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Understanding a Brazilian High School Blended Learning Environment from the Perspective of Complex Systems

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The use of technological resources has the potential to make viable new and less traditional methodologies of teaching that take into account student differences. Blended learning can be a way to rethink classes so that students have more freedom in their processes of learning. The goal of this article is to understand a blended learning environment from the perspective of complex systems. We observed the classroom as a complex unit emerging from collective class member interactions. Data from one of two mathematics classes of first year high school students, in São Paulo, Brazil were used in this article. The results suggested that a high school blended learning environment, when seen as a complex system, not only frees students to make personal meaning in their learning processes, but it also provides for collective learning in virtual and face-to-face groups. Features of online discussion groups contributed to the teachers' knowledge about the collective learning, providing them valuable information for formative assessment and pedagogical actions. The blended learning environment seen from a complexity perspective provided evidence that such classrooms demand a different relationship between the teacher, the learner, and the curriculum than relationships observed in the traditional class.

UNDERSTANDING A BRAZILIAN HIGH SCHOOL BLENDED LEARNING ENVIRONMENT FROM THE PERSPECTIVE OF COMPLEX SYSTEMS

Ever since the introduction of the microcomputer, mathematics and science educators and researchers have been challenged to understand the implications of digital technologies for the classroom (see for example the first volume of the International Journal for Computers in Mathematics and Science, 1981). From Logo programming (el.media.mit.edu/logo-foundation/) to GeoGebra (www.geogebra.org/), from the graphing calculator to spreadsheets, from YouTube videos (https://www.youtube.com) to massive open online courses, there are many possibilities for using technology for mathematics teaching and learning. Yet, how different are the vast majority of mathematics classrooms today than they were 100 years ago? Papert (1993) asked what difference would a traveller from an earlier century notice in various situations today. He concluded that a traveller to a medical operating theatre would have a great deal of difficulty recognizing it; however, upon entering most classrooms a traveller would find him or herself in a familiar space. Almost 25 years have passed since Papert wrote *The Children's Machine*. In spite of advances in technology, the secondary school mathematics classroom is still much like it was a century ago, with the chalkboard replaced by an interactive whiteboard, and the potential presence of a computer that in many mathematics classes function as a tool for the teacher's work (including instructional presentations) and less commonly found on the child's desk (Maltempi & Mendes, 2016). In spite of statistics from the Organization for Economic Co-operation and Development (OECD, 2015) indicating an average of 1 computer to 5 students in Canadian classrooms, these are not on students' desks. They are in computer rooms or at the back of the classroom. Javaroni and Zampieri (2015) noted the use of computers in Brazil is almost always restricted to computer labs. Thus, it is not surprising as Borba, Silva, and Gadanidis (2014) noted, that despite the intensification of the use of the internet in all sectors, there is a resistance in the classroom.

Contrast this scene in the contemporary classroom with what one might see on the street, in a café, or in the home of students. The availability and use of handheld technologies by youth outside of the classroom is pervasive. All the while developing technological habits that could be leveraged in the classroom and other formal learning situations (Maltempi & Malheiros, 2010). Ninety-two percent of American youth reported they are online at least once per day on digital technologies (Lenhart, 2015). Studies from the Brazilian Institute of Geography and Statistics (IBGE, 2016) showed that the use of the internet in Brazil has a direct relationship with people's years of study. "For people with up to 7 years of schooling, the proportion was lower than the national average (57.5%), while for those with 8 years or more the proportion was higher. The highest was observed in the

15-year-old population or more study (92.3%)" (p. 49). Digital technologies may provide pathways to mathematical learning and thinking both inside and outside of the 21st century classroom.

Maltempi and Malheiros (2010) reported that with the commonplace and everyday use of digital technologies that students and teachers bring with them to school, "We may someday see the end of the strong dichotomy between distance and face-to-face education" (p. 300). In the scenario where almost half of the world population has digital technology available for them (Borba, Askar, Engelbrecht, Gadanidis, Llinares, & Sánchez Aguilar, 2016), where there is substantial advancement in the access to telecommunications globally and in the enhancement of the internet (Borba, Silva & Gadanidis, 2014), and with massive growth in the number of internet users (Maltempi & Malheiros, 2010), digital technology provides an opportunity for access to high-quality experiences for all learners. Currently there are more possibilities than ever before, particularly for developing countries where internet access has grown and is more pervasive than ever before. For instance, Maltempi and Malheiros (2010) found that at the beginning of 2008, 21% of the Brazilian population were internet users and by "the use of the internet exceeded 50%" (Brazilian Institute of Geography and Statistics, 2016, p. 48). In the Brazilian context this is an opportune moment for educators to rethink pedagogical practices in mathematics classes, as is also the case in many countries where it is now possible to incorporate digital technology to transform the mathematics classroom.

