialogic talk as a methodological instrument to analyze in fine grain the interactional events when two or more individuals work together to solve a mathematical task. Taking dialogue as a medium to observe cognitive learning, Díez-Palomar and others explore how learners justify their statements when working with peers and/or the teacher. Learners may use dialogic talk (defined as a type of talk in which participants use valid claims to justify their answers, that can be verified by everyone who is involved in the interactive event), or non-dialogic talk (which is the kind of talk grounded on power claims emitted by someone who is using his/her position of “power” to justify his/her statements). Evidence suggest that learning is more likely to appear when within an interactional event dialogic talk is predominant, rather than non-dialogic one.

MDGs are spaces where participants should use dialogic talk when sharing their thoughts regarding a mathematical idea coming out from one reading in mathematics. Next, I will define MDGs and how they work.

Mathematics Dialogic Gatherings

The Dialogic Literary Gatherings (DLGs) created by Flecha and a group of [mostly] women, without any academic degree, in 1978, in Barcelona, inspires MDGs. DLGs are one of the successful educational actions (SEAs) identified in the research project INCLUD-ED. Strategies for inclusion and social cohesion from education in Europe (2006-2011). This research project has transformed the social and political impact of educational research all over Europe, since most of its findings provoked the creation of new educational propositions approved by the European Parliament, European Council and parliaments from diverse member states in Europe (Flecha, 2014). DLG is a dialogic reading activity where participants read the classics (like Shakespeare, Cervantes, Kafka, Wilde, Woolf, Alighieri, Austen, Homer, Hugo, Goethe, Lorca, etc.).

![Scheme of how a MDG works](image)

*Figure 1. Scheme of how a MDG works.*
Then, they met once a week to share their reading (questions, curiosities, further information, personal narratives, etc.) sharing words, meanings and reflections. They use the dialogic methodology which state that every person must invoke validity claims to justify his/her words within the dialogue.

Figure 1 displays how DLGs work. In doing so, the participants within the DLGs are exposed to elaborated codes (in Bernstein’ terms), but they also have the time and the support to connect such words to non-formal ways to say the same idea. In this sense, DLGs become spaces for people to share their previous knowledge, and learn new ideas making meaningful bridges between their notions. Participants become literate in using high quality texts. This is the reason of using classic readings: because this type of book contains an established quality text including appropriate vocabulary and grammar. Using these readings, adult learners have more chances to improve their literacy skills.

In a similar vein, in the MDGs we use classic readings in mathematics (and sometimes sciences as well). We read Euclid, Archimedes, Copernicus, Galileo, Kepler, along with Boyer, Klein, Jean-Paul Collette, etc. Participants choose a classic reading in a topic, for instance: history of the number systems. Then, they agree on the number of pages to read at home. Next week everyone meets again, to share his/her reading. The facilitator asks the participants who wants to share his/her “paragraph,” because everyone highlights sentences or paragraphs at home, for sharing. Then, the discussion begins. The facilitator selects the order of those speaking. If someone who never participates raises his/her hand to share something, this person has the priority rather than those participants who always talk. Egalitarian dialogue is the rule. Everyone can share a sentence (a question, a comment, etc.), and everyone’s opinions are respected. All participants should draw their comments on validity claims (susceptible to be verified by the rest of the group). In this way, if someone makes an error, another participant can ask for clarification until the justification or the argument is mathematically correct. MDGs are perfect examples of what Bakhtin (2010) called dialogism and polyphony. According to him, speech acts include others’ voices, styles, references and assumptions (polyphony); hence dialog is a complex cultural situation in which participants share their own voices (previous knowledge, personal experiences and narratives, assumptions, etc.). Using this theoretical approach, we can assume that learning is the sum of all these voices, but in a context of egalitarian dialogue (Flecha, 2000) where everyone uses dialogic talk.

Methodology

In this article I’m discussing a case study using qualitative methods based on an ethnographical approach, using communicative methodology (Aubert, 2015; Gómez & Munté, 2016) as a framework. Ethnography attempts to understand social and cultural situations including the perspective of the participants involved in the study. It involves a close relation between the researcher and the participants. The researcher becomes a member of the community observed. I collected the data during the school year 2015-2016. The setting for the study was an adult school placed in a working class neighborhood in Barcelona. I arrived at this school seventeen years ago, in 1999. I served as a volunteer to help adult learners to develop their skills in mathematics. Along the way, I worked with individuals who never attended a school before, individuals with few notions about “school-mathematics”, and individuals who had a strong mathematical background in formal mathematics. Eventually I became member of the community. I taught them and I learnt from them. I was able to understand many different ways to do and talk about mathematics. Being part of the community, I was invited to create a MDG using classic readings in mathematics, to learn from the classics with people that have not any academic degree. I asked them to allow the presentation of their dialogues in the annual conference of ALM. They agreed, so I did it.

