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LATENT CLUSTER ANALYSIS OF INSTRUCTIONAL PRACTICES REPORTED BY HIGH- AND LOW-PERFORMING MATHEMATICS TEACHERS IN FOUR COUNTRIES

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Abstract

Using Trends in International Mathematics and Science Study (TIMSS) 2011 eighth-grade international dataset, this study explored the profiles of instructional practices reported by high- and low-performing mathematics teachers across the US, Finland, Korea, and Russia. Concepts of conceptual teaching and procedural teaching were used to frame the design of the current study. Latent cluster analysis was applied in the investigation of the profiles of mathematics teachers' instructional practices across the four education systems. It was found that all mathematics teachers in the high- and low-performing groups used procedurally as well as conceptually oriented practices in their teaching. However, one group of high-performing mathematics teachers from the U.S. sample and all the high-performing teachers from Finland, Korea, and Russia showed more frequent use of conceptually oriented practices than their corresponding low-performing teachers. Another group of U.S. high-performing mathematics teachers showed a distinctive procedurally oriented pattern, which presented a rather different picture. Such results provide useful suggestions for practitioners and policy makers in their effort to improve mathematics teaching and learning in the US and in other countries as well.

Keywords: Latent Cluster Analysis, International Comparison, Mathematics Teaching, TIMSS, Instructional Practices

Abstrak

Penelitian ini mengeksplorasi profil praktik instruksional yang dilaporkan oleh guru matematika berkemampuan tinggi dan rendah di seluruh Amerika Serikat (AS), Finlandia, Korea, dan Rusia menggunakan data TIMSS tahun 2011. Konsep pengajaran secara konseptual dan prosedural digunakan untuk merancang penelitian ini. *Latent cluster analysis* diimplementasikan dalam penyelidikan profil praktik instruksional guru matematika di empat sistem pendidikan di masing-masing data tersebut. Hasil penelitian ini menunjukkan bahwa semua guru matematika di kelompok berkemampuan tinggi dan rendah menggunakan praktik prosedural dan konseptual dalam pengajaran mereka. Namun, satu kelompok guru matematika berkemampuan tinggi dari sampel di negara AS dan semua guru berkemampuan tinggi di Finlandia, Korea, dan Rusia menunjukkan penggunaan praktik konseptual yang lebih sering daripada guru yang berkemampuan rendah. Kelompok guru matematika berkemampuan tinggi lainnya di AS menunjukkan pola prosedural yang berbeda, yang menyajikan hasil yang sedikit berbeda. Hasil tersebut memberikan saran yang berguna bagi praktisi dan pembuat kebijakan dalam upaya mereka untuk memperbaiki pengajaran dan pembelajaran matematika di AS dan di negara lainnya.

Kata kunci: *Latent Cluster Analysis*, Perbandingan Internasional, Pengajaran Matematika, TIMSS, Praktik Pengajaran

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Since the past two decade or so, researchers and policy makers in the US and around the world have realized the crucial role that teaching quality plays in improving students' mathematics achievement and thus promoting a nation's economic competitive edge in the global economy (NCTM, 1989, 1991, 2000; OECD, 2004, 2005;

Prahmana, Zulkardi, & Hartono, 2012; Abdullah, 2017). Efforts have thus been made to improve the teaching practices of mathematics teachers in the US by introducing new type of teaching backed up by empirical evidences (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). Although such efforts have led to the improvement of U.S. students' mathematics performance over the years, the improvement is slight and the gap remains (Aud et al., 2010; Gonzales et al., 2009), largely due to the teaching gap between the US and those high-achieving education systems in East Asia and Europe (Stigler & Hiebert, 2009). Therefore, there is still pressing need to further improve mathematics teachers' instructional practice in order to boost student achievement.

One important perspective to take in this endeavor is to analyze the teaching practices of high-performing and low-performing mathematics teachers. Examining and comparing the profiles of the instructional practices used by high-performing and low-performing mathematics teachers is able to provide important understanding about the teaching gap and how to improve the teaching quality of mathematics teachers in the US and elsewhere. However, the majority of the existing studies that examined the types of quality instructional practices in the current literature focused primarily on the US (Le et al., 2009). Furthermore, the existing international comparative studies had quite divergent results for the same practice used by teachers among different countries (e.g., Hamilton & Martinez, 2007). Additionally, these studies often included the whole student and teacher sample from a certain country without differentiating the high-performing teachers from those low-performing ones in order to provide useful empirical evidence for identifying quality instructional practices (Zuzovsky, 2013).

Addressing the limitations in the existing literature, this study is aimed to explore the profiles of the mathematics instructional practices of high- and low-performing mathematics teachers across Finland, Korea, Russia, and the US. Using the international data from Trends in International Mathematics and Science Study (TIMSS) 2011, this study investigates, first, the profiles of mathematics teachers' instructional practices; and second, whether such profiles vary across the four countries. Results from the study can provide useful suggestions for practitioners and policy makers in order to improve mathematics teaching and learning in the US as well as globally.

