mathematical mnemonics

Finding meaning in mathematical mnemonics

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Abstract

A mathematical mnemonic is a visual cue or verbal strategy that is used to aid initial memorisation and recall of a mathematical concept or procedure. Used wisely, mathematical mnemonics can benefit students' performance and understanding. Explorations into how mathematical mnemonics work can also offer students opportunities to engage in proof and reasoning. This article will firstly illustrate how dissecting the so-called "butterfly method" for adding and subtracting fractions can deepen a student's comprehension of fraction arithmetic. Secondly, the article will exemplify how the same dissection process can be applied to other mathematical mnemonics.

Finding meaning in mathematical mnemonics

A mathematical mnemonic is a visual cue or verbal strategy used to aid initial memorisation and recall of a mathematical concept or procedure. Cross-multiplication, FOIL, and PEMDAS are common examples of mathematical mnemonics. Used wisely, mathematical mnemonics can increase performance and understanding, particularly within special education populations, about which the majority of research on mathematical mnemonics has been published (Greene, 1999; Kavale & Forness, 2000; Mastropieri, Scruggs, & Levin, 1987). In my experience, though, mathematical mnemonics have worked well in general student populations, too.

However, on occasion, mathematical mnemonics can be over-emphasised at the expense of conceptual fluency (Karp, Bush, & Dougherty, 2014; Kilpatrick, Swafford, & Finell, 2001). This is symptomatic of a difficult situation in some classrooms: "... too much focus is on learning procedures without any connection to meaning, understanding, or the applications that require these procedures" (National Council of Teachers of Mathematics, 2014, p. 3). Dr. Diane J. Briars (2014, July), the past president of the National Council of Teachers of Mathematics (NCTM), believes the overuse of mathematical mnemonics can impede some students' progress. When mathematical mnemonics are detached from conceptual understanding, they can become colourful tricks that usually get the right answer but distract from the meaning of mathematics.

Kalder (2012) and many others have seen first-hand how mathematical mnemonics can sometimes mislead students to view mathematics as a magical set of formulas to be memorised and applied without much thought to the matter. That is why educators such as Cardone (2013) have recently suggested eliminating mathematical mnemonics entirely from curricula. However, such a drastic approach is impractical and overlooks many of the positive findings of research (For example, Everett, Harsy, Hupp, & Jewell, 2014; Manalo, Bunnell, Stillman, 2000).

Indeed, students create and exchange mathematical mnemonics with one another quite naturally. Not only that, but encouraging students to explore how mathematical mnemonics work can provide them with opportunities to engage in proof and reasoning and, in fact, deepen their conceptual understanding.

Picture, for example, the following scenario culled from my various teaching and tutoring experiences over the years. This scenario is not a straightforward retelling of my interaction with a single student; rather, this scenario is an exercise in seeing how an instructor could use a mathematical mnemonic to initiate deeper discussions about mathematical meaning. It imagines a discussion with an introductory calculus student named John about his "butterfly method" for adding and subtracting fractions.

Dissecting the butterfly

John brings the butterfly method to my attention during a tutoring session. We are working on simplifying the rational expression:

$$\frac{x^2}{x-2}+\frac{3x+1}{x^2}$$

John draws curves and ellipses around the expression (see Figure 1) on a dry-erase board, and then writes in another place, without intermediate steps:

$$\frac{x^2}{x-2}+\frac{3x+1}{x^2}=\frac{(x-2)(3x+1)+(x^2)(x^2)}{(x-2)(x^2)}.$$

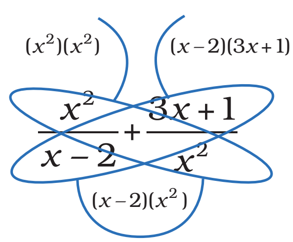

Figure 1. John applies the butterfly method to add rational expressions.

John is asked what he has done. "The butterfly method," John replies. "Did I get the right answer? I mean, it looks right. You said rational expressions work just like fractions, except you don't know what the *x* is. So, I thought you could just add them just like fractions, and you always add fractions using the butterfly method."

Ignorance about the butterfly method is expressed. The curves and ellipses drawn were taken as examples of fraction with no variables.

"Sure, no problem. It's easy."

He smiles and begins to write (see Figure 2).