I collected data every week. Participants include six women between 40-years-old and over seventy-years-old. I audiotaped all the sessions during the last semester in 2015-16 (from May to July 2016). I also took field notes in my diary, from the classroom observation. This set of data
was transcribed partially. The communicative methodology underpinned the data collection, analysis and interpretation. I worked assuming a position of “egalitarian dialogue” with all the participants in the MDG. In the next section I try to build on their voices.

Table 1.
Analysis of the speech acts. Types of talk

<table>
<thead>
<tr>
<th>Interaction Type 1</th>
<th>Interaction Type 2</th>
<th>Interaction Type 3</th>
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</thead>
<tbody>
<tr>
<td>Exchange of information</td>
<td>Non-dialogic interaction</td>
<td>Dialogic interaction</td>
</tr>
<tr>
<td>No argumentation</td>
<td>Arguments are based on power claims</td>
<td>Arguments are based on validity claims</td>
</tr>
<tr>
<td>Example: memorization</td>
<td>Example: authoritarian order</td>
<td>Example: egalitarian dialogue</td>
</tr>
</tbody>
</table>


In terms of data analysis, I used discourse analysis focusing on the relationship between language and cognition—in mathematics. Discourse analysis grew up during the 1960s and 1970s. At that time linguistics were more interested in understanding single sentences. But in 1952 Harris published an article titled ‘Discourse analysis’ looking on the links between text and its social context. Later on, other scholars provided seminal works on the study of speech in its social setting (Hymes, 1964 or Searle, 1969). Linguistics and socio-linguistics were concerned not only with the grammatical and lexical forms of what is said, but, more importantly, on what people can do with words (Austin, 1962). People with words can actually create “opportunities for learning” for other people. They can create ZPD where other peers can receive support and develop their cognitive potential. For this reason, I used the codes presented in table 1 to analyze dialogue during the interactions occurred during the meetings. I used utterances (from the dialogues) as unit of analysis. Within these utterances, I looked for interactions of type 1, 2 or 3.

**Interactions type 1** are defined as interactions in which individuals use language to share information. They do not explain, nor justify, their statements. They just exchange information. The type of learning associated with interactions type 1 is memorization, because it is low demanding in terms of cognition. The interaction type 1 does not require any “understanding” of the idea transmitted.

**Interactions type 2** are defined as non-dialogic interactions. Participants use language to express mandate, order. Justification of the correctness, veracity, and truth of the statement is based on the power position that occupies the person who pronounces the sentence. This is the case of a statement like “2 plus 2 equals four, because I am the teacher and my authority is based on my position of power in front to the students.”

**Interactions type 3** are defined as dialogic ones because participants always use validity claims to justify their arguments. Correctness, veracity or truth are based on valid claims emitted by the speaking person. The audience can verify those valid claims. For example, “2 plus 2 is four, because I’m placing 2 pieces of paper on the table, then I’m adding 2 more pieces of paper, and then I’m counting with you all the pieces, being four the last number that I pronounce when finishing all the pieces of paper over the table.” Interactions type 3 may create opportunities for participants to build on those valid claims to understand the mathematical concepts discussed within the dialogue.
Results

In this paper I discuss the interactions occurred during a session about *The Number System*. We were reading “Historia de las Matemáticas” the Spanish translation of Jean-Paul Collette (1979) book “Histoire des Mathématiques.” At the beginning of the first volume, Jean-Paul Collette introduces the origins of the mathematics (Prehistory, Babylonians, Greeks, Romans, etc.). He talks about the different forms to represent numbers, as well as different number systems. Along the pages, we (the participants in the MDG) held a discussion about the first marks in a bone found in Ishango that archeologist believe are tally marks. I shared with the women that the marks seem to be grouped keeping records of 28 days, which scientist belief that correspond to lunar cycles.

The conversation came along, and a genuine interest about where the numbers come from appeared. Someone noticed that there are different types of numbers: Sumerian, Egyptian, Greek, Roman, Hindu-Arabic, etc. Ancient people used different ways to represent numbers, but using tokens (in Sumeria) was a huge advance since it allowed people to perform easily the first arithmetic calculations, adding or removing some tokens from the full set. I noted that using tokens was also important because you can use an object (a token) to represent an abstract idea, in this case, the number. In this sense, numbers are “connected” to their physical representations in a bi-univocal relationship. We discussed what does the word “bi-univocal” mean, using different numbers as examples to illustrate it. We used pencils to represent numbers, and then I wrote down on the whiteboard a series of marks and their link to its numeral, from one to five. We noteded that ancient people discovered that relation (Aida claimed that “ancient people were smarter than us, because we use numbers, but they discovered numbers.”) In so doing, concepts like *numeral*, *quantity*, *cardinality*, *LGHD* (in Plato’s sense) emerged in our dialogues.

