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Improving Mathematics Teaching as Deliberate Practice through Chinese Lesson Study

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This study examined how a ninth grade teacher improved an Algebra I lesson through a lesson study approach. We used multiple data sources to investigate the improvement of the lesson towards student-centered mathematics instruction, perceived benefits of the teacher, and factors associated with the improvement of teaching. The lesson group substantially improved the lesson through appropriately launching and effectively implementing worthwhile mathematical tasks and strategically orchestrating students' work. The teacher improved his teaching skills and reflection ability, changed his views about mathematics teaching, and acknowledged the importance of repeated teaching, expert feedback, and self-reflection in improving his teaching.

Currently there is a nationwide endeavor to prepare teachers for effectively implementing the Common Core State Standards for Mathematics (CCSSM hereafter; National Governors Associate Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010). The subsequent teacher-training projects tend to focus on helping teachers understand what the CCSSM entails but often neglect

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an essential component: how to develop a lesson to meet the expectations of the CCSSM (Editorial Projects in Education Research Center, 2013). By adopting a lesson study (LS) approach, we examined ways of progressively improving a lesson toward better implementation of the mathematical practices described in CCSSM and documented how a practicing teacher benefited from participating in LS. This LS group collaboratively worked through three cycles of preparing, delivering, and reflecting on a single lesson focused on justification and proof in algebra. The final lesson exemplified features that support students in achieving the Standards for Mathematical Practice (NGA & CCSSO, 2010). This study addressed the following research questions:

1. How did the practicing teacher make substantial changes across the enacted lessons?
2. What did the practicing teacher perceive that he learned from the continued improvement of the lesson?
3. What were the major factors associated with the improvement of the lesson?

Theoretical Framework

This section begins with a description of lesson study and its effect on teachers' learning. Then, we introduce the notion of deliberate practice, which provides a theoretical lens for this study.

Implementing Reform-oriented Curriculum through Lesson Study

LS, in general, is a job-embedded, research lesson-oriented, student-focused, collaborative professional development model that has demonstrated its power in improving teaching and promoting teachers' growth (Hart, Alston, & Murata, 2011; Huang & Shimizu, 2016; Lewis, Perry, & Hurd, 2009). In LS, teachers set goals, study curriculum and research on student learning, plan a lesson, teach the lesson, and debrief using the data collected during the lesson. As revealed by Lewis et al. (2009), LS can change teachers' knowledge and beliefs, promote development of professional learning communities, and develop teaching resources.

To implement effective mathematics teaching practice in this study, we focused on three key aspects of the mathematics teaching practices described in *Principles to Actions* (National Council of Teachers of Mathematics, 2014): to launch high cognitively demanding mathematical tasks and engage students in rigorous mathematical content (Jackson, Shahan, Gibbons, & Cobb, 2012; Weiss & Pasley, 2004); to maintain the cognitive demand of tasks during implementation and provide opportunities for students to engage in high-level thinking and reasoning throughout an instructional episode; and to provide opportunities for students to explain and discuss their mathematical thinking, reasoning, and solutions (Boston, 2012; Stein, Engle, Smith, & Hughes, 2008).

LS is a job-embedded, system-wide teacher professional model in China and Japan (Huang & Han, 2015; Murata, 2011), yet there has been a challenge in implementing LS in schools on a larger scale in the US (Akiba & Wilkinson, 2015; Yoshida, 2012). Little research has focused on uncovering the process of how LS helps teachers improve mathematics teaching and develops teachers' instructional expertise. By focusing LS on a specific topic, *exploring patterns embedded in a calendar*, we aimed to document the process of developing an exemplary lesson that reflected mathematical practices recommended in CCSSM and what the teacher learned during the process.

Improving Teaching through Deliberate Practice

To identify and examine possible ways of achieving top performance, cognitive scientists have found that participation in self-improvement activities is an important factor for continued improvement and attainment of expert performance (Ericsson, 2008; Ericsson, Krampe, & Tesch-Romer, 1993). Ericsson et al. (1993) defined deliberate practice as special activities developed for and repeatedly pursued by individuals with feedback from experts. Being engaged in deliberate practice means that one has a task with a well-defined goal, is motivated to improve, and receives feedback and ample opportunities for repetition, all of which result in gradual refinements of their performance (Ericsson, 2008; Ericsson et al., 1993). In general, deliberate practice is defined by four characteristics (Bronkhorst, Meijer, Koster, & Vermunt, 2013; Ericsson et al., 1993; Ericsson, 2006; Gog, Ericsson, Rikers, & Pass, 2005). First, the practice involves activities designed for self-improvement. In LS, knowledgeable others (e.g., teachers, coaches, and trainers) offer guidance with regard to the sequence and challenge of these activities. Second, the individual repeats the practice to enable successive improvement and refinement. In LS, the teacher repeats teaching a continually improved lesson. Third, the individual receives immediate and informative feedback concerning different aspects that underlie the practice. In LS, the knowledgeable others provide the feedback to the teacher after the lesson. Fourth, the practice requires significant effort and concentration. In LS, the teacher reflects on the enacted lesson and feedback from team members and revises the lesson plan.