The use of technological resources has the potential to make viable new and less traditional methodologies of teaching that take into account student differences in ways that were not possible with traditional teaching methods (Mizukami, 1986). Maltempi (2008) explained, "Technologies represent an opportunity for change in education, particularly from the teacher-centered practice (or traditional) to the student-centered one, in order to meet their [student] knowledge desires and demands" (p. 60). Accordingly, classes can be rethought in a way that are more student-centered and that give students more freedom in their processes of learning.

Researchers have shown different ways of teaching mathematics that allow students to make their own choices and/or traverse their own way in the learning process (Barros, 2013; Dalla Vecchia, Maltempi & Borba, 2015; Rosa, 2004). These studies made use of technology (Barros & Amaral, 2015), but the results were limited in what they offered teachers because the studies were not conducted in regular high school classes. Research from the Clayton Christensen Institute highlighted that blended learning models as hybrid innovation which incorporate the internet have helped K-12 students to achieve more student centered experiences to attain success in their studies (Staker & Horn, 2012). Educators are creating blended learning environments—online learning with traditional face-to-face classrooms—

to explore further the value of hybrid models of blended learning in K-12 classrooms (Staker, 2011; Bacich, Tanzi, & Trevisani, 2015). Pytash and O'Byrne (2014) suggested the blended learning environment still requires further examination and research, primarily in classrooms.

To contribute to the knowledge of blended learning approaches and their implications, we report on research that created blended learning environments in two mathematics classrooms of first year public high school students (16 to 18 years of age), in São Paulo, Brazil. Our goal with this article is to investigate and interpret data collected from one of these classrooms from the perspective of complex systems. This classroom-based, qualitative study (Bogdan & Biklen, 1999; Araujo & Borba, 2004) involved collaborative work (Fiorentini, 2004), between the first author of this article and the two classroom teachers. It is an action research project to which both the researcher and teachers brought their professional experiences (Tardiff, 2012) to inform the planning, teaching, and evaluation of student learning in the creation of the blended learning environment.

We interpret the classroom as a complex system that emerges from the collective and self-adaptive organization of the members of the class (Davis & Simmt, 2003). As we explicate the blended learning approach in this class, we describe the conditions of complexity within the system that were implicated in the mathematics teaching and learning (Davis & Simmt, 2003) and illustrate how the class can be seen as a self-organizing system (Davis & Sumara, 2006). The value to educators in understanding the classroom as a complex self-organizing system is discussed in relation to rethinking teaching methods that use multimedia and social media in blended learning approaches.

LITERATURE REVIEW

In this section we present the concepts that form the foundations of this study. Both complexity and blended learning approaches will be described. With this literature and using the data from the research we will develop the notion of the self-organization of the blended learning math class.

Complexity

Complexity thinking or complexity science (Davis & Sumara, 2006) is interdisciplinary work that studies collective phenomena that demonstrate self-organization, emergence, and adaptation. The complexity thinking educational researcher recognizes these phenomena as nested systems that learn (Davis & Simmt, 2003). For example, the child is a learning system and her body contains an immune system that is a learning system (Varela, 1994). The class she participates in is also a learning system (Simmt, 2015), and the educational organization that the class is part of is a learning system (Preciado-Babb, Metz, & Marcotte, 2015). Complexity thinking offers a different perspective of learning from many contemporary learning theories. In so far as contemporary theories direct their primary focus at the individual human learner, complexity thinking considers the collective phenomena that may be subsystems of the individual learner or supra-systems of which the individual is a component. To repeat, complexity science is concerned with a range of nested learning systems that include the co-implicated products, relationships, and dynamics of individual and collective knowledge generation.

Davis and Sumara (2006) suggested complexity science could be described as a science of learning systems, where the focus of study is the learning system that self-organizes to the adaptive behavior that co-emerges with the interactions among the components of the systems and with the interactions with the environment. According to Davis and Sumara, a phenomenon can be considered complex if it is self-organizing, adaptive, and emergent. Collective learning systems arise from the interaction of multiple agents where learning is understood as adaptive behavior with which the system maintains its coherence in a dynamical environment (Davis & Sumara, 2006).

Davis and Sumara (2006) noted, "the behaviours of simple and complicated systems are mechanical. They can be thoroughly described and reasonably predicted on the basis of precise rules, whereas the rules that govern complex systems can vary dramatically from one system to the next" (p. 11). Complex systems emerge from the relationships and interactions among the parts but are not the sum of those parts. In complex systems the parts of the system interact and combine in ways such that a new unity can be observed to emerge; it cannot be explained as the sum of the parts, but rather a transformation as a result of their interactions (D'Amour, Khan, Davis, & Metz, 2014; Davis & Simmt, 2014).

Working from the science of complexity we are particularly interested in the emergent phenomena that arise out of the interactions of participants in a mathematics classes. We see the possibility of a *mathematics class* as a collective unity emerging out of the interactions and dynamics of the agents of the mathematics class.