"Say you want to add three quarters and two fifths. First, draw the wings and butt of a butterfly. Times the numbers in each wing, and put the answers in the antennas. Then, draw the butt of the butterfly and times the denominators. Put the answer inside the butt. Finally, add the antennas to get the numerator. The denominator comes straight from the butt. So, the answer's twenty-three twentieths. If instead you wanted to do three quarters minus two fifths, you'd just minus the antennas and get seven twentieths. And that's how you add fractions with the butterfly method."

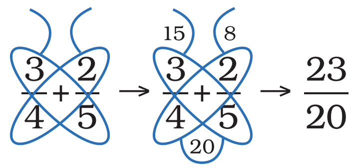

Figure 2. John applies the butterfly method to find $\frac{3}{4} + \frac{2}{5}$.

We sit in silence for a moment. It's easy; I think to myself. But it doesn't make much sense to me. However, my opinion of it doesn't matter. What really matters is, does John know how his method gets the right answer? There does not seem to be any mathematical sense-making undergirding John's initial explanation of the method.

"Can you explain to me how your method works?" I ask.

"I just did," John says. "You just draw the butterfly."

"No, I mean, how does it get the right answer? What's the logic?"

"That's just how you add fractions." Confusion floods his expression.

"When one adds fractions one always figures out the smallest number that's divisible by all the denominators and then change the numerators accordingly."

"Huh? What do you mean?"

"Well, let's take your example, three quarters plus two fifths. To use something concrete, this means we've sliced one apple into four equal pieces and taken three of them, then sliced another apple into five equal pieces and taken two of them. We want to know how much apple we've taken in terms of whole apples. Do you follow?"

"Yeah, but the slices are two different sizes. So, that would be hard."

"Not if you cut both the apples into the same number of equal pieces."

We sit for another moment of silence.

"Oh," John's eyes light up. "To make the slices, all the same, size! Then we can put them together! If we cut each of the apples into twenty equal pieces, the first three pieces would turn into fifteen pieces, and the other two pieces would turn into eight pieces. We'd have twenty-three same-size slices altogether. And the denominator is … twenty. The number of equal pieces we cut each apple into."

"That's right. And the same principle works with rational expressions. We want to find the first division of the whole into equal parts to add the pieces. In other words, we're looking for the least common multiple of the denominators."

"That makes sense," John says, "So what about the butterfly method?"

"Your butterfly method is not necessarily a bad way to help remember how to add fractions. I mean, it saw you all the way to calculus, right?"

"Yeah." He looks up at me. "I guess you're right. It did."

"The important thing is, do you know how it works?"

He stares at the dry-erase board for a few seconds. "Going back to three quarters plus two fifths, when you times the numbers in each wing of the butterfly and put the answers in the antennas, what you're doing is figuring out what the numbers on the top are going to be when you find a least common multiple of the denominator."

"You mean, you're recording the numerators in the antennas?"

“Yeah,” he nods. “Exactly. Because to get the least common multiple of the denominators, you times three-quaters by five-fifths, and you times two-fifths by four-quarters. So, that’s why fifteen and eight go in the antennas because those are the numbers on top, the numerators. And twenty goes in the butt, because that’s the number on the bottom, the least common multiple of the denominators. Then you just add or subtract the numbers on the top.”

“I see how it works now,” I smile. “That’s cool.”

“Yeah,” he returns a confident grin. “I guess the butterfly method is just a way to keep track of stuff when you’re finding the least common multiple of the denominators. It’s kind of like when you’re subtracting things, you cross numbers out to show where you’ve borrowed tens ... thanks; you helped me out!”

“I’m glad. But before we move on, are you sure the butterfly method always works?” I write five-sixths minus two-thirds on the dry-erase board. John pauses briefly and then applies the butterfly method.

“Okay, so you get three eighteenths. That seems right. But doing your way of adding fractions, finding a least common multiple of the denominators ... you get one sixth. Okay, but that’s just the same thing. Oh! So the butterfly method doesn’t always give you the fraction in smallest form.”

“You mean reduced form?”

“Yeah, it doesn’t give it to you in reduced form. So you still have to check that.”

“Now, what if you had to do three fractions?” I write:

$$\frac{5}{6}-\frac{2}{3}+\frac{3}{4}$$

He thinks for a moment.