The concepts of conceptual teaching and procedural teaching were used in forming the theoretical framework of the current study. Conceptual teaching, which was promoted by NCTM since the late 1980s, has its central focus on the conceptual understanding and problem solving capacity of students learning K-12 mathematics (NCTM, 1989, 1991, 2000). To achieve the goal, NCTM argued in its published standards documents that teachers need to use student-centered, inquiry based type of teaching so that students have the opportunity to positively struggle in learning new concepts and skills, and in solving math problems. Recommendations were also provided by NCTM for school mathematics teachers in their instructional practices that they should engage the students in using five process standards in learning and doing mathematics. Specifically, instead of being passively driven in mathematics learning, students need to actively persevere in solving non-routine problems, justifying and communicating their problem solving approaches with appropriate tools and representational devices, and at the same time, making connections between relevant topics within mathematics, and relating what they learn in mathematics classroom to what is happening in their daily life. As a bold attempt to improve the mathematics teaching and learning in the US that has been lagging behind other high performing education systems, conceptual teaching has its strong support from constructivist approach of teaching and learning and is believed by NCTM to be able to help U.S. students learn mathematics better in the ever-changing society that needs more proficient problem solvers with adequate critical thinking skills in the workplace (Hiebert et al., 1996; Romberg, 1990; Thompson, 2001).

On the other hand, the concept of procedural teaching has been having its strong presence in mathematics education. With its origin traced to Behaviorist learning theory, procedural teaching has its focus on the primary role the teachers in their use of direct instruction to foster students' basic mathematics knowledge and skills due to their foundational role in students' learning to use mathematical reasoning, solve complex problems, and engage in more advanced mathematics topics (Gamoran, 2001; Geary, 1994; Wu, 1999). Proponents of this type of teaching argue that procedural teaching approach has a unique role in helping students develop high-level conceptual understanding of mathematics ideas and gain better mathematics problem-solving skills through laying down solid foundation of basic mathematics knowledge and skills, which can be developed through procedural mathematics teaching that emphasizes solid memorization of algorithms, facts and rules, routine computational drill, procedural skill practice, and using algorithms, facts, rules and concepts to solve simple, routine problems (Gamoran, 2001; Geary, 1994; Wu, 1999).

The debate between the role of conceptual teaching and procedural teaching in U.S. K-12 school mathematics teaching and learning culminated in the 2000s (National Math Advisory Panel, 2008). Although the debate has been rescinded and it was recommended against the exclusive use of only one type of teaching in teachers' actual classroom instruction, what combination of the two types of teaching can better help students learn mathematics is still an important question to be explored. Using the two types of teaching conceptualizations, this study is useful in examining the profiles of high- and low-performing mathematics teachers in a cross-cultural setting to get a better understanding of the role of instructional practices in students' mathematics achievement.

METHOD

Data

We used the eighth-grade international data from the Trends in International Mathematics and Science Study (TIMSS) 2011 as the data source of the study. A primary reason for using this data set is because each participating education system in TIMSS 2011 has a large student sample obtained from a two-stage, nonrandom sampling design to represent that country's whole student population (Martin & Mullis, 2012). Therefore, study results from using such data set have potential for better generalizability to inform policy makers and practitioner (Schneider, Carnoy, Kilpatrick, Schmidt, & Shavelson, 2007). Another consideration in using TIMSS 2011 as the data source of the study is due to the fact that TIMSS 2011 solicited mathematics teachers' responses to their classroom instructional practices as well as teacher background information such as gender and teaching experience in the teacher questionnaire, which provided important information that meets the focus of the current study (IEA, 2011). Furthermore, in TIMSS 2011, the mathematics achievement information of a student can be linked to the mathematics teacher who taught that student (Foy, Arora, & Stanco, 2013; IEA, 2011). Such linkage is vital in differentiating mathematics teachers into high and low performance categories based on the class mean in order to answer the research questions.

To generate the two groups of mathematics teachers, we first selected four countries from TIMSS 2011 international dataset. Then using IDB Analyzer that is designed to accommodate the sampling weight of TIMSS 2011, we calculated the class mean of the mathematics teachers who taught the same group of students. We then used the international average score of 500 on the class mean to divide the mathematics teachers into high-performing and low-performing groups. Although TIMSS 2011 provided 550 as the high benchmark score and 600 as the advanced bench mark score (Mullis et al., 2009), neither of them were used as the cut off points for categorizing the high- and low-performing teacher groups in order to obtain a larger sample size for the high-performing teachers, which is necessary for the statistical analysis (Tabachnick & Fidell, 2007).

Measures

Following the conceptual and procedural teaching framework, we selected ten in-classroom instructional practices from the teacher questionnaire (see Table 1) (Gamoran, 2001; Geary, 1994; NCTM, 1989, 2000; Wu, 1999). The selection of the ten items, especially the items indicating conceptual teaching, was supported by prior studies (e.g., Desimone, Smith, Baker & Ueno, 2005; Hamilton & Martinez, 2007) and the assessment framework of TIMSS 2011 which used NCTM's standards documents as a guide in constructing the teacher questionnaire (Mullis et al., 2003, 2005, 2009). These instructional practices include five procedurally oriented practices that are 1) teacher explaining problem-solving, 2) ask students to memorize rules, 3) ask students to work on problems guided by teachers, 4) ask students to work problems together in the whole class with direct guidance, and 5) ask students to apply facts, concepts, and procedures to solve routine problems. Additionally, another five instructional practices were selected according the conceptual teaching framework. These are 1) ask students to work problems while teacher is occupied by other tasks, 2) ask students to explain their answers, 3) ask students to relate what they are learning in mathematics to their daily lives, 4) ask students to decide on their own procedures for solving complex problems, and 5) ask students to work on problems with no obvious solution. When surveying the mathematics teachers how often they use the ten instructional practices, the teachers' responses were measured at four levels that include 1) every or almost every day, 2) once or twice a week, 3) once or twice a month, and 4) never or almost. To prepare for data analysis, the four levels were reversed in the process of variable recoding so that the higher levels would indicate more frequent use of a certain practice while low levels would indicate less frequent use of such practice.