Then, one of the participants, Carlota, raised another topic for discussion: she shared a paragraph from the book talking about how decimal numbers travelled from one civilization to another (see lines 5 and next in the transcript).

[1] Carlota: The number system...
[3] Carlota: Page 12… almost at the end of page 12...
[5] Carlota: it says... “The decimal system is well know and used by the Arabs, who passed us in the Iberian Peninsula during the period of Al-Andalus and then it was disseminated through the whole Europe. In turn, the Arabs took it from the Hindus, as we can see in the figure.” This caught my attention because I did not think that it was that old...
[6] Volunteer: Aha. And what the rest of you think? It came to your attention the same issue?
[7] Cèlia: yes, yes... I do. It’s like in the Roman times, when... When... Explaining the numbers... Or when on TV they start to cross out the numbers on a piece of wood
[8] [referring the notches on the Ishango bone]
[9] Carlota: The Arabs seems to have three more numerals than Roman people... Can it be?
[10] Volunteer: I don’t know, I had not ever thought... Let’s see, could you further explain...
[11] Carlota: Romans seems to have, one, five, ten, fifty, hundredth and thousand. And they [the Arabs]...
Carlota makes an interesting point in line 11. She wonders if Arabs had more numerals than Romans. That attracted my attention since I thought that this question was a very important one. Carlota, in fact, was noticing that there are more numerals in the Hindu-Arabic number system, than in the Roman one (see Table 2).

We counted that whereas in the Hindu-Arabic number system we have 9 different numerals (at this point there was no mention of zero), whereas in the Roman one we only have 7 numerals. The next question was “how can they count with only 7 numerals?” Someone said that ancient Romans were “troublemakers” with such a numerical system. “Our numbers are easier” Célia said. “Why?” I asked. Alba said that “our” numbers are easier because we are used to them.

Table 2

<table>
<thead>
<tr>
<th>Hindu-Arabic</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roman</td>
<td>I</td>
<td>V</td>
<td>X</td>
<td>L</td>
<td>C</td>
<td>D</td>
<td>M</td>
<td></td>
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</tr>
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</table>

During the discussion, we discovered that in order to represent quantities like 2, with Roman numbers, we should use the symbol I two times (II); but to represent quantities like 4, then we have to do some calculations using the Roman numerals (5-1=4; hence four in Roman numerals is IV). We discovered that the rules to represent the numbers with the Roman numerals were somehow complex (never use the same numeral four times in a row; repeating a numeral up to three times means addition; a small numeral to the left of a larger numeral means subtract the larger minus the smaller, but on the opposite means addition).

Then Carlota contributed again by mentioning “zero.” She realized that zero was not among the numerals that they were discussing. She repeated that there are more numerals among Hindu-Arabic system than in the Roman one. She was estimating that “our” numerals are three times more than the Roman ones…, which was a bit high as estimation. Then, I asked Carlota “what are those numbers?” (line 26) She guessed that zero may be one of them. Then, Alba jumped into the discussion and said that 3 and 7 should be also part of “those numbers [numerals].” She mentioned another interesting idea “10-based system.” (line 29) This notion added a new layer to our concept of “number system.” It seems that number systems, in addition to numerals, quantities, cardinality, and so on, also have something called “base.”

[19] Carlota: Zero, I do not know…
[20] Fe: This is ten, isn’t?
[21] Alba: What are the numbers you mentioned?
[22] Carlota: Look, the one… those are the Roman [numbers]: the one, the five, the ten,
[23] the “L” means fifty, “C” is a hundredth, “D” is five hundredth, and “M” a thousand.
[24] Many at the same time: Yes!
[25] Carlota: But the Arab [numbers] it seems that they have three more numbers.
Volunteer: So, what are those numbers?

Carlota: One must be the zero... I do not know...

Alba: And the three as well. Of course. And the seven. It says so here. Three, seven and zero. Zero, three and seven. Well... I see that... They used a 10-based system... I mean... Everything is 10-based... Ten, twenty, thirty... They move from ten to ten...

Or from twenty to twenty... Or from a hundredth to hundredth... But the base is ten...

