Using Concrete Manipulatives in Mathematical Instruction

"A little creativity and enthusiasm are as effective as cutting edge tools for teaching mathematics to young children. Physical mathematics teaching aids can engage children's minds in valuable ways that result in high retention of the information."

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With the ever-increasing shift in the accountability movement, teachers and parents are constantly in search of the "key" to high-quality mathematical instruction. Teachers and parents believe they must compete with lively television shows, faddish computer applications, and popular video games in order to capture student attention and interest; however, using concrete manipulatives in math instruction can generate student interest in mathematics (Moch, 2001; Moyer, 2001). This truth applies whether instruction is being performed in a formal learning setting, such as a classroom, or an informal learning setting, such as a family's kitchen.

Manipulatives are "physical objects that are used as teaching tools to engage students in the hands-on learning of mathematics" (TeacherVision, 2009, p. 1). Manipulatives can be particularly effective in further developing conceptual understanding in mathematics (Witzel and Allsopp, 2007), because they help students relate concrete ideas to abstract ideas, as well as link informal approaches with formal approaches (Uribe-Flórez & Wilkins, 2010). Using hands-on, concrete manipulatives throughout math instruction can lead to higher retention rates and a more positive student attitude toward education in general. The early childhood years, from ages 0-8, are critical in terms of development (McGuire, Kinzie, & Berch, 2012), so it is important to explore instructional strategies that align with and cater to the young child's growing understanding of the world. As Smith (2009) writes, "A good manipulative bridges the gap between informal math and formal math. To accomplish this objective, the manipulative must fit the developmental level of the child" (p. 20).

Concrete, Representational, Abstract Instruction

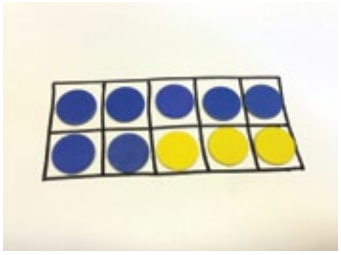
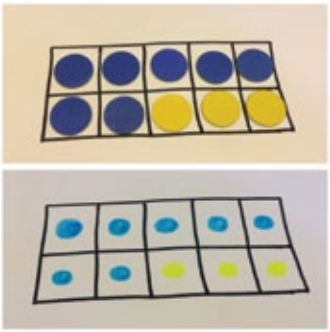
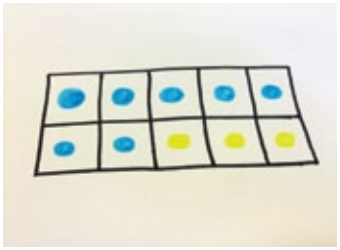
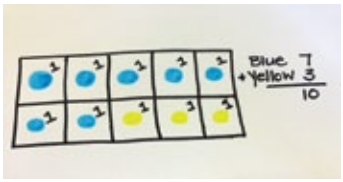
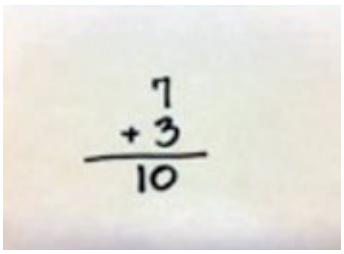
Concrete, Representational, Abstract (CRA) instruction is a process for teaching and learning mathematical concepts. Starting with manipulation of concrete materials (counters, beans, Unifix cubes), the process moves students to the representational level (tallies, dots, stamps), and peaks at the abstract level, at which numbers and symbols are used to demonstrate understandings (Witzel, 2005). With the purpose of giving students a thorough understanding of math concepts, CRA instruction allows students to make associations from one stage of the process to the next. When students are allowed to first develop a concrete understanding of the math concept/skill, they are much more likely to perform that math skill and truly understand math concepts at the abstract level. Specific information for the stages of CRA can be found in Table 1 (*next page*).

Children understand math better when they can see and touch it

Student Engagement and Accessibility using CRA Instruction

Witzel, Smith, & Brownell (2001) suggest that interactive experiences with concrete manipulatives can increase

Table 1: CRA instruction materials and practice.

Stages	Key Elements	Sample Problem	Explanation
Concrete	Chips, Unifix cubes, base ten blocks		Here, a ten frame with colored counters is used to show the equation 7 + 3 = 10.
Transition to Representational	Use of concrete and representational materials together		Once the concrete materials have been used, students begin to draw their own ten frames using the concrete model as a guide.
Representational	Tallies, dots, circles, stamps		At the representational level of CRA, the student is comfortable using pictures to solve the problem.
Transition to Abstract	Use of representational and abstract materials together		Students now start using abstract symbols (numbers in standard form) with their drawing to explain their reasoning.
Abstract	Numbers, mathematical symbols		Students at the abstract level of CRA no longer need pictures or manipulatives to solve the problem.

the relevance of mathematical material for students. Instead of delving straight into an abstract concept, the use of hands-on interaction with concrete manipulatives allows students of all mathematical levels to begin instruction on a level playing field. Accessibility for students is of utmost importance: students must be able to connect and engage with the mathematical concepts through direct, hands-on participation (Witzel et al., 2001; Devlin, 2000; Maccini & Gagnon, 2000). Recognizing that abstract concepts can be illustrated and manipulated using everyday objects gives students easier access to mathematical knowledge (Witzel et al., 2001; Devlin, 2000; Maccini & Gagnon, 2000).