Table 1. Instructional Practice Items in TIMSS 2011

Items and shortened name used in analysis (How often teachers do the following…)
1) Teacher explaining problem-solving (explaining)
2) Ask students to memorize rules (memorizing)
3) Ask students to work on problems guided by teachers (individualworkproblem)
4) Ask students to work problems together in the whole class with direct guidance (wholeClassworkProblem)
5) Ask students to apply facts, concepts, and procedures to solve routine problem (solveroutineproblem)
6) Ask students to work problems while teacher is occupied by other tasks (workproblem)
7) Ask students to explain their answers (explaininganswers)
8) Ask students to relate what they are learning in mathematics to their daily lives (relatingtolife)
9) Ask students to decide on their own procedures for solving complex problems (solvecomplexproblems)
10) Ask students to work on problems with no obvious solution (workonproblems)

Data Analysis

In order to answer the research questions of the current study, latent cluster analysis (LCA) (Collins & Lanza, 2010) was applied to examine the group characteristics of the mathematics teachers based on their use of in-classroom instructional practices. LCA is a person-centered multivariate approach to identifying distinct subgroups of individuals. This person-centered approach is different from a variable-centered approach, such as factor analysis. LCA is centered on identifying clusters of participants while factor analysis is focused on identifying clusters of variables.

The model for LCA was presented in Figure 1. One latent variable (i.e., latent cluster) represented by an oval was loaded on ten instructional practices (i.e., cluster indicators represented by squares labeled y1, y2, …, and y10). Note that the latent variable is a categorical latent variable with k cluster. The circles containing the letters e1, e2, …, and e10 represent the error components associated with y1, y2, …, and y10, respectively. For each country, we evaluated models with k equal to one to three by Lo-Mendell-Rubin Likelihood Ratio Test (LMR-LRT) (Lo, Mendell, & Rubin, 2001) to determine the optimal number of clusters k. The LMR-LRT indicates the difference in model fit between a model with k cluster and a model with $k – 1$ cluster. If the LMR-LRT suggests the difference in model fit is not statistically significant, the model with smaller k is retained; otherwise, the model with larger k is preferred. We also looked at entropy which quantifies the degree of separation among the clusters based on posterior probabilities; values closer to one, indicating greater classification accuracy, are desirable (Ramaswamy, DeSarbo, Reibstein, & Robinson, 1993). After the optimal number of clusters was determined, we further explored various characteristics of membership (e.g., gender, highest educational level, major in mathematics, and teaching experience) within the same cluster. All analyses were conducted using Mplus V7.11 (Muthén & Muthén, 1998-2015).

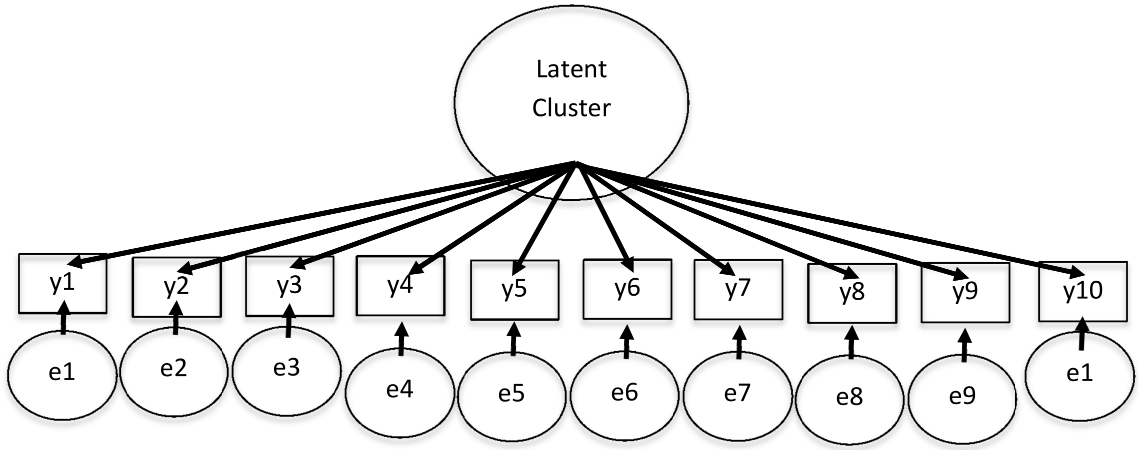

Figure 1. The model specified for LCA.

Note. Latent cluster (*k*) represented by an oval is a categorical latent variable with *k* cluster. The observed indicators are represented by squares labeled y1, y2, …, and y10. The circles containing the letters e1, e2, …, and e10 represent the error components associated with y1, y2, …, and y10, respectively. y1 = Explaining; y2 = Memorizing; y3 = Individualworkproblem; y4 = WholeClassworkProblem; y5 = Solveroutineproblem; y6 = Workproblem; y7 = Explaininganswers; y8 = Relatingtolife; y9 = Solvecomplexproblems; and y10 = Workonproblems.

RESULTS AND DISCUSSION

Results from latent cluster analysis of the data indicate that only one group was generated for the high-performing and low-performing mathematics teachers respectively for the Finish, Korean, and Russian samples. Nonetheless, for the U.S. sample, two groups were identified for the high-performing teachers, while only one group was generated for the low-performing teachers. In the following, we report the results obtained from the analysis of each of the four countries' data.