Initially, researchers studied the concept of deliberate practice in chess, sports, and music; however, researchers have explored deliberate practice in other professions including teaching and teacher education (e.g., Bronkhorst et al., 2011, 2013; Dunn & Shriner, 1999; Gog et al., 2005). By analyzing daily activities of experts and experienced teachers, Dunn and Shriner (1999) found that deliberate practice could account for teaching expertise and likely results from meaningful engagement in the teaching-evaluation-revision cycle. Further, Bronkhorst et al. (2011) found that deliberate practice could be conceptualized as activities intended not only for developing teachers' expertise, but also for fostering student learning and development.

Within a context of LS in China, Han and Paine (2010) found that teaching mathematics as deliberate practice provided opportunities to improve instructional practices related to both pedagogy and content. Chinese LS, unlike Japanese LS, requires repeated teaching and knowledgeable others' involvements (Huang & Han, 2015), and typically includes cyclic phases of setting a goal, selecting a topic, planning a research lesson, teaching and observing the research lesson, debriefing, and revising the lesson. Furthermore, Huang and Li (2014) argued that, as a process of deliberate practice, Chinese LS provides a mechanism that enables participating teachers to make continued improvements in teaching. As a product of high-leverage practice, namely, exemplary lessons, Chinese LS creates opportunities for teachers to learn how to teach using certain instructional products (e.g., annotated lesson plans).

Methodology

In this section, we first introduced the formation of the LS group and selection of the topic for the research lesson. Then, we describe the improvement process for the research lesson, data collection, and data analysis.

The Lesson Study Group

Researchers have suggested the importance of including knowledgeable experts in LS groups (Groth, 2012; Huang & Han, 2015; Yoshida, 2012). Intentionally, our LS group consisted of two mathematics education professors (as knowledgeable others), two graduate students, and two practicing teachers (see Table 1).

The table shows that both experts were quite knowledgeable in reform-oriented mathematics teaching at the high school level, whereas the participating teachers and graduate students had some experience in teaching mathematics at the high school level. Although the participating teacher who taught the lesson, Mr. Jobs, was an Algebra II teacher, the lesson featured in this research focused on an Algebra I topic. Therefore, Mr. Jobs taught the lesson in Ms. Cambrin's Algebra I classes. Ms. Cambrin arranged for her students to attend the lessons and observed all of the lessons. When the LS took place, Mr. Jobs was taking a graduate course taught by Dr. Ross, who took a mentoring role during the LS process.

Table 1
Background Information of The Members of The LS Group

Name	Background
Dr. Annette	Veteran high school mathematics teacher; Extensive research experience in reform-oriented mathematics teaching
Dr. Ross	Veteran high school mathematics teacher; Extensive research experience in teacher learning through the LS approach
Ms. Philborn	Doctoral student; High school mathematics teaching experience
Ms. Holt	Doctoral student; High school mathematics teaching experience
Mr. Jobs	Algebra II teacher with 4 years of teaching experience; Second year master's student
Ms. Cambrin	Experienced Algebra I teacher

The Topic

The calendar task (see a version of the task in Figure 2) used in this lesson has been used to introduce algebra symbolically (Thornton, 2001) and to develop discovering and justifying in algebra learning (Friedlander & Hershkowitz, 1997). Yet, literature does not document specifically how to implement the task in classrooms. In general, research suggests that to develop students' algebraic reasoning, it is crucial for students to experience the entire process of reasoning from observation of a pattern to making and proving conjectures (Friedlander & Hershkowitz, 1997; Reid, 2002); to work on deliberately designed and varying mathematical tasks to promote their reasoning (Blanton & Kaput, 2005); and to have productive discourse to scaffold their reasoning (Stylianou & Blanton, 2011). Therefore, the instructional objectives of this lesson included (a) analyzing simple quantitative relationships, recognizing patterns within the context of real-life situations, (b) using variables to represent unknown numbers, (c) connecting algebraic expressions to daily life, and (d) developing algebraic reasoning and justification ability.

Process of Developing an Exemplary Lesson

Dr. Ross provided Mr. Jobs with a lesson plan that had been developed by a LS group in China (Huang, Su, & Xu, 2014). Mr. Jobs, with help from other group members, attempted to make improvements on the lesson through three cycles of repeated teaching (see Figure 1).

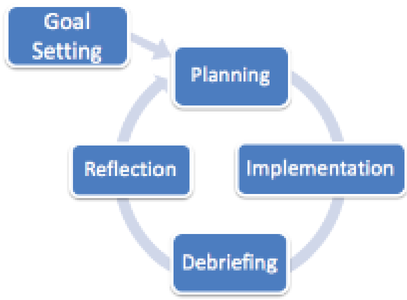


Figure 1. Cycle of the lesson study.