“You know, you could use the butterfly method on the first two, then take that and add it to the third. But I don’t think I’m going to use the butterfly method anymore, now that I see the logic behind it. You can just see that the smallest number six, three, and four go into is twelve. So that’s your least common denominator.” He writes:

$$\frac{10}{12}-\frac{8}{12}+\frac{9}{12}$$

“Now you can do the math easily, and the answer is eleven twelfths”

Reflecting on the butterfly

As the scenario with John illustrates, asking students to reflect on the mathematical mnemonics that they use can provide a fruitful exercise in reasoning and proof. Initially, John was unable to explain why the butterfly method got the right answer. He exhibited little knowledge of what fractions mean or how they work, and in this regard, he is not unlike a significant portion of students—even students enrolled in college-level mathematics courses. However, by the end of the scenario, John had a much firmer grasp on fraction arithmetic and was able to see some limitations in the butterfly method. For example, he recognised that the butterfly method could not handle more than two fractions at a time. John entered into a deeper understanding of fraction arithmetic through the butterfly method, as if through a gateway. Once inside, he left the mathematical mnemonic behind.

We can do better than leaving students like John to struggle through mathematics using tricks and shortcuts that they do not understand. It is true that it may take time for some students to make sense of the concepts behind mathematical procedures, and mathematical mnemonics can act as a useful support system until full understanding is achieved (Karp, Bush, Dougherty, 2014). However, there is no good reason that John had to do fraction arithmetic without understanding it for so many years—without even knowing that he should understand it. Instructors must use mathematical mnemonics wisely and attach them to the conceptual understanding whenever possible.

Other common mathematical mnemonics

Fraction arithmetic is, of course, not the only haunt of mathematical mnemonics. Almost every mathematical procedure can be presented using mnemonics—and thus, the potential for overuse is always present. However, procedural and conceptual fluency can, and, in fact, must, be taught as a pair (NCTM, 2014). They are the two wings on which mathematical proficiency takes flight. Mathematical mnemonics can act the part of the fuselage.

Another common mathematical mnemonic is the action of "borrowing" during the standard American vertical subtraction algorithm. Remember, mathematical mnemonics can be as simple as a single line marked on a piece of paper to serve as a visual cue. Suppose we want to find the differecne between 313 and 217. Most of us would probably write the following:

$$\begin{array}{r} {}^{2\,10\,13} \\ \not{3}\not{1}\not{3} \\ -\;217 \\ \hline 96 \end{array}$$

But what does borrowing mean? Why all the dashes and scribbles? Very few students are able to explain this.

In the case of borrowing, a student is rewriting numbers as sums of magnitudes of ten and then reordering. Borrowing means noticing that 313 can be expressed as 300 + 10 + 3 or 200 + 100 + 13. Thus, 313 – 217 becomes:

$$\begin{array}{rrrrrr} & 200 & + & 100 & + & 13 \\ - & 200 & - & 10 & - & 7 \\ \hline & & & 90 & + & 6 \end{array}$$

Showing students the inner-workings of borrowing has the double benefit of developing number sense and contextualising the standard subtraction algorithm. It gives math meaning. And that is the primary goal of mathematics education—to make mathematics meaningful for students (NCTM, 2014).

A word of caution must be added here, lest this article give the wrong impression. Students should not expect all mathematical mnemonics to be attached to a deeper mathematical meaning. For example, the acronym BODMAS (or BOMDAS, or DAMNUS, or BIMDAS, depending on the instructor's inclinations) exists to help students acculturate to the linguistic structures of modern mathematical notation, which is in some ways arbitrary (Watson, 2010). Other orders of operation could have been chosen, but the modern notation and way of doing things is what it is. The fictional Native American Chief, SOH CAH TOA, fills a similar niche as a mathematical mnemonic that helps students in the United States and other countries to remember the definitions for trigonometric ratios.

Conclusion

Mathematical mnemonics have their proper place in mathematics education; they can be pleasant sights along the safari of learning (Greene, 1999; Kavale & Forness, 2000; Mastropieri et al., 1987). However, difficulties can occur when these mathematical mnemonics distract students from spotting larger mathematical creatures such as conceptual understanding, problem-solving, and reasoning and proof (Kilpatrick et al., 2001). We need to focus more on these larger creatures while not discounting the smaller ones (NCTM, 2014).

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