When looking at the U.S. sample, significant differences were mostly found in the use of procedurally oriented practices instead of the conceptually oriented practices among the three groups (see Table 2). The low-performing group of teachers reported to have used more frequently the five procedurally oriented practices compared with teachers from Group 1 of the high-performing category. However, this group indicated less frequent use of four of the five procedurally oriented practices compared with teachers from Group 2 of the high-performing category. No significant difference was found in teachers' use of all conceptually oriented practices among the three groups except asking students to decide on their own procedures for solving complex problems and working problems while teacher is occupied by other tasks (see Table 2). Among the three groups of teachers, Group 1 of the high-performing category showed the least frequency in using the five procedurally oriented practices, followed by the low-performing group. Group 2 of the high-performing category reported the highest frequency in their use of the five procedurally oriented practices.

For the Finnish sample, high-performing and low-performing teachers only showed significant difference in their use of the last two conceptually oriented practices. Specifically, Finnish mathematics teachers in the high-performing group reported to have used more frequently two of the five conceptually oriented instructional practices than teachers in the low-performing group, “asking students to decide on their own procedures for solving complex problems” and “asking students to work on problems with no obvious solution” (see Table 4). No significant difference was found in their use of the other three conceptually oriented practices as well as all the five procedurally oriented practices (see Table 4).

For the Korean sample, high- and low-performing teachers only showed significant difference in their use of one procedurally oriented practice, “asking students to work problems together in the whole class with direct guidance” as well as one conceptually oriented practice, “work on problems with no obvious solution” (see Table 6). Korean mathematics teachers in the high-performing group reported to have used more frequently the above two conceptually as well as procedurally oriented instructional practices than teachers in the low-performing group. No significant difference was found in their use of all other practices (see Table 6).

For the Russian sample, the high-performing teachers showed significant difference in using four of the five conceptually oriented practices but not the procedurally oriented practices (see Table 8). Russian mathematics teachers in the high-performing group reported to have used more frequently four of the five conceptually oriented instructional practices than teachers in the low-performing group.

Table 2. Descriptive Statistics of Class Indicators Presented by Groups (U.S. sample)

	Score ≥ 500			Score < 500		
Cluster Indicators	Subgroup 1 (n = 49)	Subgroup 2 (n = 164)	Difference Score[a] (Cohen’s d)	n = 199	Difference Score[a] (vs. Subgroup 1) (Cohen’s d)	Difference Score[a] (vs. Subgroup 2) (Cohen’s d)
Explaining	2.31 (0.55)	3.93 (0.25)	-1.62** (d = -3.21)	3.44 (0.82)	1.13** (d = 1.83)	-0.49** (d = -0.79)
Memorizing	2.14 (0.74)	2.89 (0.84)	-0.85** (d = -0.98)	2.75 (0.84)	0.61** (d = 0.81)	-0.14 (d = -0.16)
Individualworkproblem	3.02 (0.80)	3.82 (0.46)	-0.80** (d = -1.08)	3.70 (0.58)	0.68** (d = 0.89)	-0.12** (d = -0.23)
WholeClassworkProblem	2.67 (0.77)	3.74 (0.61)	-1.07** (d = -1.15)	3.55 (0.70)	0.88** (d = 1.23)	-0.19** (d = -0.29)
Solveroutineproblem	3.08 (0.79)	3.64 (0.65)	-0.56** (d = -0.83)	3.51 (0.72)	0.43** (d = 0.59)	-0.13** (d = -0.20)
Workproblem	1.86 (0.87)	2.61 (1.13)	-0.75** (d = -0.81)	2.28 (1.16)	0.42* (d = 0.46)	-0.33 (d = 0.29)
Explaininganswers	3.59 (0.67)	3.49 (0.74)	0.10 (d = 0.01)	3.49 (0.69)	-0.10 (d = -0.14)	0.00 (d = 0.01)
Relatingtolife	2.98 (0.85)	2.74 (0.83)	0.24 (d = 0.29)	2.95 (0.82)	-0.03 (d = -0.04)	0.21 (d = 0.26)
Solvecomplexproblems	3.06 (0.80)	2.77 (0.87)	0.29* (d = 0.35)	2.83 (0.83)	-0.23 (d = -0.28)	0.06 (d = 0.07)
Workonproblems	2.45 (0.94)	2.29 (0.84)	0.16 (d = 0.18)	2.35 (0.84)	-0.10 (d = -0.11)	0.06 (d = 0.08)

Note. [a]T-test has been conducted for group mean comparison. *$p < .05$; **$p < .01$.

Table 3. Descriptive Statistics of Demographic Characteristics Presented by Groups (U.S. sample)

	Score ≥ 500		Score < 500
Demographic Characteristics	Subgroup 1 (n = 49)	Subgroup 2 (n = 164)	n = 199
Gender			
Male	27.66%	29.88%	29.44%
Female	72.34%	70.12%	70.56%
Highest Edcational Level			
ISCED Level 5A, First	34.04%	40.85%	37.24%
ISCED Level 5A, Second	65.96%	59.15%	62.76%
Major in Mathematics			
No	19.15%	28.22%	32.49%
Yes	80.85%	71.78%	67.51%
Teaching Experience (year)	14.38 (9.04)	14.85 (10.85)	12.05 (9.26)