After the implementation of each lesson, the LS group met to debrief. In these meetings, each of the group participants provided their thoughts, both written and verbal, on the lesson's strengths and weaknesses and made suggestions for future improvements. Mr. Jobs taught and refined the calendar lesson on three separate occasions to three sections of Algebra I with an average class size of 24 students and lasting approximately 47 minutes. Because the mathematical content underlying the calendar task included broad notions such as introducing letters to represent variables and operations of algebraic expressions, the teacher presented the lesson as an independent, problem-solving class. Although Mr. Jobs delivered the three lessons within a three-week period, the independent nature of the lesson did not drastically affect Ms. Cambrin's teaching schedule.

Data Collection

We collected a set of data including three lesson plans and relevant student materials, videotaped lessons, members' written comments (WC) for each lesson, and a semi-structured interview (Galletta, 2013) with Mr. Jobs after completion of the third lesson (IV). We conducted the interview to solicit Mr. Jobs' understanding of mathematics teaching and learning and his perceived changes of the instructional design, classroom teaching, and knowledge for teaching. Based on a detailed examination of lesson plans and videotaped lessons, the LS group identified 11 major changes that occurred across the three lessons. Mr. Jobs confirmed and revised the list of changes and explained why and how he made these changes in his reflection report (RR). Mr. Jobs included what he gained from the LS process. In addition, after each class, the participating students completed a quiz to assess their understanding.

Data Analysis

Process of making substantial changes across lessons. We looked at three components of deliberate practice within the context of LS: well-defined instructional goals for the lesson, teaching of the lesson and immediate feedback on it, and enactment of the revised lesson. Throughout the LS, the instructional goals remained the same as the first lesson, with minor clarifications. The data analysis focused on what significant changes were made to achieve the instructional goals in each lesson and what factors resulted in these changes.

We collaboratively analyzed the self-reflection report and interview to answer why and how the major changes were made to the lesson. The researchers classified these changes into three categories: launching tasks, implementing tasks, and orchestrating students' discussion. Moreover, based on explanation of these changes in the self-report and interview, the researchers classified the major causal factors for each change.

Instructional paradigms. To capture the overall changes in classroom instruction, the researchers analyzed the videotaped lessons (Stigler, Gallimore, & Hiebert, 2000) using Studio code software by focusing on four types of classroom activities that occurred. The classroom activities included teacher-led classwork, student-oriented classwork, individual seatwork, and collaborative group seatwork. Teacher-led classwork referred to teacher explanation and discussion carried out in a whole-class setting. Student-oriented classwork referred to students presenting their answers on the board/screen (with or without teacher assistance to clarify student work). Individual seatwork described students working on tasks without discussion. Collaborative group activity referred to two or more students working on tasks, discussing their thoughts or solutions. The first two authors coded the last lesson individually, with an inter-rater agreement of 90%. The authors resolved disagreements through extensive discussions. The first author coded the remaining lessons, and the second author verified and resolved minor disagreements. In addition to identifying the shift of classroom instruction, the researchers analyzed students' post-lesson quizzes to capture students' learning outcomes, which included: (a) methods of justifying the pattern, (b) judgment of varying patterns, and (c) proof of the pattern.

Participating teacher's perceived benefits. Through constant comparisons (Corbin & Strauss, 2008) the researchers analyzed the self-report and interview with a focus on what the teacher learned through participating in the LS process.

Results

The following sections present the results, organized according to the research questions. We first report the major changes across the research lessons and then present the teacher's perceived gains through the LS process. Finally, we identify the factors associated with the improvement of the research lesson.

The Process of Making Major Changes

Based on lesson plans and videotaped lessons, the researchers identified the major changes made to the lesson after the first and second teachings. The sections that follow illustrate why and how these changes were made to the lesson.

We include a description of the implementation of the lesson for each of the three teachings to provide context for these changes.

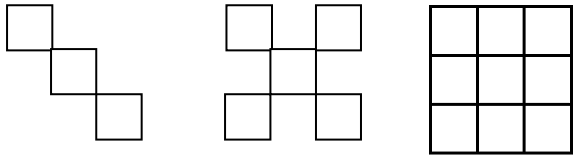
Adopting an existing lesson plan: First teaching. The first teaching of the lesson primarily involved implementing the existing lesson plan. In this section, we present the implementation and results of this first teaching of the lesson.

Implementation. The first teaching of the lesson took place in Ms. Cambrin's room, where the desks were arranged in rows. The lesson included three major phases: creating a learning situation, self-exploration, and summary.

Creating a learning situation. The lesson began with three basic questions about the calendar (e.g., How many days are there in a week?). After hearing responses, Mr. Jobs displayed a month from a calendar and asked students to find patterns.

