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Fraction Intervention for Students With Mathematics Difficulties: Lessons Learned From Five Randomized Controlled Trials

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Abstract

In this article, the authors summarize results from 5 randomized controlled trials assessing the effects of intervention to improve the fraction performance of fourth-grade students at risk for difficulty in learning about fractions. The authors begin by explaining the importance of competence with fractions and why an instructional focus on fractions magnitude understanding may improve learning. They then describe an intervention that relies strongly on this type of understanding about fractions instruction, and they provide an overview of the intervention's overall effects. This is followed by an overview of 5 intervention components for which the authors isolated effects. They conclude by discussing some of the lessons learned from this research program.

Keywords

fractions, fraction magnitudes, intervention, math difficulty, risk, mathematics difficulty

Competence with fractions is important for success with more advanced mathematics and in the American workforce (Geary, Hoard, Nugent, & Bailey, 2012; National Mathematics Advisory Panel [NMAP], 2008; Siegler et al., 2012). Yet understanding about fractions and skill in operating with fractions are difficult for many students, especially those who have experienced difficulty with whole-number concepts and operations (Namkung & Fuchs, 2016; Seethaler, Fuchs, Fuchs, & Compton, 2011). Due to the challenge that fractions present, the NMAP assigned high priority to improving fraction instruction to increase success in algebra and beyond. The purpose of the present article is to provide an overview of results from a series of five randomized controlled trials (RCTs) examining the effects of a fraction intervention on understanding of and procedural skill with fractions for students who begin fourth grade with poor whole-number computational skill. In this article, we refer to this population as *students at risk for difficulty in learning about fractions* (at-risk [AR] students).

The fraction intervention that we developed and evaluated for use with AR fourth graders focuses mainly on fractions magnitude. This type of understanding is often represented with number lines (Siegler, Thompson, & Schneider, 2011). Although such understanding can be linked to children's experiences with measuring, it depends largely on formal instruction that explicates the conventions of symbolic fraction notation (e.g., what the 3 and 4 mean in ¾), the inversion property of fractions (e.g., fractions with the same numerator become smaller as denominators increase), and the infinite density of fractions on any segment of the number line.

The other form of understanding relevant at fourth grade is the part-whole interpretation of fractions. This involves understanding a fraction as one or more equal parts of a single object (e.g., two of eight equal parts of a cake) or a subset of a group of objects (e.g., two of eight cakes). Such understanding is typically represented with an area model, in which a region of a shape or a subset of objects is shaded. It is more intuitive, based on children's experiences with sharing, and observed in young children (Mix, Levine, & Huttenlocher, 1999).

The NMAP (2008) hypothesized that improvement in fractions magnitude understanding is more critical than the part-whole interpretation in developing competence with fractions, in part because the part-whole interpretation encourages separate counting of numerator and denominator

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segments, which reinforces children's tendency to conceptualize a fraction as two separate whole numbers. By contrast, an emphasis on fractions magnitude understanding encourages relational thinking about numerators and denominators as determinants of a single number.

Even so, part-whole understanding about fractions continues to dominate American schooling (e.g., Fuchs, Malone, et al., 2016). Therefore, in our series of RCTs, a key distinction between study conditions was that a focus on fractions magnitude understanding dominated instruction in the intervention conditions (designed and conducted by the research team). By contrast, the part-whole interpretation dominated fraction instruction in the business-as-usual control condition (designed and provided by the participating schools). (Additional distinctions between the intervention and control groups are described later in this article.)

The first study of the 5-year series was a two-condition RCT contrasting the initial iteration of the core fraction intervention against the control group. In the subsequent 4 years, each RCT had three arms: a control group and two variants of the intervention. Both variants in each RCT relied on the same core program, which we iteratively improved over the years. However, the two variants of the intervention differed by one component, with the goal of isolating the effects of those two components. Isolating the effects of key intervention components was important to optimize the design of (i.e., determining which components to include in) the intervention program to optimize student outcomes.

This article is organized in three sections. First, we describe the core fraction intervention and its overall effects when compared with the control condition. Then, we provide an overview of the intervention components for which we isolated effects. We conclude by discussing some of the lessons learned about intervention for AR students.