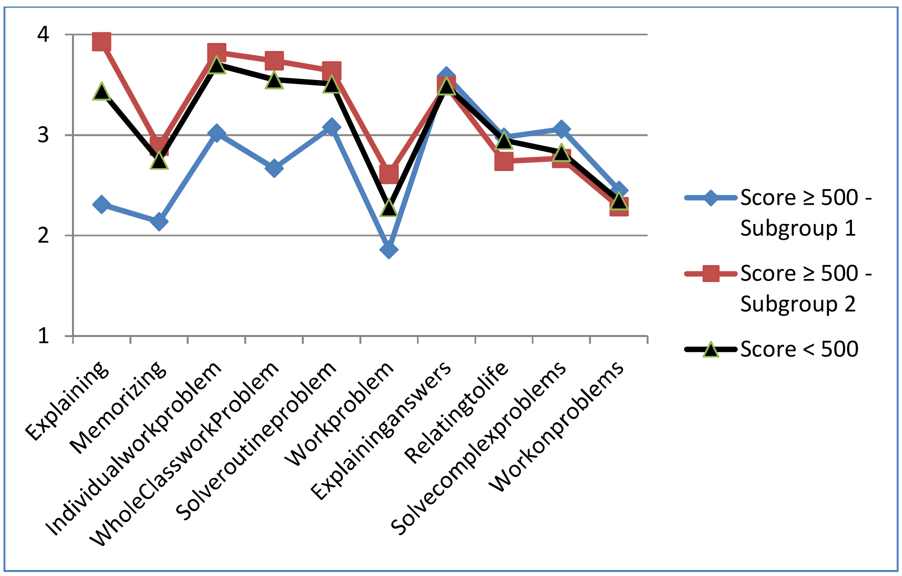

Figure 2. Instructional Patterns for Three Groups (U.S. sample)

Table 4. Descriptive Statistics of Cluster Indicators Presented by Groups (Finnish sample)

Cluster Indicators	Score ≥ 500 n = 147	Score < 500 n = 93	Difference Score[a] (Cohen's d)
Explaining	3.80 (0.51)	3.78 (0.49)	0.02 (d = 0.04)
Memorizing	2.57 (0.73)	2.74 (0.79)	-0.17 (d = -0.22)
Individualworkproblem	3.78 (0.52)	3.74 (0.55)	0.04 (d = 0.07)
WholeClassworkProblem	2.76 (1.02)	2.57 (0.96)	0.19 (d = 0.19)
Solveroutineproblem	3.16 (0.83)	2.92 (0.97)	0.24 (d = 0.26)
Workproblem	1.63 (0.86)	1.76 (0.88)	-0.13 (d = -0.15)
Explaininganswers	3.07 (0.80)	2.96 (0.86)	0.11 (d = 0.14)
Relatingtolife	2.55 (0.66)	2.61 (0.77)	-0.06 (d = -0.09)
Solvecomplexproblems	2.80 (0.79)	2.45 (0.85)	0.35** (d = 0.42)
Workonproblems	2.07 (0.61)	1.79 (0.66)	0.28** (d = 0.44)

Note. [a]T-test has been conducted for group mean comparison. *$p < .05$; **$p < .01$.

Table 5. Descriptive Statistics of Demographic Characteristics Presented by Groups (Finnish sample)

Demographic Characteristics	Score ≥ 500 n = 147	Score < 500 n = 93
Gender		
Male	45.64%	47.87%
Female	54.36%	52.13%
Highest Edcational Level		
ISCED Level 3	2.04%	3.23%
ISCED Level 5B	0%	2.15%
ISCED Level 5A, First	19.73%	17.20%
ISCED Level 5A, Second	78.23%	77.42%
Major in Mathematics		
No	22.88%	61.86%
Yes	77.12%	38.14%
Teaching Experience (year)	17.52 (10.90)	13.26 (9.53)

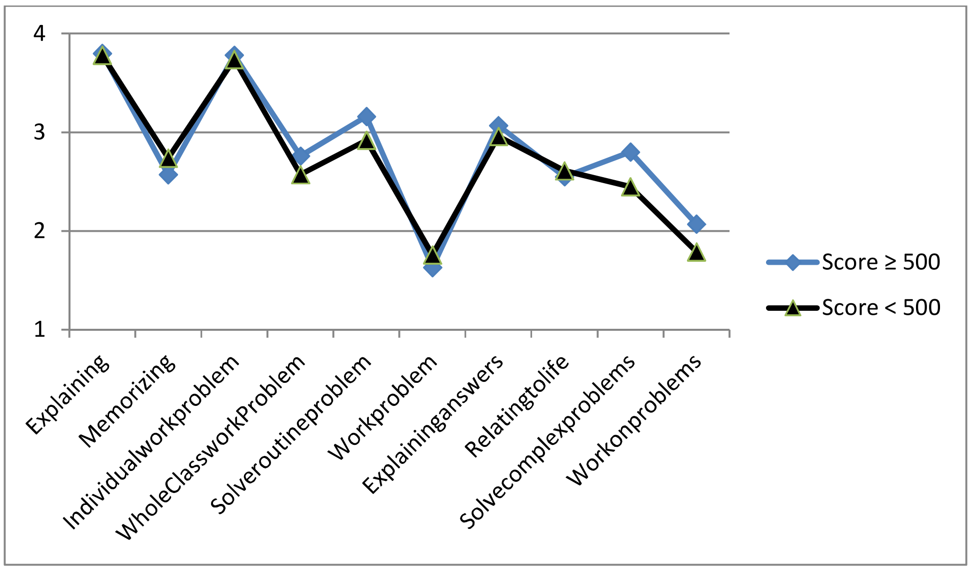

Figure 3. Instructional Patterns for Each Groups (Finnish sample)

Table 6. Descriptive Statistics of Cluster Indicators Presented by Groups (Korean sample)

