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Exploring the opinions about the concepts of "formula" and "rule" in mathematics

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The purpose of this study is to draw attention to the concepts of "formula" and "rule" in mathematics, thereby revealing the views of pre-service teachers relating to these concepts by exploring their knowledge in, and their capacity to exemplify these concepts. The study is important in that it would reveal how pre-service teachers see the concepts of "formula" and "rule", and to what extent they associate these concepts with mathematics, and further contribute to the field as there is currently no similar study in the literature. For this purpose, researchers prepared a diagnostic test consisting of ten questions. Nine (9) of the questions were open-ended, and 1 was multiple-choice. With this diagnostic test, pre-service teachers were expected to explain the concepts of formula and rule, reveal the formula and rule that first come to their mind, express the formulas and rules they learned during secondary and high education, describe what sort of a mathematics without formula and rules would be, and finally answer the question "is mathematics without formula and rules possible?". The sample of the research was composed of a total of 159 pre-service mathematics teachers including Grades 1, 2 and 3 students in Kars. The data were analysed by qualitative analysis and results were tabulated to indicate frequency and percentage values. It has been concluded that pre-service teachers associate the concepts of formula and rule with mathematics, they mathematically internalize and correctly interpret these concepts, they are accustomed to and familiarized with these concepts, and they believe mathematics without formulas and rules is not possible. Mathematics without formula and rule would be meaningless and difficult, take long time to study, and would not be possible, and mathematics can only be implemented through solving with individually proven concrete concepts and describing the rationale behind them.

Key words: Mathematics education, pre-service mathematics teachers, formula, rule.

INTRODUCTION

Identifying how students perceive formula and rule, the concepts that typically come to mind first as far as mathematics is concerned, especially what formulas and rules are permanent in their minds, how their relationship with mathematics is perceived, and what students think about these concepts, is critical in establishing the mathematics education programs as well as the teaching methods to be employed by teachers. It is also believed

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that formula and rules involved in mathematics lessons have a typical role in negative attitudes and concern developed by majority of the students towards mathematics. The student feels that if the necessary formula or rule cannot be remembered, (s)he would not be able to do anything due to the lack of essential infrastructure, and this leads to a dislike of the lesson. Taking these facts into consideration, it is important to question these concepts that are frequently involved by teachers in mathematics education programs.

A mathematician typically strives to reach correct results by employing the rules and formula of logic and mathematics. However, the formula referred to here actually serve as tools only. In other words, mathematics does not mean a huge pile of formula (Altintas, 2005). What is important in equipping students productively and raising them towards success in real life is not simply to teach mathematical formula and rules, but rather to teach how to integrate mathematics into life. Students may be expected to know mathematical formula and rules, and use them where necessary. However, it is not very logical to dictate the memorization of this data and making assessments based on such data. Rather, by teaching when, where and why to use such knowledge throughout life, effective curricula could be built that deliver essential knowledge and skills instead of non-useful mass of information (Yildiz and Uyanik, 2004).

The goal in mathematics lessons should not be to memorize the theories, and formula and then to solve questions based on what have been memorized because, in this case, students are urged to act without knowing why they solve the particular problem (Nasibov and Kacar, 2005). For example, in their study, Sisman and Aksu (2009) found that students have difficulty in effectively using various formulas. What is important here is the ability to think comprehensively, considering all existing conditions, to understand what consequences would arise after certain circumstances, and to learn and teach how to think logically and systematically (Nasibov and Kacar, 2005).

Formulae are typically short algebraic expressions used to show relations between quantities. In other words, they summarize the relations between quantities through mathematical symbols (Isik et al., 2005). According to Turkish Language Association (2017a), a formula is a set of symbols describing a general phenomenon, rule or principle. It is a mathematical expression serving to calculate a quantity linked with one or several quantities. Additionally, according to Turkish Language Association (2017b), a rule is defined as a principle and guideline which is fundamental to and guiding an art, a science, and a system of thought and behaviour. Furthermore, it is also defined as a general judgement established after long research and trial in a specific field, and a guideline demonstrating how to achieve a good result in a process (Turkish Language Association, 2017c).

The purpose of the mathematics curriculum for primary grades 6 to 8 is to lead students to principally create the basic concepts, relations-correlations and formula through structured activities concerning circumferences and surface areas of planar figures including squares, rectangles, triangles, parallelograms, rhombuses, trapezoids and circles, as well as the surface areas and volumes of geometrical figures including cubes, rectangular prisms, squares and triangular prisms, spheres, cones, cylinders and pyramids. Moreover, in all other subjects involving formula, the fundamental principle is to associate the formula with previous subjects rather than directly presenting them, so that students can principally investigate and learn them (Ministry of Education, 2009). Nevertheless, the concern of getting the result fast in practice takes students and teachers closer to a formula. The problem here is that formulae are presented directly to the students without building on previous knowledge, and therefore they are not persistently established in students. The same also applies to rules in mathematics. The rules are not just the data to be delivered through memorization, but information that should be presented to students in a logical sequence.