Overall Effects of Intervention When Compared With Control Condition

We begin this section by describing major distinctions between the fraction intervention and the control group. In the next section, we summarize key features of the RCTs, including participants, fidelity, and study measures, and we report the efficacy of the fraction intervention as contrasted against the control condition. We do this for each RCT, by measure, and provide an aggregate effect size (ES) for each measure across the RCTs. Due to the brevity of this article, readers should consult primary study reports to obtain complete information for each of the five RCTs: for the Year 1 study, Fuchs et al. (2013); the Year 2 study, Fuchs et al. (2014); the Year 3 study, Fuchs, Schumacher, et al. (in press); the Year 4 study, Fuchs, Malone, et al. (2016); and the Year 5 study, Fuchs, Malone, Sterba, and Wang (2015).

Key Distinctions Between Fraction Intervention and Business-as-Usual Control Condition

The control group represented the schools' business-as-usual fraction instructional program, which involved classroom instruction as well as intervention for most control group participants. The fraction intervention and business-as-usual control conditions differed along five dimensions. Readers should consult the primary studies just referenced for additional information on the specifics of these conditions; the richest description of the control condition is found in Fuchs, Malone, et al. (2016). Also, an intervention manual, *Fraction Face-Off!* (Fuchs, Schumacher, Malone, & Fuchs, 2015), includes materials and guides for the 36 lessons.

The first distinction, already mentioned, was that fraction intervention primarily emphasized fractions magnitude understanding, whereas the control condition relied primarily on the part-whole interpretation of fractions. The second distinction involved the relative emphasis on concepts versus procedures. Although both conditions focused on concepts as well as procedures, the intervention focused more on understanding, whereas the control condition focused more on procedures and more often relied on procedures to obtain solutions to conceptual tasks (e.g., cross multiplying to compare the value of fractions).

The third distinction involved the scope of topics covered. The control condition addressed a greater range of fractions, with more challenging denominators; it addressed more fraction topics, including estimation and word problems. By contrast, the intervention condition restricted fraction coverage to reduce calculation demands, did not address fraction estimation, and only began addressing word problems in Year 3. The fourth distinction concerned the duration, frequency, location, and format of fraction intervention that was added to classroom instruction. Our intervention occurred outside the classroom for 12 weeks in groups of two to four students, three times per week, for 30 to 35 min per session. The duration, frequency, location, and format of the control condition's intervention were more variable.

Finally, our fraction intervention relied more on systematic, explicit instructional principles to (a) maximize the clarity of instruction; (b) address foundational skill deficits and the kinds of cognitive/linguistic limitations that AR students often experience (e.g., poor language comprehension and limited working memory); (c) optimize student attention, participation, motivation, and perseverance; and (d) ensure responsive tutor feedback.

Key Study Features, Overall Efficacy, and Mediation

In the five RCTs, we defined risk as performance <35th percentile at the start of fourth grade on a broad-based

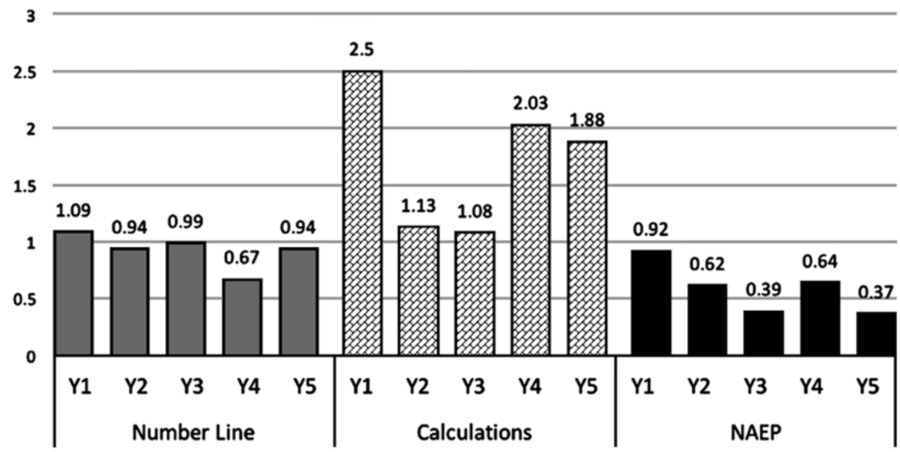

Figure 1. Effect sizes for number line, calculations, and National Assessment of Educational Progress (NAEP) by study year (Y).

nationally normed computation test (*Wide Range Achievement Test–4*; Wilkinson & Robertson, 2006). At the start of fourth grade, this measure largely taps whole-number computational skill. To ensure strong representation across the range of scores <35th percentile, we systematically sampled AR students from more versus less severe risk strata (<15th percentile vs. 15th–34th percentile). We also administered the two-subtest *Wechsler Abbreviated Scale of Intelligence* (Wechsler, 1999) to students who met the risk criterion, and we excluded those with T scores <9th percentile on both subtests. The reason for this exclusion was that the series of RCTs was geared to address the needs of students with mathematics difficulty and learning disabilities, not intellectual disability. We sampled three to nine AR students per class, stratifying by more versus less severe risk.