Cluster Indicators	Score ≥ 500 n = 333	Score < 500 n = 25	Difference Score[a] (Cohen's d)
Explaining	3.79 (0.49)	3.56 (0.77)	0.23 (d = 0.23)
Memorizing	3.27 (0.78)	3.36 (0.71)	-0.09 (d = -0.06)
Individualworkproblem	3.52 (0.77)	3.28 (0.79)	0.24 (d = 0.16)
WholeClassworkProblem	3.70 (0.57)	3.32 (0.85)	0.38* (d = 0.33)
Solveroutineproblem	3.65 (0.59)	3.44 (0.71)	0.21 (d = 0.18)
Workproblem	3.02 (1.02)	2.68 (0.90)	0.34 (d = 0.17)
Explaininganswers	2.82 (0.80)	2.61 (0.64)	0.21 (d = 0.14)
Relatingtolife	2.53 (0.72)	2.56 (0.82)	-0.03 (d = -0.02)
Solvecomplexproblems	2.55 (0.85)	2.40 (0.81)	0.15 (d = 0.09)
Workonproblems	2.36 (0.76)	2.04 (0.98)	0.32* (d = 0.21)

Note. [a]T-test has been conducted for group mean comparison. *$p < .05$; **$p < .01$.

Table 7. Descriptive Statistics of Demographic Characteristics Presented by Groups (Korean sample)

Demographic Characteristics	Score ≥ 500 n = 333	Score < 500 n = 25
Gender		
Male	27.33%	40.00%
Female	72.67%	60.00%
Highest Edcational Level		
ISCED Level 5A, First	61.45%	72.00%
ISCED Level 5A, Second	38.55%	28.00%
Major in Mathematics		
No	2.71%	0%
Yes	97.29%	100%
Teaching Experience (year)	12.67 (10.72)	10.64 (10.55)

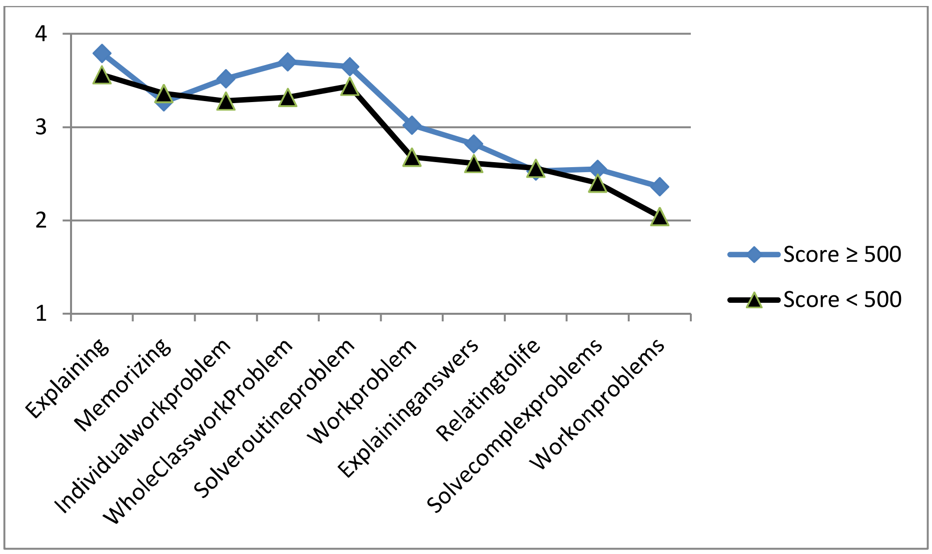

Figure 4. Instructional Patterns for Each Groups (Korean sample)

Table 8. Descriptive Statistics of Cluster Indicators Presented by Groups (Russian sample)

Cluster Indicators	Score ≥ 500 n = 170	Score < 500 n = 58	Difference Score[a] (Cohen's d)
Explaining	3.63 (0.65)	3.66 (0.55)	-0.03 (d = -0.04)
Memorizing	3.18 (0.77)	3.00 (0.79)	0.18 (d = 0.20)
Individualworkproblem	3.30 (0.72)	3.36 (0.74)	-0.06 (d = -0.08)
WholeClassworkProblem	3.35 (0.73)	3.48 (0.78)	-0.13 (d = -0.15)
Solveroutineproblem	3.66 (0.64)	3.51 (0.76)	0.15 (d = 0.20)
Workproblem	2.66 (0.77)	2.33 (0.66)	0.33** (d = 0.40)
Explaininganswers	3.69 (0.56)	3.38 (0.83)	0.31** (d = 0.43)
Relatingtolife	2.70 (0.74)	2.48 (0.75)	0.22 (d = 0.25)
Solvecomplexproblems	2.06 (0.60)	1.74 (0.61)	0.32** (d = 0.46)
Workonproblems	1.94 (0.52)	1.66 (0.51)	0.28** (d = 0.48)

Note. [a]T-test has been conducted for group mean comparison. *$p < .05$; **$p < .01$.

Table 9. Descriptive Statistics of Demographic Characteristics Presented by Groups (Russian sample)

Demographic Characteristics	Score ≥ 500 n = 170	Score < 500 n = 58
Gender		
Male	4.71%	3.45%
Female	95.29%	96.55%
Highest Edcational Level		
ISCED Level 4	1.18%	0%
ISCED Level 5A, Second	98.82%	100%
Major in Mathematics		
No	2.94%	0%
Yes	97.06%	100%
Teaching Experience (year)	24.81 (10.16)	23.97 (14.82)