Self-organization of Emerging Collectives

Research has identified dynamics and features that are normally present in complex phenomena (Mitchell, 2009). Some of these include self-organization; bottom-up emergence; short-range relationships; nested structures; ambiguous boundaries; organizationally closed; structure determinism; and states far-from-equilibrium (Davis & Sumara, 2006). In this research we are particularly interested in aspects of self-organization; that is, the spontaneous organization of groups of individuals that arise out of the actions and interactions of autonomous agents that come to be interlinked and co-dependent (Mitchell, 2009; Davis & Sumara, 2006). Because of sensitivity to initial conditions and the self-organizing nature of the complex systems, they are not repeatable. However, systems can be recognized by their overall organization. Hence the *mathematics class* that emerges as an entity that demonstrates learning can be observed in multiple circumstances.

Self-organization or emergence—as Davis and Sumara (2006) equated—is the most important feature of complexity for the purpose of educational research but they cautioned it is the most difficult to appreciate. One of the difficulties they suggested is "the specific conditions and mechanisms of its [emergence] occurrence can vary dramatically across situations" (Davis & Sumara, 2006, p. 81). In spite of being difficult, self-organization should not be ignored. Davis and Simmt (2003) explained, "[T]he addition of the idea of self-organization is critical in the move from descriptions of complex activity to efforts to affect the activities of complex units" (p. 144). Davis and Simmt (2014) later asserted that complexity research has become more pragmatic in its emphases, not just identifying emergence of complex phenomena, but with "more deliberate efforts to trigger them into being, to support their development, and to sustain their existence" (p. 469). There is a need for work that attempts to make the shift from descriptions of learning systems to recommendations for creating the conditions for it in the classroom. Hence, the focus of this research was not to simply describe what happens in a blended learning classroom but instead illustrate how the design teaching methodologies that prompted the class as a learning system.

Decisions around planning for teaching and learning are more about setting boundaries and conditions for activity than about predetermined actions, means, and outcomes (Davis & Simmt, 2003). Hence planning requires a proscriptive rather than a prescriptive orientation. Whereas prescriptive is understood as actions that are allowed in the environment, proscriptive is understood to specify only what is forbidden (anything not forbidden is allowed). Proscriptions set the bounds of behavior at the same time as they allow for an expansion of the sphere of behavioural possibility (Davis & Simmt, 2003). Considering this in the context of a community of learners does not imply "an abandonment of constraints, but a shift in thinking about the sorts of constraints that are necessary for generative activity" (Davis & Simmt, 2003, p. 155). Because the planned lesson is an anticipated overarching site for actions and interactions among individual learners, if conditions of complexity are in place the *lived lesson* can trigger the emergence from a group of individuals to a collective complex unity. Planning lessons can be reconsidered based on the loose boundaries and conditions that the teacher performs.

In addition to the proscriptive logic, there are a many common features that are associated with complex systems (Johnson, 2001; Davis & Simmt, 2003). We focus on five that are particularly relevant to our investigation: internal diversity, redundancy, decentralized control, organized randomness, and neighbouring interactions (Table 1). These five conditions serve to maintain the system's fitness within a dynamic context (Davis & Simmt, 2003). For more conditions of complex phenomena please see Johnson (2001) and Kelly's (1994) work.

Table 1

Conditions of complexity as adapted from Davis and Simmt (2003)

Features	Description
Internal diversity	The capacity for intelligent action of a complex system is based, in part on the diversity agents, products and interactions within that system. The internal diversity also enhances the system's viability and adaptability in its environment.
Redundancy	Redundancy among the agents and their actions is needed for the robustness of a system and the capacity for interaction among agents, as well as making possible for agents to compensate for others' failings.
Decentralized control	In complex systems control is largely based on local interactions rather than on direction from some central control agent. Decentralized control allows for emergent standards of acceptable activity and response.
Organized randomness	This is a reiteration of the proscription versus the prescription notion. Loose boundaries are in place that allow for diverse contributions. Davis, Sumara and Luce-Kapler (2000) reframe this feature as liberating constraints.
Neighboring interactions	Units of knowledge must somehow be made to interact with one another within the spaces dedicated to collective knowledge.

Valuing the possibilities that are afforded by collectives for teaching and learning of mathematics in terms of promoting mathematical actions of individuals and groups, we are interested in understanding how the teaching methods in blended learning environments make possible the emergence of groups as complex learners. Hence our focus is on a blended learning approach in a high school classroom that has a tradition of direct instruction and individual study of mathematics.

Blended Learning Approach

According to Staker (2011), a growing number of schools are introducing programs that go beyond the traditional forms of online learning. In these schools blended learning environments are created where students experience at least part of their instructional time online and part of it in face-to-face classrooms. Skater and colleagues have studied the emerging patterns of blended learning models seen among schools (Staker, 2011).