The annual sample, depending on year, was approximately 250 students from 50 classrooms in 15 schools. Mean pretest computation performance approximated a standard score of 75; mean IQ was in the low 90s. The sample was predominantly African American and Hispanic, and about 90% received subsidized school lunch. We randomly assigned AR students at the individual level, stratifying by classroom and risk severity, to two study conditions in Year 1 (intervention vs. control) and to three conditions in Years 2 through 5 (control vs. Intervention Variant 1 vs. Intervention Variant 2). Each year, we audiotaped every intervention session and randomly sampled 20% of recordings such that tutor, student, and lesson were sampled comparably. These tapes were coded to identify the percentage of essential points that the tutor implemented, which exceeded 95 in each RCT.

In this article, we report Hedges *g* ESs, comparing the intervention group against the control group for the Year 1 RCT. In the remaining four RCTs, where we had a control group and two variants of the intervention, we report the mean ES comparing students across the two intervention conditions against the control group. We do this for three study measures, all from the *Vanderbilt Fraction Battery* (Schumacher, Namkung, Malone, & Fuchs, 2013). Test-retest reliability or alpha, calculated on the study samples, exceeded .80 on all study measures, including those we refer to later, when discussing the effects of the intervention components in Years 2 through 5.

Number line estimation. The first measure, number line estimation, indexed performance on an aspect of fraction knowledge to which the intervention condition allocated more attention than the control condition: fractions magnitude understanding about fractions. On the number line estimation task, students place proper fractions, improper fractions, and mixed numbers on a number line marked with the endpoints 0 and 2. The score is the mean absolute value of the difference between where the child places the number and where the number belongs. On this task, intervention students significantly outperformed control students each year (see Figure 1 for ES by study year; i.e., by RCT). Across RCTs, the mean ES was 0.93. The fact that large effects were found on this computer-administered number line task, which is structured differently from the intervention's instructional activities, suggests that students who received intervention transferred their understanding about fractions to a more general representation, the fractions magnitude understanding. This is noteworthy also because performance on the number line task is a strong predictor of fraction learning across Grades 3 to 5 (Jordan et al., 2013) and more advanced mathematics achievement, including algebra (e.g., Siegler et al., 2012).

Addition and subtraction of proper fractions and mixed numbers. The second outcome measure indexed addition and subtraction of proper fractions and mixed numbers. Because the control condition allocated more attention than the intervention condition to this topic, we had expected effects to favor the control condition. Yet, intervention students again significantly outperformed control students each year (see Figure 1 for ES by study year; i.e., by RCT). Across RCTs, the mean ES was 1.72. Moreover, when aggregated over the five RCTs, the achievement gap of intervention students narrowed by 0.99 *SD* from pre- to postintervention on the calculation measure (on which we had normative data on not-AR classmates). By contrast, the achievement gap for control group students remained approximately constant, with a mean ES increase in achievement gap of 0.11 *SD* from pre- to postintervention. This is an advantage of 1.10 *SD* in narrowing the achievement gap. The large effects favoring the intervention over the control group, even though the control group allocated more instructional time to computational procedures, suggest that understanding of fractions is important for learning computational procedures involving fractions, as shown in earlier work (Hecht, Close, & Santisi, 2003; Mazzocco & Devlin, 2008; Rittle-Johnson, Siegler, & Alibali, 2001).