The study of Ahuja et al. (1998) states that teachers should develop formulae with students to involve thinking rather than just dumping the formula on them. They must encourage students to think more – "don't spoon feed and don't encourage them to memorise". In their study, Isik et al. (2005) explored the ability of pre-service mathematics teachers to recognize and remember some mathematical concepts. One of these mathematical concepts is formula. Pre-service teachers were asked "What do you think a formula is?" and answered as follows:

1. Fixed expressions
2. Shortcuts to solve problems
3. Short solutions previously proven
4. Rule, and
5. Others.

In addition, they were asked to answer the question "Which of the following describes the formula concept?" and pick one of the following options:

1. Shortcuts to solve problems
2. Short solutions previously proven
3. Summarized form of a subject with mathematical symbols, and
4. None.

An analysis of the answers revealed that the subjects failed to state the expected answers in open-ended questions, and could answer correctly only 5.6% of the multiple-choice questions. And it was concluded based on the findings that recall rates of the subjects were relatively higher, despite being low from the general angle, compared to recognition.

In their study, Altay and Umay (2011) argued that mathematics is not just a set of rules to be applied, and therefore it is necessary to emphasize and discuss meanings beyond the teaching of rules. In their study where the effect of realistic mathematics teaching on student achievement, as well as students' opinions are explored, Ozdemir and Uzel (2011) concluded that students should be guided not by formula but reasoning structured under three main headings: imparting the ability to interpret, avoiding mere memorization, and focussing on the subject and its aspects. Moreover, it was further found that when students develop formulas by themselves, try different methods and associate their learnings with daily life, a huge contribution is made to understanding the subject.

In his study, Karakus (2014) found that 82.53% of pre-service teachers attempted to learn geometry during their past study by memorizing the formula or rules and solving problems. This reveals that memorizing rules and formula and solving problems represent the most preferred method of learning geometry. Accordingly, it may be concluded that without formula and rules, pre-service teachers will have difficulty in learning mathematics because they associate mathematics, especially geometry, with formula and rules.

In their study, Ozudogru (2016) concluded that conceptual learning is not fully realized in students, learning is limited to the operational level, and students attempt to solve problems on functions and find the result based on memorized rules.

The conclusions from the study of Pale (2016) are as follows: Approximately 58% of students viewed learning mathematics as mostly memorizing formulas and rules less than half of them who viewed mathematics as interesting. Teachers must provide students with learning opportunities in which they experience the excitement that comes from making sense of mathematics instead of memorizing formulas and rules. Instead of focusing on formulas and rules, mathematics teachers should help their students make sense of the mathematics they are learning.

The purpose of this study is to draw attention to the concepts of "formula" and "rule" in mathematics, thereby revealing the views of pre-service teachers relating to these concepts by exploring their knowledge in, and their capacity to exemplify these concepts. The study is important in that it would reveal how pre-service teachers see the concepts of "formula" and "rule", and to what extent they associate these concepts with mathematics, and further that it would contribute to the field as there is currently no similar study in the literature.

Taking the views of pre-service teachers about the place and use of the concepts of "formula" and "rule" in mathematics is also important as it would reveal their opinions in mathematics. The present study is also important in that it could provide the opportunity to interpret the root causes of negative attitudes, anxieties and prejudices developed against mathematics. In addition, it is also believed that the findings of this paper would also serve as a guide for structuring the mathematical curricula and hence the course contents and also guides the teachers to elaborate their lesson plans and educational methods. A similar study could be found neither in local nor the foreign literature. In this study thus representing an unprecedented one, particularly the data gathered and their interpretations were involved extensively.

Based on these descriptions, the problem statement of the research is as follows: What is the capacity of pre-service teachers to define and exemplify the concepts of "formula" and "rule" in mathematics, and what are their opinions in these concepts?

METHODOLOGY

Research model

The content analysis from the qualitative data analysis approaches was used in the present study. The intentions and perceptions of subjects often enjoy a privileged position in qualitative research, because of the access they can give us to the meaning of action for particular observers. Qualitative research often seeks to illuminate the ways individuals interact to sustain or change social situations. Through analysis, a fresh view of the data can be obtained. Progress can be made from initial description, through the process of breaking data down into bits, and seeing how these bits interconnect, to a new account based on reconceptualization of the data. The data is broken down in order to classify it, and the concepts created or employed in classifying the data, the connections made between these concepts, provide the basis of a fresh description. The core of qualitative analysis lies in these related processes of describing phenomena, classifying it, and seeing how these concepts interconnect (Dey, 1993). Content analysis is a widely used qualitative research technique. Rather than being a single method, current applications of content analysis show three distinct approaches: conventional, directed, or summative. In the present study conventional content analysis is used. The advantage of the conventional approach to content analysis is gaining direct information from study participants without imposing preconceived categories or theoretical perspectives (Hsieh and Shannon, 2005). While making content analysis, codes are created from the data and common directions between codes are found. Through the codes the data are categorized (Yildirim and Simsek, 2011). The answers gotten from pre-service math teachers were analysed qualitatively, and percentage and frequency values were given in Tables.