Christensen, Horn, and Skater (2013) defined blended learning as a:

> formal education program in which a student learns at least in part through online learning with some element of student control over time, place, path, and/or pace and at least in part at a supervised brick-and-mortar location away from home. (p. 9)

Blended learning models, according to Christensen et al. (2013) are emerging in many schools "as a hybrid innovation that is a sustaining innovation relative to the traditional classroom" (p. 5). At the same time "other models of blended learning appear disruptive relative to the traditional classroom" (p. 5). These authors divided blended learning models into two categories: hybrid innovation models and *disruptive* innovation models.

The hybrid innovation models are those that enjoy the best of old technology and the best of new technology, offering sustaining enhancements to classrooms (Christensen, et al., 2013). For instance, hybrid innovation models harness the benefits of traditional classroom at the same time as providing online learning experiences. In contrast, disruptive innovations or disruptive models, are ones in which the old technology is replaced by the new technology focusing on making online learning systems more customizable, affordable, and convenient to the consumers (Christensen et al., 2013). That is, the disruptive innovations break with the structure of the traditional classroom.

The study presented in this article used the rotation model (Christensen et al., 2013) as a hybrid innovation for the teaching design; hence it did not abandon the traditional classroom, but added the internet as sustaining innovation for student learning both out of class and in class. The rotation model is defined as follows:

> The rotation model is one in which within a given course or subject (e.g., math), students rotate on a fixed schedule or at the teacher's discretion between learning modalities, at least one of which is online learning. Other modalities might include activities such as small-group or full-class instruction, group projects, individual tutoring, and pencil-and-paper assignments. The rotation model has four sub-models: station rotation, lab rotation, flipped classroom, and individual rotation. (Christensen et al., 2013, p. 28)

In the station rotation, "students rotate within a contained classroom" (Christensen et al., 2013, p. 28). Students work through content by visiting multiple stations in which content is offered in different ways—for example, different activities (at least one online) and small groups with the help of tutors (students or teacher), as needed. This gives students the opportunity to make sense of the content in situations that differ in the way the content is offered (hence the notion of a station) and at the same time are complementary to each other. In the flipped classroom, "the rotation occurs between the school for face-to-face teacher-guided practice (or projects) and the home or other off-site location for online content and instruction" (Christensen, et al., 2013, p. 28). Students come into contact with the content prior to class through interactions with digital materials suggested by the teacher through (for example) the internet.

Although a common understanding of the flipped classroom involves delivering content via the internet (for example using lectures from YouTube) followed by face-to-face classroom time to deeply explore the concepts with practical work and critical discussions, we understand the flipped classroom in a broader sense where the critical discussions and practical work could also be done out of the class and online. In this article we report episodes that involved flipped classroom and station rotations and present results of those experiences.

THE STUDY CONTEXT

The research presented here is part of a qualitative doctoral research study, conducted by first author of this article based on collaborative work (Fiorentini, 2004) between her and the classroom mathematics teacher. The study was conducted in a public school located at Vinhedo City, Brazil. The survey began with 37 students and it is important to emphasize that the irregular attendance of the students was an issue that worried all the teachers of that class. The mathematics classes took place on Wednesday, Thursday, and Friday evenings each week. On Fridays there were two mathematics periods. This situation provided more time for the teacher and researcher to collaborate and facilitate student investigations.

The research involved an instructional intervention in which learners were immersed in fourteen weeks of lessons on the introduction of the concept of function as a relationship of interdependence, its different representations, and its characteristics through multiple modalities. We selected episodes from the first six weeks of the study to illustrate the classroom dynamics that emerged in this blended learning environment. The researcher and teacher collaboratively developed lessons and co-taught the classes. Due to the collaborative nature of the work, when we use teachers (in plural form) we are referring to the teacher and the researcher. We are using pseudonyms for students' names to preserve their identity.

Research Design

The classes were designed based on Christensen et al.'s (2013) hybrid innovative blended learning models: flipped classrooms and station rotation. Facebook was chosen as learning platform for the out-of-school lessons and interactions. According to Decuypere and Bruneel (2012) and Ferreira, Machado, and Romanowski (2013), Facebook is a tool capable of providing concrete educational possibilities that facilitate learning. The platform was a space for interactions among teachers and students, among students themselves, and between the teachers. Before the classroom sessions on functions students worked with materials offered on Facebook; videos, photographs and software were shared via this platform. After the face-to-face class, teachers and students posted questions on Facebook and those posts fostered further discussions.

Data collection included the following: audio recordings of face-to-face sessions in the classroom; the digital data trail left on the Facebook group page; student questionnaires (pre and post study); two semi-structured interviews with the teacher; and written reflections made by the teacher and the researcher after each class. Data were analysed using standard techniques for general qualitative studies (Merriam, 2009); beginning with transcriptions of the data, data reduction and organization into categories, followed by interpretation from multiple perspectives (Fiorentini & Lorenzato, 2006; Bogdan & Biklen, 1999). Through those actions we sought to qualitatively describe complex phenomena emerging in the classroom. We begin by describing how the flipped classroom and station rotation models were designed and incorporated into the lessons to constitute the blended learning environment.