Generalized learning about fractions. Our third measure indexed generalized learning about fractions, with a strong focus on fraction concepts. This measure was comparably different from the focus of instruction in both conditions and addressed fractions magnitude understanding and the part-whole interpretation of fractions with equal emphasis. We administered 19 released items from the 1990–2009 National Assessment of Educational Progress (NAEP): easy, medium, or hard fraction items from the fourth-grade assessment and easy from the eighth-grade assessment. Here, too, intervention students significantly outperformed control students each year (see Figure 1 for ES by study year; i.e., by RCT). Across RCTs, the mean ES was 0.58. So effects were reliably stronger for the two intervention conditions over the control group on this more general outcome. Even so, NAEP effects were statistically significant, and the ESs were substantively important—and in some years, large—according to the What Works Clearinghouse's guidelines. Moreover, when aggregated across the RCTs, the NAEP achievement gap narrowed markedly for intervention students (a decrease in ES of 0.52 *SD* units from pre- to posttest), while the gap remained approximately constant for the control group (an increase of 0.04 *SD*). This is an advantage of 0.56 *SD* in narrowing the achievement gap.

Based on the NMAP's (2008) hypothesis that improvement in fractions magnitude understanding mediates the effects of fraction intervention, we tested for such effects in the first three RCTs. Each year, we found evidence suggesting that fractions magnitude understanding, but not part-whole understanding about fractions, was a mediator of the intervention's effects. For example, in Year 1, we conducted three related analyses. In the first, the outcome was the NAEP total score, on which half the items assess fractions magnitude understanding and the other half assess the part-whole interpretation of fractions. The mediator variable was improvement on the number line task (an index of fractions magnitude understanding). We created a stringent test of the hypothesis by controlling for improvement in adding and subtracting fractions, on which effects were a substantial 2.50 *SD*. Despite this stringent control, the intervention's indirect effect (via improvement in fractions magnitude understanding) was significant, with fractions magnitude understanding partially mediating the effects of fraction intervention on the NAEP total score, a general outcome.

We could not use an analogous method to assess the mediating role of improvement in part-whole understanding on the NAEP outcome, because our only index of part-whole understanding was the subset of NAEP part-whole items. We therefore conducted two complementary analyses. These assessed (a) whether improvement on the NAEP items indexing fractions magnitude understanding mediated intervention effects on the NAEP part-whole items and (b) whether improvement on the NAEP part-whole items mediated intervention effects on the NAEP items indexing fractions magnitude understanding. Results showed that improvement in fractions magnitude understanding completely mediated the effects of intervention on the part-whole understanding outcome but that improvement in part-whole understanding did not mediate the effects of intervention on the outcome of fractions magnitude understanding.

Mediation effects are correlational. But, when combined with results favoring intervention over control on each of the three major study outcomes (documented in the context of RCTs), findings suggest a causal role for an emphasis on fractions magnitude understanding on fraction learning.

Fractions magnitude understanding is less intuitive than part-whole interpretation, which has dominated American schooling (and was thus emphasized in the RCTs' business-as-usual control groups). Fractions magnitude understanding, by contrast, depends more on formal instruction. The NMAP (2008) hypothesized that improvement in fractions magnitude understanding is an important mechanism in the development of fraction knowledge, and it recommended that fraction instruction be reoriented in this direction. Our findings provide support for this hypothesis.

Contribution of Five Intervention Components

As noted, each year in Years 2 through 5, we ran a three-arm RCT that not only relied on a control group but also incorporated two versions of the core fraction program. The two versions differed by including two contrasting intervention components. In each RCT, 25 min in each intervention session were identical across the two conditions; instructional methods differed for the other 5- or 7-min component (5 min in Year 2; 7 min in Years 3–5). To estimate the effects of these two components, we compared the intervention components with each other. This created a stringent test to evaluate each intervention component, because the contrast condition had high-quality relevant fraction instruction (via the same core program), with the same amount of intervention time provided in the same small-group format.

Different Forms of Practice Benefit Students Differentially in Intervention, Depending on Working Memory Capacity

In Year 2, we compared the same core program with two contrasting forms of practice. We referred to one of these forms of practice as the *fluency practice condition*, in which students completed strategic speeded activities to build fluency on four topics central to fractions magnitude understanding. The four topics were as follows: identifying fractions equivalent to ½; identifying which of two proper fractions is greater; identifying whether numbers are proper fractions, improper fractions, or mixed numbers; and identifying which of two fractions is greater (one a proper fraction, the other an improper fraction). We referred to the other form of practice as the *conceptual practice condition*, in which students explained their reasoning, with the aid of manipulatives, about the same topics. The fluency condition was designed to help students automatize steps for deriving accurate solutions on four topics central to fractions magnitude understanding. The conceptual condition was designed to consolidate the ideas represented in those topics. We found no significant difference between conditions on the number line task, fraction calculations, or NAEP fraction items.