Next, Mr. Jobs posed a game by asking students to compute the sum of three consecutive days in a row and give the sum to him. Mr. Jobs then revealed their original three numbers immediately. Students worked individually and as a class to figure out his process. Only a few students participated in this discussion. With scaffolding, the students saw the pattern: the sum divided by three is the middle number. The teacher asked students to express the three days algebraically. Students used multiple variables to represent the three numbers, but Mr. Jobs redirected them to write the three days as $m - 1$, m, and $m + 1$. Mr. Jobs then altered the situation to three adjacent days in a column to see if students could express those algebraically. With scaffolding, students were able to determine it was the same process and wrote $\frac{s}{3} = m$ with the three days being $m - 7$, m, and $m + 7$.

Self-Exploration. Mr. Jobs provided varying grids for the students to analyze, see if the pattern held, and make comparisons with previous grids. He first gave them a three-in-a-diagonal grid (Figure 2a) to express algebraically. Next, students worked independently on the "x"-box (Figure 2b). Finally, students worked independently on the 3 x 3 grid (Figure 2c) then discussed with their classmates. Mr. Jobs then orchestrated a discussion that connected this grid to all the previous grids. One student observed, "This one has all the other patterns inside of it."



(a) (b) (c)

Figure 2. Calendar grids for students to analyze.

Summary. Mr. Jobs asked the class: (a) What do you think was most meaningful? (b) What do you think the major steps of exploring patterns are? Students shared by saying, "Finding the middle number given the sum" and "Finding all the sub-patterns."

Feedback and intended changes. After the first teaching of the lesson, the LS group met to discuss improvements. Mr. Jobs intended to make five major changes based on feedback and self-reflection.

Classroom setting. Both Mr. Jobs and group members agreed that placing students into small groups would facilitate collaborative group work. Specifically, Ms. Philborn felt that "you might have gotten more students involved if they had discussed in groups of two or three before offering comments for the whole class" (WC). In addition, Mr. Jobs realized that he would be more comfortable teaching in his classroom, where the desks were already placed in groups.

Introductory questions. Dr. Annette offered this suggestion: "I would suggest starting with an open-ended question such as, "Take a look at the calendar. Write down at least three things that you notice" (WC). Mr. Jobs realized the change would require students to validate their prior knowledge. He reflected, "The questions in the first teaching were too easy and too narrow. This open-ended question allows for creativity and connects to students' existing knowledge."

Student-created game. Dr. Annette further suggested changing the game to make it more student-focused, suggesting that the teacher say:

> I had a student in a different class who noticed a pattern when looking at three consecutive days. But he wouldn't tell us the pattern. Instead, he said if you picked any three numbers and found their sum, he could tell you the three numbers based on the sum. Let's try it. (WC)

Mr. Jobs discerned this scenario would advance students' thinking without devaluing the previous ideas offered by students. As a result, students might confidently and critically consider the idea because it was a student's idea and not the teacher's idea (RR).

Process of justification. Dr. Annette commented, "The lesson does not require students to justify why dividing by 3 gives the middle number" (WC). Mr. Jobs stated he had discussed justifying in this first lesson, but needed to make it explicit and emphasize the entire process of justification. He reflected: "We mostly looked at specific cases to justify . . . I need to do a better job of leading students to prove this conjecture works in all cases. After all, that is the main reason why we need to represent the days algebraically" (RR).

Orchestrating student work. Both the experts and graduate students offered suggestions for providing opportunities for students to display their work in order to enhance student understanding by critiquing each other. Ms. Holt suggested "let [ting] each group work on a different pattern from the calendar, and then have each group show and explain to the class what they found" (WC) which would combine multiple activities. Mr. Jobs agreed: "Each group justifying a different grid will allow for more variety, in less time. Plus, it will require the students to construct the knowledge, not me" (RR).

Making a lesson your own: Second teaching. After reflecting on the feedback from the first teaching of the lesson, Mr. Jobs made changes resulting in a lesson for which he had more ownership. In this section, we describe the implementation of the lesson and results of this second teaching.

Implementation. Mr. Jobs substantially revised the lesson plan and taught it to another Algebra I class in his own classroom. The lesson included five phases: creating a learning situation, exploration, self-exploration of varying patterns, orchestrating student work, and summary.

Creating a learning situation. Mr. Jobs began the class with the following task: “Take a look at the calendar on your desk and write down at least three patterns that you notice.” In the whole-class discussion that followed, Mr. Jobs wrote students’ observations on the board and helped reveal other important patterns that would aid student thinking in the next activity.

To focus students’ attention on a particular pattern, Mr. Jobs used Dr. Annette’s story. He asked students to compute the sum of the three consecutive days in a row. He performed the student’s “magic” writing down the three numbers along with their sum for students to see. With the examples displayed, students worked in groups to justify how it worked while Mr. Jobs facilitated small group interactions. In the discussion that followed, one student shared her thoughts: “If they give you a number like 27, you just divide by three and you get 9 and you could just put the other numbers that are above and below it.” Mr. Jobs led students in checking this conjecture with the other examples then invited the students to try to prove algebraically that the sum divided by three is the middle number.