The study group

The research was carried out in the spring semester of 2015 to 2016 academic year. The research group of the study consisted of 159 pre-service teachers of primary mathematics at 1st, 2nd and 3rd grade from Kafkas University in Kars. While determining the participants, convenience sampling was conducted for some practical reasons, such as ease of transportation, implementing the study rigorously and ease of communication because the researchers work at Kafkas University. Also the so-called semester there comprised of only 1st, 2nd and 3rd grade students in primary mathematics education department in Kafkas University in Turkey.

When subjects are chosen because of the close proximity to a researcher, that is, the ones that are easier for the researcher to access, it is called a convenience sampling (Etikan et al., 2016).

Data collection tool

A “diagnostic test” which was prepared by the researchers was used within the scope of this study. “Diagnostic test” is composed of 9 open ended and 1 multiple choice questions. The questions in the test are as follows:

1. “What is formula?”
2. “What is rule?”
3. “What is the first formula you remember?”
4. “What is the first rule you remember?”
5. “Please state a formula you learned in primary school”
6. “Please state a rule you learned in primary school.”
7. “Please state a formula you learned in secondary school”
8. “Please state a rule you learned in secondary school”
9. “Is there mathematics without formula and rule?”
10. “According to you, how does mathematics without formula and rule become?”

For validity of the diagnostic test, the views of two experts were taken and some arrangements were made in the test. In presenting the ideas of pre-service teachers about the terms of formula and rule, pre-service teachers were required to express their ideas about formula and rule. They were required to express the formula and rule they remembered first to determine which formula and rule became permanent in their minds. Regarding the question of “According to you, how does mathematics without formula and rule become?” The researchers aimed to obtain data about how the pre-service teachers adapt to terms of formula and rules with mathematics and how they envisage mathematics.

Data analysis

Nine open-ended questions in diagnostic test were analysed qualitatively. One multiple-choice question answers were analysed as “Yes” or “No”. By using content analysis, answers given by the pre-service math teachers were categorized and the categories obtained were given on tables by giving frequency (f) and percentage (%) values. Inter-coder reliability was made confirmable. For consistency, double-coding method of Miles and Huberman (1994) was used. The consistency value was found as 0.78. It shows that there is harmony between researchers. For transmissivity, the answers of participants were used in the present paper. For cogency, expert views were used. The researchers are mathematics instructors and expert on the so-called matter. Also the researchers made literature research about which examples in mathematics are formula or rule. Hence, they determined which answers are formula or rule and categorized the answers as true or false.

RESULTS

In the results part, tables were provided for analysis made in view of the data at hand. As shown in Table 1, answers of pre-service teachers to the question “What is formula?” are grouped into 23 categories. These categories can be sorted by a number of answers as follows: “Practical information for the simple solution of a problem”, “Solution method used for a specific subject”, “Stereotyped statement requiring memorization with no underlying rationale illustrated”, “There is nothing called formula”, “Concept formed by symbols”, “Conclusion part of the proofs”, “Provable rules”, “Proven form of the theories”, “Set of symbols relating to the phenomenon, principle or rule”, “A correlation”, “Concrete form of abstract thoughts”, “Fixed form of specific aspects” “Theoretical part of mathematics, the abstract side”, “Summary of the subjects, the theme”, “Simplified form of statements”, “Shortcuts finally proven after tests”, “Frequently used mathematical statements”, “The vital aspect for the meaning of mathematics”, “Operations invented later in time”, “A term used in mathematics”, “Something with informative quality”, “Home to mathematics”. 4.40% of pre-service teachers skipped this question.

As shown in Table 2, answers of pre-service teachers to the question “What is rule?” are grouped in 26 categories. These categories can be sorted by number of answers as follows: “The path to the solution of a problem”, “A principle that people should respect”, “Shortcuts to simplify complicated operations”, “Conclusive”, “Ensures that the lesson is studied in a systematic manner”, “A precondition and necessity in problem solving”, “Theory”, “Anything previously proven and accepted to be true”, “Limits, borders and criteria specific to each matter”, “Something indispensable for the matter”, “The rationale, core of a subject”, “An element that makes the maths meaningful”, “Something that cannot be proven and changed”, “The path to implementing the formula”, “Fixed form of results derived from certain aspects”, “An essential behaviour”, “Shows the method and place of a mathematical process”, “A pretext to impose the intended action”, “A must-know statement”, “Fixed form of the formula”, “The formula leading to the result”, “Discipline and responsibility”, “Student’s assistant”, “Necessity to do what’s required”, “A principle fundamental to and guiding a science and art”. In addition, 9.43% of pre-service teachers skipped this question.