We did, however, find a moderator effect. On the number line task, this interaction showed that conceptual practice was superior to fluency practice for students whose working memory capacity (measured at the start of the study) was very low but that fluency practice promoted better learning for students with more adequate working memory. For example, at the sample's 10th percentile on working memory, the ES favoring conceptual over fluency practice was strong (0.61). The opposite was true at the sample's 90th percentile on working memory, with an ES of 0.52 favoring fluency over conceptual practice.

This finding suggests the potential for personalizing intervention in line with a student's cognitive profile at the start of intervention, although additional research is clearly required before recommending that schools implement such an approach. It is, however, interesting to consider why different forms of practice produce varying effects depending on children's working memory capacity. In this study, as part of the larger intervention, students in both conditions received instruction focused strongly on fraction understanding, but the larger program also taught strategies for executing tasks central to fractions magnitude understanding: for example, segmenting fraction comparisons into a series of steps, each of which is less resource demanding, and considering fractional values in relation to benchmark fractions, such as one-half. Only students in the fluency condition, however, completed speeded activities to build automaticity with such strategies.

The goal of this fluency practice was to reduce demands on (or compensate for poor) working memory. Yet, as the results show, a minimum amount of working memory does appear required to benefit from the speeded practice. It is also possible that the conceptual understanding underlying the four topics central to fractions magnitude understanding was less strong for students with very low working memory. In either case, however, a larger proportion of students benefited more from fluency than conceptual practice. So, if schools must decide between the two forms of practice, fluency would be preferred for the majority of students. For this reason, in subsequent iterations of the intervention (Years 3–5), we adopted fluency practice in the core program. Practitioners should, however, be mindful that for a small number of students (those with severe working memory deficits), conceptual practice appears superior.

Multiplicative Fraction Word-Problem Instruction Benefits AR Fourth Graders More Than Additive Fraction Word-Problem Instruction

In Year 3, we contrasted two types of word-problem intervention, each involving fractions and each integrated within the core program. We targeted word problems because the best school-age predictor of employment and wages in adulthood is word problems (see, e.g., Every Child a Chance Trust, 2009; Parsons & Bynner, 1997). A combined focus on fractions and word problems therefore represents an important instructional target.

The word-problem intervention was designed to enhance performance on multiplicative word problems in one condition and on additive word problems in the other condition. We were primarily interested in multiplicative word problems for two reasons. First, multiplicative thinking is central to fraction knowledge, as reflected in the fact that finding equivalent fractions requires multiplying or dividing the numerator and denominator in one fraction by the same quantity. Second, multiplicative thinking with fractions can be difficult to achieve, in part because multiplying two proper fractions results in smaller quantities and dividing a proper fraction by a proper fraction produces larger amounts.

The multiplicative word-problem intervention component focused on "splitting" and "grouping" word-problem types, examples of which follow, respectively:

1. Matthew has 2 watermelons. He cuts each watermelon into fifths. How many pieces of watermelon does Matthew have?
2. Keisha wants to make 8 necklaces for her friends. For each necklace, she needs ½ of a yard of string. How many yards of string does Keisha need?

The contrast additive word-problem condition focused on fraction "increase" and "decrease" word-problem types, examples of which follow, respectively:

1. Maria bought $1^4/_{10}$ pounds of candy. Later she bought another $^3/_{10}$ of a pound of candy. How many pounds of candy does Maria have?
2. Jessica had $^5/_6$ of a cake. She gave $^2/_6$ of the cake to her friend. How much cake does Jessica have now?

The instructional approach in both word-problem intervention components was schema-based instruction (e.g., Fuchs et al., 2010; Jitendra & Star, 2012), in which students learn to identify word problems as belonging to word-problem types that share structural features (splitting and grouping word-problem types in the multiplicative word-problem condition; increase and decrease word-problem types in the additive word-problem condition). With schema-based instruction, students are also taught to represent the underlying structure of the word-problem type with a number sentence (e.g., Fuchs et al., 2003; Fuchs et al., 2009; Fuchs et al., 2010) or visual display (e.g., Jitendra & Star, 2012; Jitendra et al., 2009). Schema-based design principles constituted 64% of the word-problem instructional emphasis in the two intervention conditions. By contrast, the control group allocated no attention to schema-based instructional design principles and instead relied heavily on key words.