Exploration. After Mr. Jobs led students in proving the conjecture for three days in a row, he presented students with a variation of the conjecture involving three days in a column. Similarly, students worked in small groups to try to justify the conjecture. Finally, the class collaborated to prove the conjecture was valid by presenting students’ work on the screen.

Self-exploration of various patterns. To develop students’ flexibility in making and justifying conjectures, Mr. Jobs assigned each group different grids that varied with regard to the number of days (three to seven) and the shape of grids (Figure 1). Groups explored relevant patterns and made/justified their conjectures. Groups displayed their work on large pieces of poster paper. Once again, Mr. Jobs visited each group to assist and advance student thinking.

Orchestrating student work. To discuss findings, Mr. Jobs displayed the work of two groups and identified some main features. For example, the idea that grids with an even number of days (four in a row) did not have a middle number and, thus, the average produced the median.

Summary. Mr. Jobs outlined the major steps of pattern seeking: observing a pattern, making a conjecture, and justification.

Feedback and intended changes. Based on suggestions offered by written comments and his own reflections, Mr. Jobs attempted the following changes: concise and coherent focus, time management, and selecting groups to present.

Concise and coherent focus. Both experts agreed that "the process of discovering the pattern should be made clear: observation, conjecture, and justification" (WC). Mr. Jobs appreciated the comments because they helped provide a central theme for the lesson:

> This really helps focus the content and practices that I need to be developing during the lesson. I need to be clearer about what the conjecture is as well as talk in more detail about the process of justifying an observed pattern. (RR)

Time management. The experts and graduate students made various constructive comments and suggestions to Mr. Jobs concerning the use of a timer to prevent a tendency to "stop and stay" with a group when circulating the room and diminish the cognitive nature of the group work. Mr. Jobs' concern was with groups who were unfinished and asked to present their work. The expert consensus was to let the class assist that group in helping them reason out what was needed to finish their work.

Selecting certain groups to present. Mr. Jobs realized a weakness: "Obviously, [all] the groups were not able to present, which is one of the most important aspects of the lesson. The students were not able to compare and contrast the grids, share their work with others, nor critique each other's work" (RR). In the essence of time management, Mr. Jobs decided to select and sequence certain grids to illuminate certain ideas. With Dr. Ross's assistance, he selected four grids that would create the most variation and lead to meaningful class discussion (including three in Figure 4 and one counterexample of grid with a four consecutive days in a row).

Making a lesson for student learning: Third teaching. As before, Mr. Jobs revised the lesson based on the feedback from the second teaching. With the lesson goal more clearly articulated (i.e., the process of observing patterns, making conjectures, and justifying their conjectures), Mr. Jobs' lesson seemed to focus on student learning. In this section, we describe the third teaching of the lesson and the resulting reflections.

Implementation. Based on the second teaching and reflection, Mr. Jobs further revised the lesson plan. This lesson included five major phases: creating a learning situation, setting the tone of algebraic reasoning, self-exploration of varying patterns, orchestrating student work, and summary.

The third lesson plan was very similar to lesson two. However, the implementation was drastically different. Using a timer, Mr. Jobs moved from task to task more efficiently and spent less time on introductory tasks. During whole class discussions, Mr. Jobs continuously referred back to the process of recognizing a pattern, forming conjectures, and seeking to justify the conjecture. He referred to the conjecture as "the sum is three times the middle number" and instructed students to try to use one variable in labeling the three consecutive days.

Mr. Jobs provided groups with the same varying grids from lesson two to analyze and present. However, in this lesson, four specific groups were asked to present their work. Two groups justified the conjecture in relation to their grid. The third group demonstrated that it did not hold for their grid, which did not have a middle number. Finally, the last group presented their work, which was limited to a single example, with the 3 x 3 grid.

The teacher asked the class to help them finish, and another student described how to use algebraic expressions to represent each of the days. This suggestion led the class to transition the justification from empirical to inductive levels, which was a main focus of the lesson. Mr. Jobs then summarized and students shared their thoughts related to the major steps in pattern seeking and what they found to be most meaningful.

Reflection on the final lesson. Mr. Jobs and group members were satisfied with this lesson and ended the revision process. When asked in the interview about the major changes, Mr. Jobs said:

> One of the most significant changes made to our lesson throughout this process was the addition of a timer. Restricting students' individual-think time and collaborative discussion time created more time for further exploration and reflection later in the lesson. Not only did it create more time, but it also forced me to stick to the schedule and spend less time interjecting in the groups. Instead, I spent this time circulating the room to listen to the various ideas, develop the organization of subsequent discussions, and pose thoughtful questions to groups that finished early. The timer kept students on task because they knew I was going to move forward with the lesson when time ran out. I was worried initially about unfinished groups, but the 3 x 3 grid group presented what they had done so far, described why they might be stuck, and finished the task with help from the class.