Flipped classroom

The flipped classroom model can be used to provide students with practical experiences that then can be used as a basis for reflection and discussion about mathematical ideas in the face-to-face classroom situation. From our own practice, we accept that a large part of a mathematics teacher's work is creating suitable experiences for learners in the classroom. Practical investigations are one type of task that can be used to generate empirical artifacts and evidence to be unpacked through subsequent discussion. The flipped classroom model was used in this study as a way to bring practical investigations into the mathematics classes.

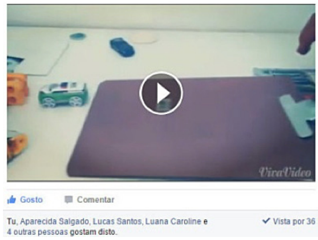
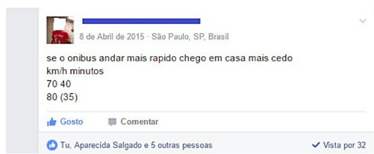
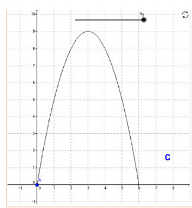
The first activity, using the flipped classroom model, was a video production of some practical investigations (experiments) involving functions. Students were assigned to groups and each group was responsible to engage in an experiment outside of class time, record it, and then share the recording on the Facebook group page. Organizing the lessons in this way provided the students choices with respect to time, place, and collaborators to engage in the learning process. The teacher then used the face-to-face classroom time to develop the functions content more systematically with mathematical ideas that emerged from the students' investigations.

To illustrate we discuss the first practical investigation students did outside of class. The students built three funnels, each with different radii (R_1, R_2 and R_3), then poured the same amount of sand (Q) in each, measuring the time taken for the sand to flow completely through each funnel. In Experiment II, students were to build a funnel with hole of radius (R) and pour three different quantities of sand, (Q_1, Q_2 and Q_3), measuring the time taken for the flow of each quantity of sand. Finally, in Experiment III the students were to build two funnels with holes of different radii, (R_1 and R_2), then pour different amounts of sand, (Q_1 and Q_2), in both the funnels, such that the sand flow should start and end at the same time. For this activity, the students could try more than once and adjust the sand distribution in each funnel—all done outside of class time. From the videos of the experiments posted to the Facebook group, all students in the class had the opportunity to share their ideas and reflections on the relations of interdependence between the quantities (variables) that they had identified: the amount of sand and time, amount of sand and area of the holes in the funnels, among others. In the face-to-face session in the classroom, students shared reflections to deepen their conceptions about the relationships between quantities. Thus, by using the internet, students were provided some flexibility in their learning time, place, pace and path. Because the face-to-face sessions began from the student's ideas shared on the internet, the classroom discussions were grounded in experiences they could relate to. Further affordances by the flipped classroom included access to the videos and online conversations as we explain later.

Station rotation

Initially, students in the face-to-face session were arranged in five groups and each assigned a station with one aspect of the concept of function. The stations focused on relations between two variables, relations of interdependence (direct and inversely proportional), geometric representations of functions, and domain and image of functions. In each station, questions were posed to the students and online materials were offered to ensure learner freedom to engage in their learning process. In Table 2 we present the description of each station.

Table 2
Stations

Station	Description	Internet[1] Resources
1	Cart and Ramp: Consider the experiment of sliding of cart on the ramp. Explain the relationship between the number of books and the height of the ramp. Stack of Dominoes: Consider a stack of dominoes. Explain the relationship between the numbers of dominoes and stack height.	 Video of Experiment of cart and ramp produced by students
2	Robert's Problem: Read the example posted by Robert on Facebook and answer questions provided.	 Comment posted by student Robert on Facebook
3	Trajectory of a Ball: The trajectory of the ball in a shot on goal describes approximately a parabola. Respond to questions using information derived from the graph.	
4	Taxi Rates: The price to be charged for a taxi ride is composed of a fixed amount, for a ride, plus a variable amount, which is directly proportional to the kilometers traveled. Use the Wolfram Alpha software to respond to questions.	WolframAlpha software
5	Proportionality: Do the video lesson on proportionality.	

1 In addition to these specific materials, the students could have chosen other information found on internet.

The station rotation class session included the possibility of interacting with the teachers, learning with and from the contributions among peers within a group, video-lessons produced by the researcher, and the freedom to access the internet as students deemed necessary. Finally, Facebook was always a possibility for students to gather and post material and discussions, in or out of the classroom.