Effects on the word-problem outcomes were as we had expected. On multiplicative word problems, the multiplicative word-problem condition outperformed the control group (ES = 1.06) as well as the additive word-problem condition (ES = 0.89). By contrast, on additive word problems, the additive word-problem condition outperformed the control group (ES = 1.40) as well as the multiplicative word-problem condition (ES = 0.29). It is, however, noteworthy that the ES comparing the two active conditions was dramatically smaller on additive word problems than the ES comparing to the two active conditions on multiplicative word problems: 0.89 vs. 0.29. Moreover, whereas the multiplicative word-problem condition outperformed the control group on additive word problems (ES = 1.10), the additive word-problem condition and control group conditions performed comparably on multiplicative word problems, with an ES of only 0.16.

Thus, schema-based intervention on multiplicative word problems produced positive overall effects on fraction word problems, including multiplicative and additive word problems. By contrast, the effects of schema-based intervention on additive word problems were limited to additive word problems. This suggests that intervention on multiplicative word problems is a more efficient instructional target for improving fractions word problems, at least at fourth grade.

Supported Self-Explaining Enhances Understanding of Fraction Magnitudes

The major purpose of the Year 4 RCT was to isolate the effects of teaching children to provide sound explanations regarding a critical indicator of fraction understanding: comparing fraction magnitudes. Evaluating the effects of self-explaining for AR learners is important because explaining is a broadly recommended instructional strategy and a strong focus in the mathematics career and college-ready standards.

Three types of self-explaining are described in the literature. *Spontaneous self-explaining* occurs when learners generate explanations without being prompted to do so. Individuals who spontaneously engage in self-explaining experience superior learning (e.g., Siegler, 2002), but not all learners spontaneously do so. With *elicited self-explaining*, learners are prompted to invent explanations. Here results are mixed. Rittle-Johnson (2006) provided insight for the inconsistency in findings, when she found that although prompting learners to self-explain promoted procedural accuracy more than a no-explanation condition, such self-explaining did not produce more sophisticated procedures or understanding. The reason was that children's self-explanations rarely included a conceptual focus and often led to incorrect procedures. So inventing sound explanations appears challenging and may depend on the cognitive processes associated with strong learning, such as reasoning, working memory, and language comprehension. These findings also suggest that the key ingredient in self-explaining may be processing and expressing high-quality explanations, not inventing explanations.

This brings us to the third form of self-explaining: *supported self-explaining*, in which learners operate on high-quality explanations already created for them. Many AR students experience limitations in the cognitive processes associated with mathematics learning and thus may be especially vulnerable to inventing subpar explanations. So, the approach that we chose to test in the Year 4 RCT was supported explaining, in which we modeled high-quality explanations, provided students with practice in analyzing and applying the explanations, and encouraged them to elaborate on and discuss important features of the explanations. The contrasting intervention condition received the same multicomponent fraction program without the explaining component. This ensured that the contrast condition had a high-quality relevant intervention on the same content. To control for intervention time, the contrast condition received the previously validated intervention component focused on multiplicative word problems.

In terms of effects on students' conceptual content knowledge, which we assessed via the accuracy with which students identify larger and smaller fractions, the explaining condition outperformed the word-problem condition with a moderate ES of 0.43. On a measure of the quality of explanations about why fraction magnitudes differ (scored for the explanations' conceptual content), effects more dramatically favored the explaining condition over the word-problem condition, with an ES of 0.93. These outcomes for the explaining condition over the word-problem condition are noteworthy because the multicomponent fraction intervention provided children in both conditions with the same instruction on the essential ideas and efficient procedural strategies for comparing fraction magnitudes. What distinguished the explaining condition from the word-problem condition was supported self-explaining.

We also found a compensatory moderator effect involving working memory. In the word-problem condition (which received the core program but without the explaining component), student outcomes correlated with working memory scores such that students with severe working memory deficits experienced poorer outcomes than those with more adequate working memory. By contrast, the supported explaining intervention compensated for working memory limitations such that students scored similarly well regardless of their working memory capacity.