While giving the answer examples of pre-service teachers, the researchers categorized them as "true" and "false" by considering the following explanations: Rules are not general statements. Also in the rule statements, the right sides of equality are the expansions of left sides. They only help researchers in arriving at the results easily. But the researcher might want to use other ways to arrive at the result. Without using the rules, the researchers can arrive at the results. But the formulas are the general statements and they contain rules. Also, the literature search was made and the expert’s idea was taken by separating formula and rule.

According to Table 3, based on answers to the question “What is the first formula you remember?”, 88.05% of pre-service teachers give true answers while 8.80% give false answers and 3.14% skipped the question. Predominantly, true answers are as follows:

Table 1. Categorization of answers to the question "What is formula?"

Categories	Frequency (f)	Percentage
Provable rules	7	4.40
Solution method used for a specific subject	25	15.72
Concept formed by symbols	8	5.03
Stereotyped statement requiring memorization with no underlying rationale illustrated	12	7.54
Practical information for the simple solution of a problem	65	40.88
A correlation	5	3.14
Conclusion part of the proofs	8	5.03
There is nothing called formula	10	6.28
Proven form of the theories	7	4.40
Empty	7	4.40
Concrete form of abstract thoughts	5	3.14
Fixed form of specific aspects	5	3.14
Shortcuts finally proven after tests	4	2.51
Frequently used mathematical statements	4	2.51
Theoretical part of mathematics, the abstract side	5	3.14
Summary of the subjects, the theme	5	3.14
Operations invented later in time	3	1.88
Set of symbols relating to the phenomenon, principle or rule	7	4.40
Simplified form of statements	5	3.14
The vital aspect for the meaning of mathematics	4	2.51
Home to mathematics	1	0.62
Something with informative quality	2	1.25
A term used in mathematics	3	1.88

"The Pythagorean relation: $a^2 = b^2 + c^2$", "Area of the square:a^2", "Distance = Speed × Time: x = V.t", "Area of the circle:πr^2", "Discriminant: Δ= $b^2 - 4ac$", "Area of the triangle: $\frac{Taban\ kenarı x yükseklik}{2} = \frac{a.h}{2}$"

Predominantly, false answers are as follows:

"$sin^2x - cos^2x = 1$", "$(x + y)^2 = x^2 + 2xy + y^2$", "r=$-\frac{b}{2a}$", "Sin2x=2sinxcosx",
"$\log\frac{a}{b} = \log a - \log b$"

According to Table 4, based on answers to the question "What is the first rule you remember?", 80.50% of pre-service teachers gave true answers while 8.80% gave false answers and 10.69% skipped the question. Predominantly, true answers are as follows:

"$sin^2x + cos^2x = 1$", "The sum of the angles in a triangle is 180 degrees.", "In triangles, the sum of two interior angles is equal to the non-adjacent exterior angle.",

"Tanx=$\frac{sinx}{cosx}$", "The big angle faces the long edge", "The L-Hospital rule", "Order of operations (parentheses-multiplication, division-addition, subtraction)", "Only a single line can pass through two points.", "Indefinite number of lines can pass through a single point", "Multiplication of two numbers gives an even number."

Predominantly, false answers are as follows:

"The Pythagorean relation", "$h^2 = p.k$", "Area of a rectangle: Short edge x long edge = axb", "Volume in prisms (Base area x height)"

According to Table 5, based on answers to the question "Please state a formula you learned in secondary school.", 67.92% of pre-service teachers gave true answers while 18.86% gave false answers and 13.20% skipped the question. Predominantly, true answers are as follows:

"The Pythagorean relation: $a^2 = b^2 + c^2$ (In a right triangle, the sum of the squares of legs is equal to the square of the hypotenuse)","Δ= $b^2 - 4ac$", "Area of the square: a.a", "Area of the rectangle: length of the short edge. Length of the long edge (a.b)", "Volume: Base area x height (V=a.b.c)".

Predominantly, false answers are as follows:

"$(x + y)^2 = x^2 + 2xy + y^2$", "Tan$\theta$=$\frac{sin\theta}{cos\theta}$",
"Cotθ=$\frac{cos\theta}{sin\theta}$", "$sin^2x + cos^2x = 1$", "Tanx.cotx=1".

Table 2. Categorization of answers to the question “What is rule?”