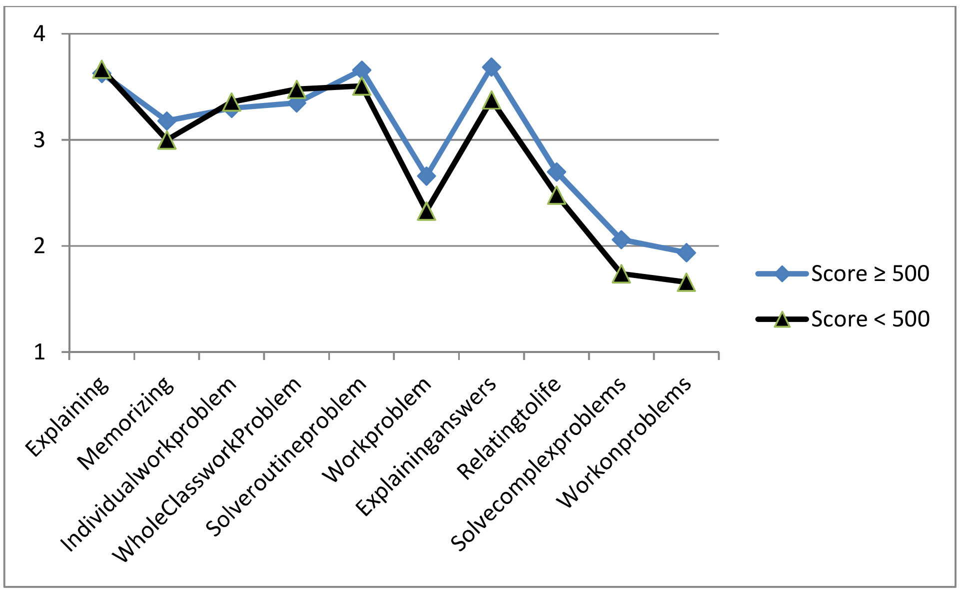

Figure 5. Instructional Patterns for Each Groups (Russian sample)

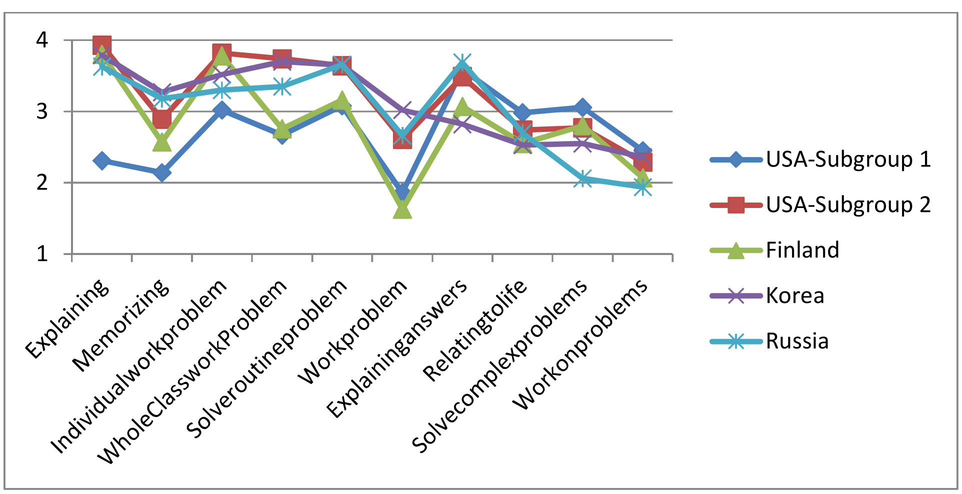

Figure 6. Instructional Patterns for ≥500 Teachers Across Four Countries

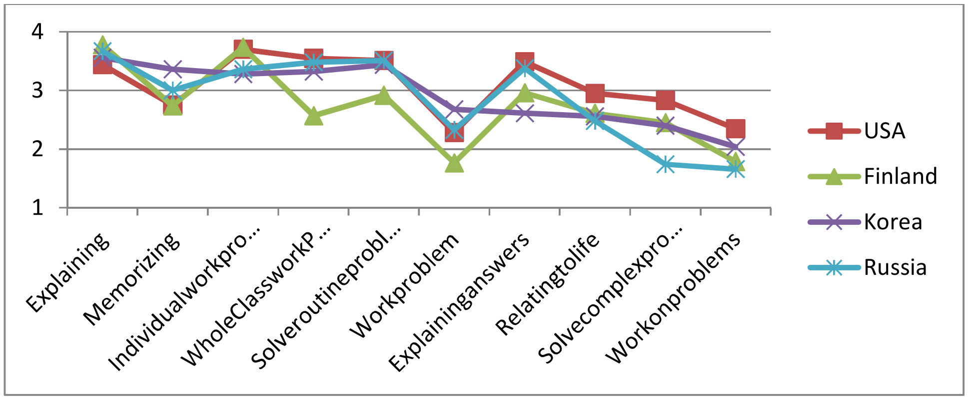

Figure 7. Instructional Patterns for <500 Teachers Across Four Countries

First of all, results from the latent cluster analysis showed that no mathematics teachers from the US, Finland, Korea, and Russia exclusively used procedurally or conceptually oriented practices. They all combined both types of teaching practices in their instruction to some degree and in some ways that are somewhat different from one another. The variations in the teachers' combination of procedurally and conceptually oriented practices led to either higher or lower student achievement in these four education systems. The result supports the recommendation provided by the National Mathematics Advisory Panel (2008) in that separation of conceptual and procedural teaching is impossible and both are useful in helping students learn mathematics. It is important to understand the similarities as well as the differences in the patterns so that recommendations can be provided for practitioners and policy makers.