Salient Changes of Instruction

In looking at the lesson from the first to last teaching, we highlight the time distribution of different activities. The time distribution (percentage) for four types of activities in the initial and final lessons is shown in Figure 3.

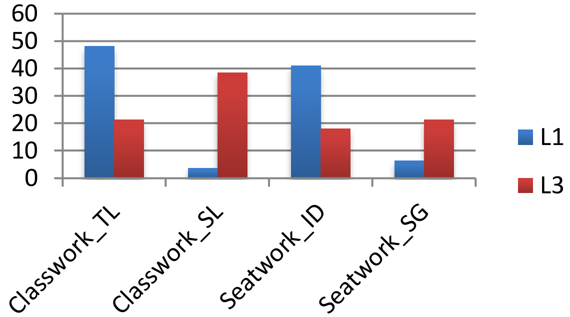


Figure 3. Time distribution in four categories in the initial and final lessons
Note. Classwork_TL: Teacher-led classroom work; Classwork_SL: Student-oriented classwork; Seatwork_ID: Student seatwork individually; Seatwork_SG: Student seatwork in groups.

There was a significant teaching pattern shift from teacher-led to student-centered. In the first teaching, almost half of the time (48%) was spent on teacher-led activities including organizing activities, explaining relevant concepts and skills, and leading student discussion, while 4% of the time was spent on sharing student-initiated solutions. Furthermore, 40% of the time was spent on students solving problems individually while only 6% of the time was devoted to group activities (partnering with students nearby). However, in the third teaching, the teacher-led activities dropped significantly from 48% to 21%, while student-led activities increased significantly from 4% to 39%. Moreover, individual seatwork decreased from 40% to 18%, while group activities increased from 6% to 21%. The data suggested that the teaching paradigms changed significantly: from a teacher-centered lesson that was dominated by teacher-led instruction and individual student responses to a student-focused instruction that emphasized student-initiated ideas and teacher summaries. The final lesson achieved its core goal: for students to experience the entire process of observation, conjecture making, and conjecture justification through variation and comparison (Huang, Prince et al., 2014).

Examining what changes the teacher made highlighted the importance of appropriately dealing with core components of high quality teaching: effectively launching mathematics tasks, maintaining the cognitive demand while implementing mathematics tasks, and orchestrating discussion. In the second lesson, Mr. Jobs was successful in regards to launching tasks (including classroom setting, introductory questions, and organizing the game) but was not effective in organizing group activity. In addition, he did not leave sufficient time to share and discuss student work. In the third lesson, Mr. Jobs effectively organized the group activity by using a timer and orchestrated and sequenced the presentation and discussion of students' work.

Huang, Prince, and Schmidt (2014) detailed how the shift of teaching paradigm represented in the third lesson supported students to: reason both abstractly and quantitatively through making conjectures based on numerical observation and prove them algebraically (Standard for Mathematical Practice [SMP] 2; NGA & CCSSO, 2010, p. 6); look for and make use of structure (SMP 7) through discovering invariant patterns within varying grids; and develop viable arguments (SMP 3) through proving and disproving. They demonstrated that the entire lesson clearly exhibited students' effort to "make sense of problems and persevere in solving them" (SMP 1) and "look for and express regularity in repeated reasoning" (SMP 8) as well (NGA & CCSSO, 2010, pp. 7–8). By engaging students in these SMPs, the third lesson met the CCSSM expectations.

In addition to meeting the expectations of CCSSM better, the revision process revealed three key differences in student learning effects, as evidenced on the post-lesson quizzes. First, in justifying "the sum of any three consecutive numbers in a column equals three times the middle number" (i.e., the average identity), the majority of students used specific cases. Only 24% of students attempted to use variables out of the three classes to represent the days during the quiz: zero students from the first teaching, five students from the second teaching, and ten students from the third teaching. Second, the classes differed in terms of their understanding of the average identity. We came to this conclusion from analyzing the question that asked students to indicate whether or not they thought the pattern would hold for four consecutive days in a row. Even though the pattern only holds for an odd number of days in a row, 48% from lesson 1, 52% from lesson 2, and 36% from lesson 3 indicated that the pattern would still hold for four consecutive days. Finally, the vocabulary the students used to explain the main steps in finding patterns differed between lessons. Students from lesson 1 used words such as "change," "relate," "rule," "formula," "observe," and "search"; whereas lesson three students used words such as "conjecture," "hypothesis," "observation," and "arithmetic sequence" to explain themselves. While some students used this vocabulary, others described the major steps for this pattern specifically. For example, a student from lesson 1 wrote, "Add the numbers & divide [the sum] by the # of numbers, to get the middle #." Whereas a student from lesson three expressed it symbolically: "middle number = *m*, how many numbers = *n*, sum = *s*, *mn* = *s*." Throughout the lesson study project, Mr. Jobs improved students' learning by moving students toward justifying algebraically and expressing using mathematical languages.

Perceived Benefits from Participating in the Lesson Study

Mr. Jobs' reflections revealed perceived benefits of his participation in LS. We coded these perceived benefits within two categories, which are described in the following sections.