Categories	Frequency (f)	Percentage
A principle that people should respect	20	12.57
An element that makes the maths meaningful	5	3.14
Something that cannot be proven and changed	5	3.14
Limits, borders and criteria specific to each matter	6	3.77
Shortcuts to simplify complicated operations	15	9.43
The path to the solution of a problem	45	28.30
An essential behaviour	4	2.51
Something indispensable for the matter	6	3.77
The path to implementing the formula	5	3.14
Fixed form of the formula	3	1.88
Theory	7	4.40
The formula leading to the result	3	1.88
Fixed form of results derived from certain aspects	5	3.14
Anything previously proven and accepted to be true	7	4.40
Conclusive	12	7.54
A principle fundamental to and guiding a science and art	2	1.25
Shows the method and place of a mathematical process	4	2.51
Ensures that the lesson is studied in a systematic manner	10	6.28
Discipline and responsibility	3	1.88
A pretext to impose the intended action	4	2.51
A precondition and necessity in problem solving	10	6.28
The rationale, core of a subject	6	3.77
A must-know statement	4	2.51
Student’s assistant	3	1.88
Necessity to do what’s required	3	1.88
Empty	15	9.43

Table 3. Categorization of answers to the question “What is the first formula you remember?”:

True		False		Empty	
Frequency (f)	Percentage	Frequency (f)	Percentage	Frequency (f)	Percentage
140	88.05	14	8.80	5	3.14

Table 4. Categorization of answers to the question “What is the first rule you remember?”:

True		False		Empty	
Frequency (f)	Percentage	Frequency (f)	Percentage	Frequency (f)	Percentage
128	80.50	14	8.80	17	10.69

Table 5. Categorization of answers to the question “Please state a formula you learned in secondary school.”:

True		False		Empty	
Frequency (f)	Percentage	Frequency (f)	Percentage	Frequency (f)	Percentage
108	67.92	30	18.86	21	13.20

According to Table 6, based on answers to the question “Please state a rule you learned in secondary school.”, 71.69% of pre-service teachers gave true answers while 6.28% gave false answers and 22.01% skipped the

Table 6. Categorization of answers to the question “Please state a rule you learned in secondary school.”

True		False		Empty	
Frequency (f)	**Percentage**	**Frequency (f)**	**Percentage**	**Frequency (f)**	**Percentage**
114	71.69	10	6.28	35	22.01

Table 7. Categorization of answers to the question “Please state a formula you learned in high school”.

True		False		Empty	
Frequency (f)	**Percentage**	**Frequency (f)**	**Percentage**	**Frequency (f)**	**Percentage**
99	62.26	31	19.49	29	18.23

Table 8. Categorization of answers to the question “Please state a rule you learned in high school.”:

True		False		Empty	
Frequency (f)	**Percentage**	**Frequency (f)**	**Percentage**	**Frequency (f)**	**Percentage**
100	62.89	9	5.66	50	31.44

question.
Predominantly, true answers are as follows:

“Inside opposite angles are equal to each other.”, “A single line passes through two points.”, “In an equilateral triangle, the length and angle of edges are the same.”, “In an equilateral triangle, all angles are 60 degrees.”, “In a right triangle, the edge facing 90 degrees is the longest and called hypotenuse.”, “The sum of angles in a rectangle is 360 degrees.”, “The sum of angles in a triangle is 180 degrees.”, “The sum of exterior angles of a triangle is 360 degrees.”, “The big angle faces the long edge”, “Numbers ending with 0 or 5 are divisible by 5.”, “$sin^2x + cos^2x = 1$”, “Multiplication of two negative numbers gives a positive result.”, “Order of operations (parentheses-multiplication, division-addition, subtraction)”

Predominantly, false answers are as follows:

“Volume of a circle = $\frac{4}{3}\pi r^3$”, “Area of a right triangle: half of the multiplication of legs.”, “Δ= $b^2 - 4ac$”, “$a^2 = b^2 + c^2$ (In a right triangle, the sum of the squares of legs is equal to the square of the hypotenuse = the Pythagorean theory)”.

According to Table 7, based on answers to the question “Please state a formula you learned in high school.”, 71.69% of pre-service teachers gave true answers while 6.28% gave false answers and 22.01% skipped the question. Predominantly, true answers are as follows:

“Volume of a sphere: $\frac{4}{3}\pi r^3$”, “Area of a circle:πr^2”, “Volume of a cube:a^3”, “Δ= $b^2 - 4ac$”, "$x_{1,2} = \frac{-b\pm\sqrt{\Delta}}{2a}$" , “Area of a trapezoid:$\frac{(a+b).h}{2}$”, "$h^2 = e.f$", "$\bar{x} = \frac{x_1+x_2+\cdots+x_n}{n}$ ", “Circumference of a circle: 2πr”, “The Pythagorean theory:$a^2 = b^2 + c^2$”, "$h^2 = p.k$", “Volume of a cylinder:$\pi r^2 h$”, “y-$y_0 = m(x - x_0)$”.
Predominantly, false answers are as follows:

"$\lim_{x\to\infty} f(x) + g(x) = \lim_{x\to\infty} f(x) + \lim_{x\to\infty} g(x)$”, “$\sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n$”, "$x^3 + y^3 = (x - y)(x^2 + xy + y^2)$”, "$\log_a b = \frac{1}{\log_b a}$”, “$(sinx)' = cosx$”, “Sin2a=2sina.cosa”, "$sin^2x + cos^2x = 1$”, “tanx= $\frac{1}{cotx}$”, “y=mx+n”, “r=$\frac{-b}{2a}$ (peak value of the parabola)”, “n!=1.2.3….(n-1).n”, "$\log_a b^a = a.\log_a b$".