Secondly, based on the results from the Finnish, Korean, and the Russian samples, the high-performing group of teachers reported more frequent use of some of the conceptually oriented practices than the low-performing group but there is no significant difference in their reported use of procedurally oriented instructional practices by the two groups of teachers. This indicates that, when both conceptually and procedurally oriented practices are used by a teacher, a more conceptually oriented teaching focus can better benefit the students learning mathematics and help bring students' achievement to a higher level. Such result tends to support the argument of conceptual teaching in that instructional practices focusing on higher order thinking skills such as problem solving and conceptual understanding is critical in helping students learn mathematics and thus can contribute to students' higher mathematics achievement (Hiebert et al., 1996; NCTM, 2000).

U.S. sample presented a rather different picture as indicated in Figure 2. Among the three groups of teachers generated from the data, the two groups of high-performing teachers represented two distinctive styles of teaching. Group 1 of U.S. high-performing mathematics teachers is more procedurally oriented as teachers of this group used more procedurally oriented practices than conceptually oriented ones. Group 2 of U.S. high-performing mathematics teachers is similar to the high performing group generated in the data analysis of Finland, Korea, and Russia and can be defined as conceptually oriented group as they used high frequency of conceptually oriented practices than the procedurally oriented ones. Positioned between group 1 and group 2 of the high performing category, the low-performing group represents a more balanced way of using both the procedurally and conceptually oriented practices. It is interesting to note that such a more balanced approach does not associate with higher mathematics achievement whereas the two high-performing groups that represent procedurally and conceptually oriented teaching approaches can lead to higher student achievement. In this sense, it seems that results from the data analysis using U.S. data support the theoretical assumptions of procedurally and conceptually oriented teaching hold true in some degree.

Overall, the above results indicate that across the four samples of high-performing teachers at the country level, there is no certain pattern identified in the teachers' use of the instructional practices (see Figure 6). A similar case was also found across the four samples of low-performing teachers in the four countries (see Figure 7). Such results indicate that teachers' use of both procedurally oriented and conceptually oriented instructional practices can be useful in helping students learn mathematics. However, it is a matter of ways of combining the two types of teaching that can eventually affect students' mathematics achievement. Results from the current study show that conceptually oriented teaching tends to be able to help mathematics teachers in different education systems achieve higher student performance, though one sub-group of high-performing U.S. mathematics teachers also used procedurally oriented practices to achieve a similar result. The result provides evidence for the recommendations of mathematics teaching in the era after U.S. math wars that procedural fluency and conceptual understanding are both important and separation of the two can be detrimental (CCSS, 2010; NCTM 2014).

Furthermore, results from the cluster analysis showed some similarities and differences in the background information of the mathematics teachers. First, the results revealed that across the samples of the four countries, whether or not math teachers had a higher degree does not associate with higher student achievement, which is consistent with what other prior studies found (Ferguson & Ladd 1996; Rivkin, Hanushek, & Kain 2005). This result, though disheartening, seriously challenges the usefulness of a graduate degree. It urges the the program designers and policy makers across the various education systems to reevaluate the graduate programs and identify any potential drawbacks that might hinder program enrollees from transferring what they learn to their actual classroom in order to help the students learn mathematics better.

Second, across the country-level samples, the high-performing and low-performing teachers share quite similar background characteristics except in one aspect: whether or not they are majored in mathematics. For the US and Finish samples, a higher percentage of high-performing teachers tend to be majored in mathematics, while a much higher percentage of low-performing teachers are not majored in mathematics. Specifically, 80.85% and 71.78% of U.S. mathematics teachers from high-performing subgroup 1 and 2 reported to be majored in mathematics, while a much lower percentage (67.51%) was found in the low-performing group (see Table 3). A much bigger percentage difference was found between the two groups of Finnish teachers. As is indicated in Table 5, while 77.12% of high-performing Finnish mathematics teachers are majored in mathematics, only 38.14% of the low-performing mathematics teachers reported to be non-mathematics majors. Majoring in mathematics is an indicator that a mathematics teacher tends to possesses more profound mathematical knowledge and skills necessary for teaching so that they can help the students learn math better, while the lack of training in mathematics will hinder the teachers from reaching their goals in their teaching. Such result confirms what other studies found about possessing a degree in math as a strong indicator for associating with higher student math achievement (Goldhaber & Brewer, 1996). It is thus recommended that the US and Finland need to strengthen the mathematics major requirements for becoming a mathematics teacher in these countries so that their students can benefit from the background training of the mathematics teachers.

However, a much different picture was identified in Korean and Russian samples. Compared with the U.S. and Finnish samples in which a high percentage of mathematics teachers are majored in mathematics and are also found in the high-performing group, almost all the participating mathematics teachers from Korea and Russia in TIMSS 2011 are majored in mathematics, though a rather small percentage (2.71% for Korean sample and 2.94% for Russian sample) of high-performing teachers are not majored in mathematics. Such difference can be explained by the varying requirements for becoming a mathematics teacher in different countries (Karp & Vogeli, 2010; Kim, Park, & Lee, 2013). Considering almost all Korean and Russian mathematics teachers must possess a math major degree and the overall higher mathematics achievements of the students in Korea and Russia, policy makers in the US and Finland are encouraged to reevaluate the requirements for licensing mathematics teachers in these countries as whether or not mathematics teachers are majored in mathematics really makes a difference in student performance.

CONCLUSION

To conclude, this study provides relevant empirical evidence for the patterns of mathematics instruction that can promote students' mathematics learning across four education systems. Results of this study regarding the combination of conceptually and procedurally oriented teaching add to the knowledge base about quality mathematics teaching. This study inspires more of this type of studies to further examine how to improve students' mathematics achievement by focusing on the teaching quality of the mathematics teachers.

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