BLENDED LEARNING CLASSROOM AS A COMPLEX SYSTEM

To describe the blended learning classroom as a complex system, we present some data taken from the development of activities on the Facebook and the subsequent corresponding face-to-face session (flipped classroom), from Experiment I. We illustrate how the class is adaptive and self-organizing: in other words, it is a complex system that can be understood as collective learning system (Davis & Sumara, 2006). We recognize the nested nature of learning systems; nervous systems learn, children learn, groups learn, classes learn (Davis & Simmt, 2003). So, our attention focuses on the individuals since this is the level of mathematical utterances but we observed for the emergence of new phenomena and patterns in the group (class) that could signal an emergent unity adapting and self-organizing out of the interactions of the agents. We begin with an analysis of data from the Facebook discussion and follow it with the face-to-face classroom activities.

Facebook Discussion

There 11 student participants on Facebook who made 37 posts and 34 visits, for the lesson based on Experiment I (comparison of size of radius with time for sand to pass through a funnel). What follows is part of a Facebook discussion that took place in relation to Experiment I.

> **Researcher Barros:** This question is for everyone. Do you agree with the conclusion of the group? Why?
> **Jesse:** Very good!
> **Researcher Barros:** Jesse do you agree with the conclusion that the group came up with? Why?
> **Peter:** I agree, yes! Because the greater the measure of the hole is, the faster the sand will pass and smaller, the sand will be more difficult to pass.
> **Researcher Barros:** Guys what do you think Peter means when he says "the measure of the hole"? What would that be? Also I wonder if everyone agrees with the conclusion of the group and why?
> **Teacher Salgado:** It is true Barros, how far is that? By the way, what instrument did Marcos's group use to make these holes?
> **Marcos:** I think the work that I and my group did was to show that math is used in everyday in our lives.
> **Marcos:** We use compass and ruler to make these holes. The measure could not pass 1cm teacher that was exactly the width of the hole of the bottle.

In various moments throughout the implementation and discussion of Experiment I, we observed some students tended to a more traditional stance awaiting questions from teachers. As we note in the interchange above, there is an apparent need for the teacher's question to prompt the discussion. In this context, we understand the issues the teacher raised were important for the students because it created a space for them to share their ideas related to the particular topic. When allowing students to act autonomously by decentralizing the control structures of the class, there is a felt challenge for the teacher to ensure members of the class learn issues related to mathematics.

In the very first teacher interview, Salgado commented about challenges of posing and responding to questions in online activities.

> **Teacher Salgado:** For example, to prompt the student to participate, ask a question that he feels challenged to respond it, that he will want to answer. I do not know how to do it… But still, I have seen that sometimes they [students] do not respond, that is, this is a challenge, do something so that students feel like—"Ah! I want give my opinion about it!" I mean, that's a challenge!

She was self-aware of the difference between the questions that emerged for her in comparison to the questions that the researcher posed to the students: "And, often you do some questions that I think - that is true! Why have I never thought about these questions?" These comments are ones that we might expect from any teacher trying something new that alters their familiar ways of controlling the learning environment. We understand that when the teacher reflected on the need for good questions, she was responding to a new emerging context for her teaching, a context that was emerging with the introduction of the internet in the class. This small excerpt suggests that control mechanisms continue to be part of the dynamic, though the control element is a potential opportunity for the class to shift from teacher centred control to emergent control mechanism that are sensitive to local feedback.

As we examine the class for signs of complexity we return to the online discussion and note the diversity in the nature of the students' participation. Until the moment of being asked the question, "Do you agree with the conclusion of the group?" Jesse's response was a hollow, "Very good." Jesse was not very involved in the interactions, or so it appeared. For example, he was not involved in carrying out any of the three experiments. Peter carried out Experiment II, and Marcos performed Experiment I, which gave rise to discussion presented above. Peter shared his reflection on the relationship of dependency between the funnel hole area and the sand flow time. While Marcos's comment related a realization of that experience as practical, it only came after the teacher's questioning. In itself this excerpt does not illustrate a collective learning system but it does illustrate the conditions were in place for its emergence.

Teacher Salgado's question can be used to illustrate how redundancy functioned in the collective. According to Davis and Simmt (2003), the redundancy is not associated with aspects that are unnecessary, but aspects that are repeated and are available to many members of the group. For instance, it would have been useful if everyone had focussed on the area between the funnel and time as Peter was doing, but as it turned out it was not necessary for the development of the discussion. The question posed by the teacher Salgado triggered the student Marcos to share his ideas associated with the practical experience of the funnel construction, which allowed focus within the collective on the radius and the diameter of the hole, as shown in the next transcript below. The balance between diversity and redundancy in a collective learning system is critical for the success of the system, as indicated by movement or progress on the task.