Improving teaching skills and reflection ability. The feedback from experts and peers provided Mr. Jobs with a way to analyze his own thought processes and better reflect on the strengths and weaknesses of the lesson. "I learned how to make a lesson my own, reflect upon a lesson, and effectively orchestrate student thinking. Group members helped me create a collection of techniques that I can use to alter future lessons" (IV)

> In the beginning, it was mainly teacher led [referring to the first teaching]. During the LS, the lesson went from focusing on the teacher with students working individually at their desk, to students working in small groups on

> individual tasks … exploring, discussing, and sharing ideas with others. (IV)

Mr. Jobs also became more aware of his role in students' learning process. He recognized students' facility in independent learning by stating, "I need to be more willing to let my students explore independently. I need to be more of a guide and just be there to clarify and expand student thinking" (IV). Mr. Jobs learned the power of students constructing new knowledge by themselves; if a student could say it, then the teacher should not.

Major Factors Associated with Improvement of the Lesson

When analyzing how the teacher made these changes, it was clear that he benefited from more knowledgeable team members' comments. It is crucial that LS teams provide critical comments, but it is even more important to provide the teacher with the time and opportunity to enact the new design and reflect on new implementation. It was the cyclic (design, teaching, and reflection) process, with knowledgeable experts' input, that helped the teacher to realize the weaknesses of the lesson and develop and implement new design.

Discussion and Conclusions

The practicing teacher made continued improvements to the lesson by adopting feedback from more knowledgeable team members and undertaking cyclic enactment and reflection. The cyclic process of design, experimentation, and reflection, with immediate feedback from experts, is the core component of *deliberate practice* (Ericsson, 2008). Meanwhile, the participating teacher must be self-motivated and willing to make great effort to improve instruction. In this study, Mr. Jobs was a master's student who had learned a lot about CCSSM and theories about teaching and learning mathematics. The theoretical knowledge may have motivated implementing reform-oriented lessons. In addition, the members of the LS group were instructors, classmates, or colleagues. Within such a learning community, the teacher felt comfortable considering and adopting comments from the team members. This study provides a vivid case of how such deliberate practice can help a teacher continuously improve teaching and make incremental growth (Huang & Li, 2014; Lewis et al., 2009).

The practice of Chinese LS, both in China (Han & Paine, 2010; Huang, Su et al., 2014) and in this case in the U.S., demonstrates the value and necessity of repeated teaching and knowledgeable others in LS from the perspective of deliberate practice. Knowledgeable others could be university mathematics educators and practice-based coaches or specialists who are knowledgeable in mathematics content, teaching and learning of mathematics in general, and conducting LS in particular. Teacher preparation programs emphasize the necessity of rehearsing a lesson multiple times (Lampert et al., 2013) and the necessity of involving the knowledgeable other in LS echoes the call from Takahashi and McDougal (2016). This study provides evidence to justify the necessity of these two components (i.e., repeated teaching and knowledgeable other's feedback) in order to maximize teachers' learning opportunity from LS from a perspective of deliberate practice. Thus, this study contributes to the conceptualization of LS and provides a perspective for examining LS. However, further research is needed to explore the mechanisms and effectiveness of implementing LS as one professional development approach on a large scale.

References

Akiba, M., & Wilkinson, B. (2016). Adopting an international innovation for teacher professional development: State and district approaches to Lesson Study in Florida. *Journal of Teacher Education*, *67*(1), 74–93.

Blanton, M. L., & Kaput, J. J. (2005). Characterizing a classroom practice that promotes algebraic reasoning. *Journal for Research in Mathematics Education 36*(5), 412–446.

Boston, M. (2012). Assessing instructional quality in mathematics. *The Elementary School Journal, 113*(1), 76–104.

Bronkhorst, L. H., Meijer, P. C., Koster, B., & Vermunt, J. D. (2013). Deliberate practice in teacher education. *European Journal of Teacher Education, 37*(1), 18–34.

Bronkhorst, L. H., Meijer, P. C., Koster, B., & Vermunt, J. D. (2011). Fostering meaning-oriented learning and deliberate practice in teacher education. *Teaching and Teacher Education, 27*(7), 1120–1130.

Corbin, J., & Strauss, A. (2008). *Basics of qualitative research* (3rd ed.). Los Angeles, CA: Sage.

Dunn, T. G., & Shriner, C. (1999). Deliberate practice in teaching: What teachers do for self-improvement. *Teaching and Teacher Education, 15*(6), 631–651.

Editorial Projects in Education Research Center. (2013). *Findings from a national survey of teacher perspectives on the Common Core*. Bethesda, MD: Author.

Ericsson, K. A. (2006). The influence of experience and deliberate practice on the development of superior expert performance. In K. A. Ericsson, N. Charness, P. J. Feltovich & R. R. Hoffman (Eds.), *The Cambridge handbook of expertise and expert performance* (pp. 683–704). New York, NY: Cambridge University Press.