According to Table 8, based on answers to the question “Please state a rule you learned in high school.”, 62.89% of pre-service teachers gave true answers while 5.66% gave false answers and 31.44% skipped the question. Predominantly, true answers are as follows:

"$\log_a b^a = a.\log_a b$”, “$sin^2x + cos^2x = 1$”, "$\log_a b$, when b=1, $\log_a 1$=0”,“Two angles of an isosceles triangle are the same.”, “The highest degree in a polynomial is the polynomial’s degree.”,“Minimum 3 points are needed for a figure to define a plane.”, “Odd exponential of negative numbers is again negative.”, “Injective and surjective functions have inverses.”, “Inscribe angle is equal to the half of the arch it faces.”, “tanx.cotx=1”, If “$\Delta < 0$ then there is no real root”, If “ACB ve BCC then ACC”, "$i^2 = -1$, $i^4 = 1$”, “The L-Hospital rule”, “Sin2x=2sincosx.

Table 9. Categorization of answers to the question “Is there mathematics without formula and rule?”

Yes		No	
Frequency (f)	**Percentage**	**Frequency (f)**	**Percentage**
59	37.10	100	62.89

Table 10. Categorization of answers to the question “According to you, how do mathematics without formula and rule become?”

Categories	Frequency (f)	Percentage
Maths would get more difficult and meaningless	22	13.83
Maths would get very abstract	2	1.25
Since subjects are interrelated in maths, the mathematical process would rely upon encoding and decoding	1	0.62
Students would be involved in the process	2	1.25
Empty	13	8.17
It would become a mix of unsolvable problems	6	3.77
I even don’t want to think	1	0.62
Then there is no maths	10	6.28
It would be excellent	3	1.88
No learning and knowledge acquiring would take place	3	1.88
It would be needed to develop solutions with individually proven concrete elements	14	8.80
We even would become unable to do the four basic operations of maths	1	0.62
Maths would be more productive	10	6.28
We could not rely upon any piece of information and everything would be baseless	3	1.88
It would be more instructive and permanent	9	5.66
It would foster curiosity	1	0.62
It may get complicated in absence of certain rules	2	1.25
Maths would be interpretation-oriented	6	3.77
It would cause loss of time	13	8.17
Memorization could be eliminated	8	5.03
Maths would get a concrete form	6	3.77
It would be more detailed and meaningful	4	2.51
It would lose its distinguishing nature	1	0.62
Students would be afraid of maths	1	0.62
It would be based on employing and illustrating the logic	14	8.80

Predominantly, false answers are as follows:

“Menelaus’ theorem”, "$\Delta = b^2 - 4ac$",“Area of a rectangle= short edge . long edge”, “The Euclidean relation”, “Area of an equilateral triangle:$\frac{a^2\sqrt{3}}{4}$.

According to Table 9, based on the answers to the question “Is there mathematics without formula and rule?”, 37.10% of the pre-service teachers answered yes while 62.89% answered no. According to Table 10, based on the answers of pre-service teachers to the question “According to you, how do mathematics without formula and rule become?” the results are grouped into 25 categories. These categories are: “Maths would get more difficult and meaningless.”, “Maths would get very abstract.”,“Since subjects are interrelated in maths, the mathematical process would rely upon encoding and decoding”, “Students would be involved in the process”, “It would become a mix of unsolvable problems.”, “I even don’t want to think.”, “Then there is no maths.”, “It would be excellent.”, “No learning and knowledge acquiring would take place.”, “It would be needed to develop solutions with individually proven concrete elements.”, “We even would become unable to do the four basic operations of maths.”, “Maths would be more productive.”, “We could not rely upon any piece of information and everything would be baseless.”, “It would be more instructive and permanent.”, “It would foster curiosity.”, “It may get complicated in absence of certain rules.", "Maths would be interpretation-oriented.", "It would cause loss of time.", "Memorization could be eliminated.", "Maths would get a concrete form.", "It would be more detailed and meaningful.", "It would lose its distinguishing nature.", "Students would be afraid of maths.", "It would be based on employing and illustrating the logic.", Pre-service teachers stated that maths without formula and rules would get complicated and become meaningless, and therefore they would have difficulty in explaining any theory, and such formula and rules are indispensable elements of maths. Maths would be very abstract, and therefore be only limited to theory and assumption. It would be a mix of unsolvable problems. They stated that just as plants would not grow without water, there would be no maths without formula and rules. That maths would be implemented by way of individually proven concrete solutions, namely proofs. That maths would be more productive, in other words, it would allow the invention of new concepts. That maths would be interpretation-oriented, and hence different solutions and different answers would arise. That this would cause loss of time, resulting in extended operations. And finally, that maths would gain a concrete nature, subjects would be taught in concrete forms, namely through physical materials or real-life examples.