During the discussion, the researcher asked Marcos about which measure he meant and Peter answered that it was the radius. Then, when the teacher asked him to explain what the radius is, Peter confused the radius and the diameter. Noting the student's doubt, the teacher Salgado questioned whether the measure could exceed 1 cm and if the radius and the diameter were the same thing, as mentioned by the student Peter. Such questioning and responding (with and without errors) triggered the participation of other students and the layering of meaning at the collective level. For example, Peter found another internet site and posted a picture of a circumference where the radius and diameter were highlighted with different colors. Peter's action suggests that the vertical relationship normally found in the classroom (Mizukami, 1986) was disrupted somewhat in the interaction. In that first instance of a student's use of an online platform to contribute to the class signalled the beginning of student-centered practice (Maltempi, 2008), in which students shared materials like video and illustrations to contribute ideas to the collective learning. Further, Peter's answer may have triggered other students thinking and contributions. For example, Paulo (who had not participated in the discussion) subsequently drew a relationship between the diameter and time.

> **Paulo:** I guess it is very well elaborated the issue of varying the quantity of sand and controlling the time.
>
> **Jesse:** Very good. The greater the measure of the hole, the faster is the sand flow, and as [the hole gets] smaller the more difficult will be the flow the sand.
>
> **Paulo:** what Jesse said is true. I fully agree.
>
> **Linda:** So, it was an experience well done and well explained. Yes Barros I agree with him, because the greater is the hole, the faster the sand will come out.
>
> **Matheus:** I agree, because the larger the diameter of the hole the faster the sand will pass.

Davis and Simmt (2003) made a point about *neighbors* in mathematics community, highlighting that in mathematics these neighbors can be ideas, hunches, queries and other manners of representation. Neighbouring interactions are a feature of a complex system that contributes to the ongoing self-organization of the system. Learners' ideas and doubts about the relations between radius and area of a circumference could be seen as neighbouring interactions. Although we might consider the neighbouring interactions the discussion among learners, in doing so we are focused on the individual as learner and that is clearly visible in the data. But if we consider the ideas as neighbors then we look for what emerges that is not specifically about the ideas under discussion but what emerges from them. In this case, for example, these interactions are important for the teacher and the learners as they become a source for anticipating and planning the next face-to-face moments in the classroom.

After analyzing student comments, it became clear to the teachers that although the collective had resolved the problem, some students still needed further clarification of the relation between the radius and area of the funnel hole. Hence, the teachers agreed that it was important to provide some more experiences with radius, diameter, and area of a circumference, in spite of the fact that such matters should already be known at that point in that school year. The researcher created an application (Figure 1) on GeoGebra for students and gave them online access to it.

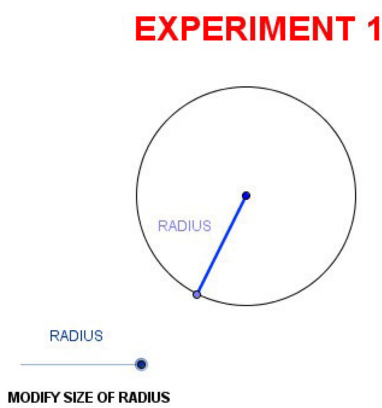


Figure 1. Relation between the radius and area of the funnel hole
http://tube.geogebra.org/material/simple/id/835225

In the application, in Experiment I, when the size of radius was modified, the surface area of the hole of the funnel was changed. In Experiment II, the size of radius could not be modified, so the area remained the same. Finally, in Experiment III students experienced two different sizes of the radius, and consequently two funnels with different surface areas of the opening. Thus, the relation between the funnel opening area and the amount of sand, and between radius and the surface area of hole could become clearer to those students who had not understood by watching the videos.

We understood the possibilities for action in the class were limited by the teaching methodology adopted. The flipped classroom design allowed more freedom in actions for the students and for the teacher in regards to the planning for instruction that could be based on the student interactions in the Facebook group. Complex systems involve a proscriptive logic, where whatever is not forbidden is allowed. Hence when situations emerge in a classroom that has become a collective learning system, the balance between redundancy and diversification among agents ensures the continuing life (learning) of that group (system). In this case, when Jesse repeated the same words of the first response by Peter, and nobody tried to give an answer about why the radius is about 1 cm, the class did not dissipate as a learning system. Rather, the teachers, also part of the complex system, addressed this in their planning. The following is an example of how the neighbouring interactions are more than the discussions the individual members had, new activity emerges from the collective learning system.

> **Researcher Barros's reflection:** During the week the students have had contact with content through Facebook and some students did experiments addressing the subject, these experiments were posted on the platform. So some questions were placed on the Facebook so that students could discuss it. I was excited because there was a good participation by some students but was apparent that some students posted comments without thinking. Also, not all students participated.
>
> **Teacher Salgado's reflection (on the same day):** During virtual conversations of the students, Peter made a comment and also posted some illustrations on radius and diameter of a circle, which needs further clarification. Then, in face-to-face class, Barros handed each student a paper sheet containing some explanatory illustrations on the difference between radius and diameter, between circle and circumference, and how to calculate the area of a circle and then gave a brief explanation on the board, this issue was a prerequisite to continue the lesson.