Do-It-Yourself Math Manipulatives

While CRA instruction can seem like an arduous task, teachers and parents, as primary instructors, do not need to purchase expensive, commercially produced manipulatives in order to impart quality CRA instruction. Rather, everyday objects found around an average classroom

or household can be used as convenient and effective mathematical manipulatives. Popular manipulatives used in mathematical instruction include: blocks, Popsicle sticks, toothpicks, Styrofoam cups, containers, Geoboards, candies, and various other counting objects. While the use of concrete manipulatives can be an effective instructional strategy, the use of manipulatives is not limited to the classroom setting alone; parents and family members can also reinforce abstract mathematical concepts with their children through the use of everyday household items.

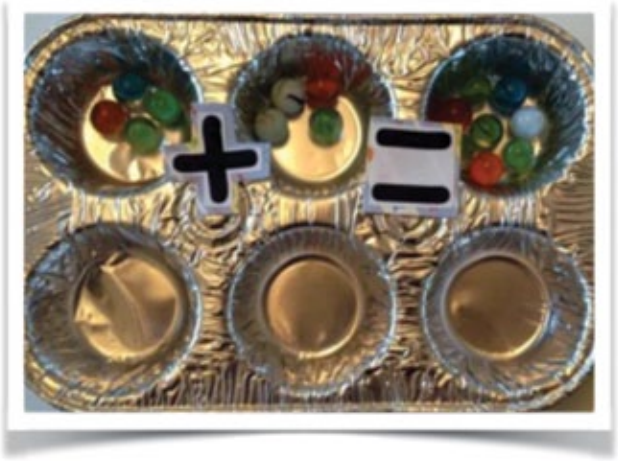
The size of objects can be determined by the age of the child— consider developmental levels and choking potential for small items-- or the quantity you intend to add. Because the intent for manipulative use is the development of number fluency, the grade and age levels for the following activities (1-6) are early-childhood levels (prekinder to second grade); however, if a student continues to demonstrate difficulty with the skills of addition or subtraction, these manipulatives can be utilized at any grade level.

The Amazing Addition Apparatus

CRA levels: Concrete & transition to representational

Materials: Pool noodles, poster board or cardboard (I used a cutting board), marbles, adhesive, basket/bowl/or box.

Description: The *Amazing Addition Apparatus* serves as a visually appealing manipulative to get the early childhood student excited about the concept of addition. The student simply drops his/her desired number of marbles through the first pool noodle and then drops more marbles through the other pool noodle. The student then adds together the two numbers by counting the number of marbles that have fallen into the collection plate. This interactive concrete manipulative allows children to simultaneously have fun and develop a sense of basic addition concepts.

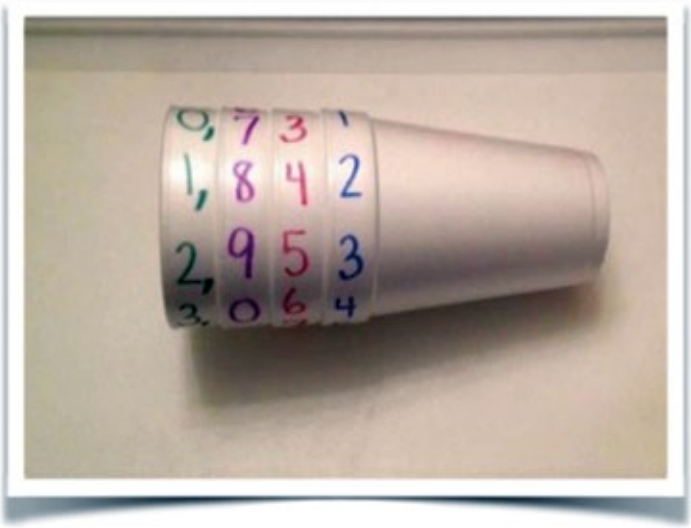


Addition/ Subtraction Muffin Tins

CRA levels: Concrete & transition to representational

Materials: Muffin tin, marbles (or other small countable objects, such as pennies), paper or foam addition, subtraction, and equal signs

Description: This simple DIY manipulative would work well for individual students in a classroom setting because of its low cost and lack of noise. It can also work well in a home setting. The beauty of this manipulative is that any small, countable object can be used in the muffin tin to practice basic addition and subtraction skills. To personalize the learning process, the early childhood learner could use his/her favorite candy as a countable object in the muffin tin! To reinforce the concept of two parts being combined, a student would need to place the object in the addend muffin holes, then move both to the equal muffin hole for counting. Note the error in Figure 2 where each muffin hole contains marbles. The teacher would want to be sure to illustrate clearly the concepts of addition by moving the marbles from the addends cups – left and center-- to the total cup at the far right. This concept can also be used for subtraction, but teachers are cautioned to ensure the concept of removing items from the first muffin hole to result in the difference.