Enhancing Understanding About Decimal Fractions

In the Year 5 RCT, we contrasted a component focused on decimal equivalents for tenths and hundredths fractions against the word-problem component. As expected, given success in preceding years of this research program for systematic, explicit interventions incorporating state-of-the-art thinking about the content area, students who received the decimal component significantly outperformed students who received the word-problem component on a task that included near- and far-transfer decimal items. The ES favoring the decimal over the contrast intervention condition was 0.97; the ES favoring the decimal over the control condition was 0.67. Although the effects of the decimal intervention were large, the effect was attributable largely to performance on near-transfer items (involving tenths and decimals), not on items demanding far transfer (involving thousandths). We therefore increased the difficulty of the decimal content addressed in this intervention component and are presently running an RCT to assess the promise of a component that addresses a greater range of decimals and additional principles involved in the understanding of decimals.

Lessons Learned and Conclusions

We conclude by sharing three of the lessons that can be derived from this series of RCTs. Lesson 1 is that fourth-grade students who are at risk for failure with the advancing mathematics curriculum, due to histories of poor mathematics achievement in the primary grades, can succeed with challenging mathematics content, if they are provided with a well-designed intervention. As the five RCTs demonstrate, an intervention that occurs in small groups and relies on systematic, explicit instructional principles, as well as state-of-the-art thinking about the content area (i.e., fractions magnitude understanding, in the case of fractions), improves AR students' performance on complex curricular content beyond what is achieved for AR control group students who receive school-based classroom and intervention programming. This is the case even when a measure of whole-number computation performance is used to screen students as AR for problems learning about fractions.

Moreover, as shown in a moderator analysis not reported in this summary of our research program (Fuchs, Sterba, Fuchs, & Malone, in press), this pattern of differentially strong outcomes for AR intervention over AR control students was similarly true regardless of the level of students' incoming whole-number mathematics achievement. Thus, more-severe-risk intervention students performed substantially better than more-severe-risk control students, even as less-severe-risk intervention students performed substantially better than less-severe-risk control students.

This brings us to Lesson 2. Although this intervention program was highly successful in terms of AR intervention students outperforming the AR control group, not all intervention students responded sufficiently to preclude the need for continuing intervention. This became clear in a disaggregation analysis (Fuchs, Sterba, et al., in press; not addressed above) only when we focused on achievement gaps with respect to not-AR classmates on the NAEP, which was not only our most challenging outcome measure but also the measure least aligned with our intervention.

On the NAEP, control group students throughout the distribution of initial (screening) achievement scores completed the intervention below the normalized performance criterion on the NAEP posttest. (Normalized postintervention performance was operationalized as one standard error of measurement above the postintervention 25th percentile of not-AR classmates.) By contrast, although most intervention students did achieve normalized performance on the NAEP posttest, those who began the study <13th percentile on the nationally normed screener (*Wide Range*

Achievement Test–4) failed to achieve normalized performance on the NAEP posttest.

This signals the need for researchers to examine the effects of interventions for students with more and less severe risk not only in terms of outperforming the counterfactual but also in terms of the size of the remaining achievement gap at the end of intervention (i.e., compared with classmates without a history of mathematics difficulty, those who began the year not-AR for fractions difficulty). In parallel fashion, schools should consider individual students' growth over the course of the intervention as well as their postintervention achievement gaps in determining whether students have "responded" to Tier 2 intervention and for formulating decisions about the need for subsequent intervention.

In a related way, Lesson 3 concerns the importance of considering moderator effects to identify AR students for whom an intervention does and does not work and to inform continued intervention development. As shown in the Year 2 RCT, although speeded strategic practice was superior to conceptual practice for the majority of students, those with very low working memory capacity profited more from conceptual practice. Space precludes discussion of each moderator effect that we identified across the 5 years, but we did identify other moderator effects that may eventually provide a path toward personalized intervention, guided by a student's profile of cognitive processes and academic skills. For example, the Year 4 RCT revealed a moderator effect in which the word-problem component was effective for the majority of students but not for those with extremely low nonverbal reasoning ability. This suggests the need to further develop the intervention to address these students' needs.

In sum, these five RCTs document the efficacy of the small-group intervention that we developed. Guided by explicit instructional principles and relying on fractions magnitude understanding, we iteratively developed and tested the effects of a core program, even as we isolated effects of a series of intervention components. Results of the component analyses indicate added value for speeded strategic practice (when students have sufficient working memory capacity to engage productively in such practice); they reveal strong added value for a multiplicative reasoning word-problem component and a supported self-explaining module; and they suggest directions for further strengthening a component focused on decimal fractions. At the same time, our intervention addresses only the fourth-grade fraction standards. Research is sorely needed to address the challenges associated with multiplying and dividing fractions as well as other complex mathematics curricular targets.

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