Ericsson, K. A. (2008). Deliberate practice and acquisition of expert performance: A general overview. *Academic Emergency Medicine, 15*(11), 988–994.

Ericsson, K. A., Krampe, R., & Tesch-Romer, C. (1993). The role of deliberate practice in the acquisition of expert performance. *Psychological Review, 100*(3), 363–406.

Friedlander, A., & Hershkowitz, R. (1997). Reasoning with algebra. *The Mathematics Teacher, 90*(6), 442–447.

Galletta, A. (2013). *Mastering the semi-structured interview and beyond: From research design to analysis and publication.* New York, NY: New York University Press.

Gog, T. V., Ericsson, K. A., Pikers, R. M. J. P., & Paas, F. (2005). Instructional design for advanced learners: Establishing connections between the theoretical frameworks of cognitive load and deliberate practice. *Educational Technology Research & Development, 53*(3), 73–81.

Groth, R. (2012). Establishing and maintaining collaborations: Factors related to the sustainability of Lesson Study. In J. M. Bay-Williams & W. R. Speer (Eds.), *Professional collaborations in mathematics teaching and learning: Seeking success for all* (pp. 285–296). Reston, VA: National Council of Teachers of Mathematics.

Han, X., & Paine, L. (2010). Teaching mathematics as deliberate practice through public lessons. *The Elementary School Journal, 110*(4), 519–541.

Hart, L. C., Alston, A. S., & Murata, A. (2011). *Lesson study research and practice in mathematics education: Learning together*. New York, NY: Springer.

Huang, R., & Han, X. (2015). Developing mathematics teachers' competence through parallel lesson study. *International Journal for Lesson and Learning Studies, 4*(2), 100–117.

Huang, R., & Li, Y. (2014). Developing high-leverage practices in mathematics through exemplary lesson development: A Chinese approach. In Y. Li, E. Silver, & S. Li (Eds.), *Transforming mathematics instruction - Multiple approaches and practices* (pp. 231–254). New York, NY: Springer.

Huang, R., Prince, K., & Schmidt, T. (2014). Exploration of patterns in a calendar. *The Mathematics Teacher, 108*(5), 337–342.

Huang, R., Su, H., & Xu, S. (2014). Developing teachers' and teaching researchers' professional competence in mathematics through Chinese Lesson Study. *ZDM Mathematics Education, 46*(2), 239–251.

Huang, R., & Shimizu, Y. (2016). Improving teaching, developing teachers and teacher educators, and linking theory and practice through lesson study in mathematics: An international perspective. *ZDM Mathematics Education, 48*(4), 393–409.

Jackson, K. J., Shahan, E. C., Gibbons, L. K., & Cobb, P. A. (2012). Launching complex tasks. *Mathematics Teaching in the Middle School, 18*(1), 24–29.

Lampert, M., Franke, M. L., Kazemi, E., Ghousseini, H., Turrou, A. C., Beasley, H., Cunard, A. & Crowe, K. (2013). Keeping it complex: Using rehearsals to support novice teacher learning of ambitious teaching. *Journal of Teacher Education*, *64*(3), 226–243.

Lewis, C. C., Perry, R., & Hurd, J. (2009). Improving mathematic instruction through lesson study: A theoretical model and North American case. *Journal of Mathematics Teacher Education*, *12*(4), 285–304.

Murata, A. (2011). Conceptual overview of lesson study: Introduction. In L. Hart, A. Alston, & A. Murata (Eds.), *Lesson study research and practice in mathematics education: Learning together* (pp. 1–12). New York, NY: Springer.

National Governors Association Center for Best Practices & Council Chief State School Officers. (2010). *Common Core State Standards for Mathematics.* Washington, DC: Author.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: Author.

Reid, D. A. (2002). Conjectures and refutations in grade 5 mathematics. *Journal for Research in Mathematics Education, 33*(1), 5–29.

Stein, M. K., Engle, R. A., Smith M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*(4), 313–340.

Stylianou, D. A., & Blanton, M. L. (2011). Developing students' capacity for constructing proofs through discourse. *The Mathematics Teacher, 105*(2), 140–145.

Stigler, J., Gallimore, R., & Hiebert, J. (2000). Using video surveys to compare classrooms and teaching across cultures: Examples and lessons from the TIMSS video studies. *Educational Psychologist, 35*(2), 87–100.

Takahashi, A., & McDougal, T. (2016). Collaborative lesson research: Maximizing the impact of lesson study. *ZDM Mathematics Education, 48*(4), 513–526.

Weiss, I. R., & Pasley, J. D. (2004, February). What is high-quality instruction? *Educational Leadership, 61*(5), 24–28.

Yoshida, M. (2012). Mathematics lesson study in the United States: Current status and ideas for conducting high quality and effective lesson study. *International Journal for Lesson and Learning Studies, 1*(2), 140–152.