That's what they decided...

Volunteer: Aha... They?

Aida: They means the Arab people. Yes, zero, six and nine... It says that it was upon a time... It was upon a time, in Florence, that people did not like those numbers [Arab numbers] because it was so easy to falsify them. It says: "At the end of the XIII Century the Florence Government passed norms against the use of those symbols because it was very easy to falsify the zero, the six and the nine." That is, they did not want the Arab numbers because that because it was very easy to falsify.

Alba provided a clear example of what 10-based system means. In lines 28 to 32 she mentions that in a 10-based number system “everything is 10-based... ten, twenty, thirty…” She was providing her justification with the statement “they move from ten to ten.” That was a clear example of interaction type 1.

Concept: base-10 number system

Statement: “They used a 10-based system…”

Validity claim: “They move from ten to ten.”

Example: “Ten, twenty, thirty…”

They continued the conversation. Aida raised another interesting aspect regarding the dissemination of number systems in the history of mathematics: Hindu-Arabic numerals were not well accepted at the beginning in Europe. This added a new layer to our discussion: the sociological approach. Numbers [numerals] are social goods. They are result of social consensus between people who agree on using a particular numeral (and not other) because a number of reasons. Aida was talking about how the Hindu-Arabic numerical system was introduced in Europe, during the Middle Age.

Concept: Hindu-Arabic number system

Statement: “It was upon a time, in Florence, that people did not like those numbers [Arab numbers]”

Validity claim: “because it was very easy to falsify the zero, the six and the nine.”

Example: 6 ↔ 9 [you just have to flip the symbol]

Discussion

The analysis of the dialogues occurred during the session reveal some important aspects related to the MDGs and how adults develop their mathematical literacy.

First, using classic readings in a dialogical way suggests that adult learners are able to discuss and understand formal mathematics. Some theories in Sociology of Education claim that learning is
stratified among individuals according to their social class. Bourdieu (1986) coined the theory of “cultural capital” to explain that certain forms of cultural capital are socially valued over others. For “cultural capital” he refers to a collection of symbolic aspects such as skills, knowledge, type of readings, taste (for books, paintings, etc.), ways to dress, etc. According to him, cultural capital comes in three forms: embodied, objectified and institutionalized. Reading classics may be a symbol of belonging to a privileged social class; hence people from the grassroots “usually don’t appreciate” this kind of readings. Our data suggest the opposite idea: the women participating in the MDGs are enjoying the best readings in mathematics, and they are maintaining meaningful dialogues drawing on such readings.

Moreover, in the 1970s, 1980s, and early 1990s Basil Bernstein published several books analyzing discourses from a social point of view. In the first volume of *Class, codes and control* (four volumes), edited in 1971, Bernstein distinguished between *elaborated codes* and *restricted codes*. According to him, the forms of spoken language are associated with particular positions in the social structure hierarchy. Elaborated codes correspond to formal discourses, distinctive of the “well educated social classes,” whereas “restricted codes” are typical of under represented social classes, using different forms of *slang*. Drawing on this approach, Paul Willis (1977) wrote an important ethnography suggesting that what children from working class families learn in the school is to be members of their social class, nothing else. Thus, they “don’t appreciate” classic readings because it does not belong to their “cultural capital.” Again, the analysis of the data collected suggests that this interpretation may be wrong.

According to Catherine Snow (2002), the crucial variable to understand an individual’s skills (she said this in respect of reading, but I also suggest the same idea in mathematics literacy) is not their social class, nor their gender; but the amount of times that a particular person has been exposed to high quality texts. In other words, the better the readings are, the better the learning is. When we provide classics to the learners, they use readings that already are high quality (because the scientific community universally claims that these readings are “classic.”) Using the best mathematical readings then, give people the opportunity to be exposed to relevant and important notions in mathematics, hence the level of their talk increases. Our data is consistent with this assumption. During the lesson the participants were able to talk about numeral, quantity, cardinality, number system, base, bi-univocal relationship, etc. All of them are important elements to understand the idea of number. I suggest that similar to work that Snow (2002) undertook with children, an equivalent effect also happens with adults: they learn more and better.

Finally, the analysis of the interactions through the discourse using *dialogic talk* as a methodological tool provides us the cognitive path that adults follow to understand the mathematical concepts. As Bakhtin (2010) suggested with his work, knowledge is a social product, hence knowing should be understood as a social process. In this lesson participants use dialogue to build on each other’s utterances. Understanding is a final stage; I would say that this “is the ultimate goal after a common path in which everyone supports each other with his/her thoughts, expressed through dialogue.”

**References**