DISCUSSION

The findings of the research are the following:

It is shown that answers of pre-service teachers to the question "What is formula?" are grouped into 23 categories. These categories can be sorted by a number of answers as follows: "Practical information for the simple solution of a problem", "Solution method used for a specific subject", "Stereotyped statement requiring memorization with no underlying rationale illustrated", "There is nothing called formula", "Concept formed by symbols", "Conclusion part of the proofs", "Provable rules", "Proven form of the theories", "Set of symbols relating to the phenomenon, principle or rule", "A correlation", "Concrete form of abstract thoughts", "Fixed form of specific aspects" "Theoretical part of mathematics, the abstract side", "Summary of the subjects, the theme", "Simplified form of statements". 4.40% of pre-service teachers skipped this question. Each student exhibits that formula represents a justification in his/her mind. In the answers, a clear conceptual definition of the formula emerges. It was found that students mathematically transform formula into a field-specific term, and are aware of the significance of formula for maths. It is evident that formula as a component of maths actually makes a scientific sense for the students.

It is shown that answers of pre-service teachers to the question "What is rule?" are grouped in 26 categories. These categories can be sorted by number of answers as follows: "The path to the solution of a problem", "A principle that people should respect", "Shortcuts to simplify complicated operations", "Conclusive", "Ensures that the lesson is studied in a systematic manner", "A precondition and necessity in problem solving", "Theory", "Anything previously proven and accepted to be true", "Limits, borders and criteria specific to each matter", "Something indispensable for the matter", "The rationale, core of a subject", "An element that makes the maths meaningful", "Something that cannot be proven and changed", "The path to implementing the formula", "Fixed form of results derived from certain aspects", "Shows the method and place of a mathematical process". In addition, 9.43% of pre-service teachers skipped this question.

While presenting mathematics to students, the main goal of teachers is to provide a quick understanding of the concepts and deliver the premises that will bring solution to the question. Perhaps the most important of these premises is the virtual keys called "rules". Because teachers generally believe that maths could be presented more easily with this strategy, answers suggest that this is not a wrong approach. The vast majority of pre-service teachers think that the mathematical logic can be reduced down to rules. This perception draws an important framework for lecturing. It is believed that the answers provided can drive the emergence of new teaching methods for mathematics. Students believe that fitting the rule sequence into a logical framework would streamline learning. Student answers support the definition of rule as simple understanding. If the teacher involves into the mental process the logical rule statements hidden in each subject, understanding would further be easier.

Based on answers to the question "What is the first formula you remember?", 88.05% of pre-service teachers gave true answers while 8.80% gave false answers and 3.14% skipped the question. Answers reveal that students mostly formulate the concept of formula correctly in their minds, and that most answers are parallel. The reason of such parallelism between answers may be argued that teachers frequently refer to these formulas conceptually during their presentations in the class, teachers create strong ties with these formulas when solving problems, and that formula strongly stick to mind. The multiplicity of similar formulas paves the way for reviewing the curriculum. The answers of pre-service teachers can be taken into consideration in fine-tuning the course contents and distribution of the questions.

Based on answers to the question "What is the first rule your remembering?", 80.50% of pre-service teachers gave true answers while 8.80% gave false answers and 10.69% skipped the question. For students, rules serve as a pill. The steps of the subject are further reinforced by rules and this approach underlines the necessity to build a discipline in presenting the rule. Student answers reveal that rather symbolic and frequently used statements are permanent in mind. Again, majority of the answers show that rule statements are dominant in subjects where the teacher has no difficulty to present. It may further be argued that the order of importance of subjects shapes student answers.

Based on answers to the question "Please state a formula you learned in secondary school.", 67.92% of pre-service teachers gave true answers while 18.86% gave false answers and 13.20% skipped the question. During secondary education, teacher should initially present more reasonable concepts, in other words concepts requiring less memorization to students. Because, in such a period when the fundamentals of maths are laid, the teacher should avoid a memorization reflex in students, and illustrate them that everything has a logical explanation. However, this is quite difficult to apply for maths because, due to interaction with individuals in social life, the student comes to class loaded with a prior assumption that maths is a series of formula. This way of thinking makes the teacher's job difficult and teachers have to destroy this wrong belief first. Especially, when parents frequently mention about their bad experiences with maths during their education life, student's perception of maths is adversely impacted because parents also perceived maths as being pinched into a formula form, thus formulating it that way in their mind. Such formulation is similarly transmitted to children. Student answers support these comments "$a^2 = b^2 + c^2$". Answers such as "$b^2 - 4ac = \Delta$" etc, may be provided as a proof of the fact that information flow is linked with conveyances. Furthermore, especially for formula expressions where students have difficulty, teacher statements like "this will have relevance in upcoming lessons" causes frequent repetition of such similar expressions.

It should be taken into consideration that the majority of answers are jointly included in the secondary and high school curricula. An analysis of the answers suggests that the Pythagorean relation, the association frequency of trigonometric symbols, identities, multiplicity of area and circumference equations of simple geometric figures or frequently repeated geometric forms as highly repeated subjects, are the basis of persistence in the minds of students.