Place Value Cups

CRA levels: Transition to abstract & abstract

Materials: Styrofoam cups (the number of cups depends on the place value being studied), permanent marker(s)

Description: This manipulative is an inexpensive and fun way to teach the concept of place value. The instructor (whether it be the parent or the teacher) gives the student/class a number, and the student would need to be able to manipulate the Styrofoam cups using their understanding of place value to illustrate that number. This manipulative would also be a fun partner activity in the class; each student could challenge their partner to create the number that he/she supplies. When combined with drawings, students in transition from representational to abstract levels can be rewarded by the game of creating the abstract number on the cups. If students have yet to demonstrate conceptual understanding of place value, it is best to start with pool noodle manipulatives or base-ten blocks before moving to the Styrofoam cups.

Place Value Sliders

CRA levels: Transition to abstract & abstract

Materials: Scrapbook paper, (or paint chips), two rulers (or two paint stirrers), permanent marker.

Description: This manipulative is perfect for frugal parents and teachers because it costs nearly nothing to create (assuming that paint chips are free at your store of choice). If you do not have access to paint chips lying around the house or if you are not able to make it to the home improvement store, you can improvise with scrapbook paper instead. Likewise, without access to paint stirrers, you can choose to use rulers. If you choose to use rulers instead of paint stirrers, you will need some paper and tape to cover up the pre-existing numbers on the rulers. If your goal is to create a manipulative that will survive years of classroom use, choose something sturdy (paint stirrer or ruler) as the backbone of this manipulative instead of something more flimsy, like construction paper. Simply write numbers 0-9 on each ruler and make sure that you cut two square windows in your scrapbook paper or paint chips. In use, students are given a number and must demonstrate their understanding of place value (ones and tens) with the given number on their manipulative. This slider could also be used as students are transitioning from representational to abstract stages of CRA by having students first draw the base-ten pictures, then create the abstract number on the slider.

Cards & Erasers

CRA levels: All

Material: Playing cards, any small countable items that are approximately the same size (The image shows dinosaur erasers from the local dollar store).

Description: What a versatile manipulative to utilize. If building conceptual understandings, you can have students create a pile (ten or less) of the same type of item. Then, find the playing card with the same number of pictures.

The abstract number on the card reinforces the student's cognitive transition to the abstract level of CRA.

To assess one-to-one correspondence, students can choose two playing cards and flip them over. Next, the students line up the correct number of small objects (in this case, the dinosaur erasers) to match the number displayed on their chosen playing cards. If placing the students in partner groups, one student can be the number creator and the other can be the number checker.

Mr. Alligator

CRA levels: All

Materials: Green construction paper, printed number cards (or small countable objects, such as pom poms), printed addition sign, printed subtraction sign, printed equal sign

Description: Before using this fun manipulative, the instructor will need to come up with a background story about Mr. Alligator. For example: *Mr. Alligator's favorite activity is eating, and he is ALWAYS hungry. Do you think he wants to turn his mouth towards the bigger number or the smaller number?*

Students who have not yet studied place value may need to use the countable objects (such as the pom poms in the bottom image) or tally marks to make number comparisons. While this manipulative can help children practice their greater than/less than skills, it can also be used as an effective tool to reinforce place value concepts.

Table 2: CRA levels of manipulatives.

	Stage of Instruction in the CRA Sequence				
Manipulative	**Concrete**	**Transition to Representational**	**Representational**	**Transition to Abstract**	**Abstract**
The Amazing Addition Apparatus		✔			
Addition Muffin Tin	✔	✔			
Place Value Cups				✔	✔
Place Value Sliders				✔	✔
Cards & Erasers	✔	✔	✔	✔	✔
Mr. Alligator	✔	✔	✔	✔	✔

Conclusion

The push for inclusivity in recent years has led to a wide variety of student academic abilities in classrooms. While this diversity provides wonderful opportunities for students to become more aware, mindful, and respectful of each other's learning differences, it can also unfortunately leave teachers feeling lost in terms of which direction to take to meet each student's individual learning needs. CRA instruction; however, has been shown to benefit students in an inclusive setting, regardless of the individual ability (Witzel, 2005). In other words, it is probable that students with various levels of arithmetic understanding will increase their knowledge of mathematical concepts by way of CRA instruction. The manipulative ideas offered in this article are useful across the spectrum of CRA (see table 2). Teachers are encouraged to employ these ideas and continue to utilize household materials in the CRA instructional sequence for math. While it is unrealistic and highly impractical for a teacher to assume that all students will advance at the same pace, each student is likely to make advancements in mathematical understanding at his/her own pace if CRA instruction is directed in the appropriate manner.

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