Based on answers to the question "Please state a rule you learned in secondary school.", 71.69% of pre-service teachers gave true answers while 6.28% gave false answers and 22.01% skipped the question. Answers reveal that pre-service teachers frequently repeat the mathematical discourses that make up the basic phenomena. In particular, it is shown that pre-service teachers are more familiar and better in the subjects of angles, divisibility and arithmetic operations. The presentation of these subjects intensively and systematically by pre-service teachers in information form may suggest the proper formulation of the secondary school curriculum. These data show that mathematical teaching can be directed to correct methods. The secondary school mathematics curriculum is aimed at ensuring that students learn by practice and answers reveal the benefits of learning by practice as well. The answers of pre-service teachers also indicate that the "rule" perception is managed properly.

Based on answers to the question "Please state a formula you learned in high school", 71.69% of pre-service teachers gave true answers while 6.28% gave false answers and 22.01% skipped the question. The multitude of examinations for future in high school tells us that the concept of formula needs to be discussed in a different platform. Because, during this period, students only focus on two key words, "time" and "correct". Time is important for them because exams are measured by performance against time, and again the "correct answer" is important for students because this defines the score. Particularly during the high school period, due to aforementioned reasons, students care more about the phenomena considered as "formula". The basic assumption of this care is that each student can easily achieve both objectives of time and correct answer if they know more "formula". In particular, the facts in Table 7 should be analysed in the light of the two reasons above. Among the answers, the multitude of $x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a}$, trigonometric expressions, limit formula and polygonal circumference concepts, are critical proofs indicating that second-degree equations, trigonometry, limit and concepts of area and circumference, respectively are experienced frequently. Also identity is accepted as one of the key subjects that can be involved in this group. Frequent mention of the aforementioned items in the answers of pre-service teachers may suggest that these subjects constitute basis to all other subjects in high school.

Based on answers to the question "Please state a rule you learned in high school.", 62.89% of pre-service teachers gave true answers while 5.66% gave false answers and 31.44% skipped the question. Based on the answers to the question "Is there mathematics without formula and rule?", 37.10% of the pre-service teachers answered yes while 62.89% answered no.

Based on the answers of pre-service teachers to the question "According to you, how do mathematics without formula and rule become?" are grouped into 25 categories. These categories are: "Maths would get more difficult and meaningless.", "It would become a mix of unsolvable problems.", "Then there is no maths.", "It would be needed to develop solutions with individually proven concrete elements.", "Maths would be more productive.", "We could not rely upon any piece of information and everything would be baseless.", "It would be more instructive and permanent.", "It may get complicated in absence of certain rules.", "Maths would be interpretation-oriented.", "It would cause loss of time.", “Memorization could be eliminated.”, “Maths would get a concrete form.”, “It would be based on employing and illustrating the logic.”,

No adequate resource could be found in literature after review within the scope of this study. Based on resources found, the answers of pre-service teachers for the question “What is formula?” point to a parallelism between this study and the study of Isık et al. (2005). Considering the answers to the question “According to you, how do mathematics without formula and rule become?”, this study shows a parallelism with the studies of Ozdemir and Uzel (2011), Altay and Umay (2011), Ahuja et al. (1998) and Pale (2016), as particularly supported by the answers of “it would be based on employing and illustrating the logic”, “memorization could be eliminated.”, and “students would be involved in the process”. Moreover, considering the opinion “no learning and knowledge acquiring would take place” stated in this study, the latter also shows a parallelism with the study of Karakus (2004). Since students perceive maths as a process of memorizing formula and rules, opinions such as “I can’t imagine maths without formula and rules”, “It would become a mix of unsolvable problems”, “We could not rely upon any piece of information”, “It may get complicated in absence of certain rules” and “Maths would be interpretation-oriented” may arise.. In this respect, the study further shows a parallelism with the studies of Pale (2016) and Ozudogru (2016). The following suggestions may be developed based on this study:

Pre-service teachers have used examples involving similar formula for many times. Accordingly, the suggestions are:

1. To review the curriculum as well as the teaching methods and techniques;
2. To restructure the distribution of topics in high school and redesign the study hours;
3. Maths teachers should teach topics as accompanied by their rationale, their origin and areas and methods of using them;
4. Teachers should encourage the use of formula in an aim to prevent students from taking the easy way out;
5. Teachers should infuse a higher level of awareness when presenting the topic;
6. Transforming the student’s belief of “formula helps to solve the problem more easily” into more logical and more educational concepts;
7. To discuss the concepts of formula and rule in maths education on subject basis. Hence, the study could be more specifically addressed.
8. To analyse how students construct in their minds the examples given by the teacher to support the concepts of formula and rule;
9. Teachers should arrive at formulas and rules collectively with their students. They must avoid spoon feeding the students and prevent them from memorizing.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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