As in any complex system, individual (agent) learning is affected by the collective, and the collective learning by the individual. As a result, it is understandable why plans would be changed and new planning take place as students move through the lessons. Student difficulties that are not previously known to the teacher can become evident through the individual and collective activity. Once they are noted, the teacher is able to take them up as her contribution to the collective activity. In the case investigated, the teachers were aware that potential difficulties within the larger class might arise after the questions and doubts of a smaller Facebook group were revealed in the face-to-face setting. In spite of working from a proscriptive logic, the teachers did not abandon common practices such as lesson planning and documenting those plans. However, because many phenomena emerge and can reveal new insights for the teachers renegotiation, reorganization, re-planning, among other actions that are not possible when the perceived rules are too rigid and do not permit changes to occur in favor the emerging interests of students.

Face-to-Face Session

In addition to the Facebook activities, the teachers collaboratively prepared for the face-to-face class, according to their flipped classroom model. The students had the freedom to choose their own groups of three to five people, but some students preferred to stay in their out-of-class investigation group. Within the face-to-face class the students had access to the internet enabling them to view the videos of the experiments, do research on websites, and access applications that were posted on the Facebook group (an example is provided in Figure 2). Moreover, each student group was required to answer seven questions that addressed relations (involved in the experiments and others) and the notions of direct and inverse proportionality as interdependencies.

The application illustrated in Figure 2 was built in GeoGebra and posted on Geogebratube. With this application the radius of a circle could be altered thus altering the area. Students were able to observe geometric and graphical representations of the relation between the circle area and the radius. Two other GeoGebra applications, beyond those described here were available online for students. One of those involved a rectangle whose side measures could be changed. The relation between two different sides of the rectangle was represented geometrically and graphically. Finally, the last application enabled students to alter various measures of a square; a number of relations could be observed, including perimeter and side of a square, diagonal side and area, among other variables.

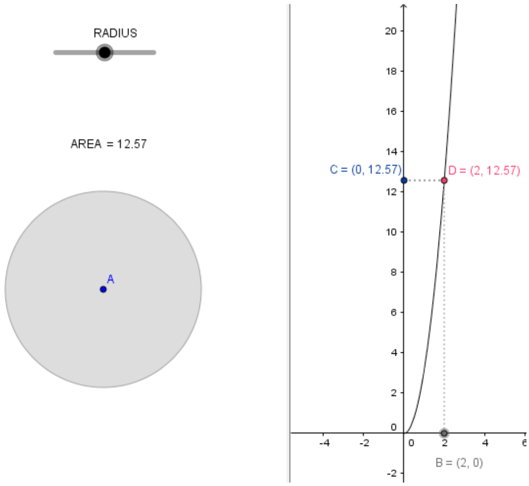


Figure 2. Relation between the circle area and the radius
http://tube.geogebra.org/material/simple/id/836605

Below is an excerpt of a discussion by a group of students as they were interacting with the app in Figure 2 in an attempt of answer some questions. Among the seven questions posed, the students were asked to respond the following: "Is there a relation between the area and the radius of circle? What? If the radius doubles, will the area of the circle double? Justify."

> **Paulo:** Yes, when...the radius, it changed the area together, right? [Pointing to the figure on the application]
> **Jonas:** You have to compare the two.
> **Peter:** Look here!
> **Paulo:** No, because when you change the radius, it is the fixed point to the other side. Seriously, let me explain you. Radius is in the fixed point in the middle to the side, whichever side you move it remains normal radius. But if you stretch it ...
> **Jonas:** It will move! Smart [the student]... it will increase.
> **Paulo:** So, but even increase you increase your circle area...

With the decentralized control in this context, we observed that the instructor-like role played by the student Paulo was a phenomenon that emerged within the group dynamic. In traditional face-to-face classrooms where the teacher is responsible for the dissemination of knowledge and for posing questions for students to answer, this role reversal may not have been witnessed. We emphasize, that which emerges within the system is locally determined by the participants. It is within the system itself that what is acceptable or not is determined (Davis & Simmt, 2003). Below is another excerpt from the same group of students but in the presence of the teachers. We observe that the teacher role in this discussion was that of a peer who also contributed to learning of that collective. She was making sense of the situation the students were discussing— she was a learner too.

> **Paulo:** Teacher, when you increase the radius, you also increase the area?
>
> **Teacher Salgado:** Increases ... how increases?
>
> **Peter:** Proportionally, right?
>
> **Paulo:** Why so here is a dot in the middle that goes from one end to another (showing application)
>
> **Teacher Salgado:** But increases proportionally? Let's see, do basic calculation there ... choose a value for the radius.

Although sharing ideas is an important action for the constitution of knowledge of a complex collective, there can be situations in which the discussion revolves around some misconception from the perspective of the observer and therefore does not advance the mathematics, as the teacher might like. The internal diversity within the group and neighbouring interactions are features of complex systems that can shape the drift of the meaning making and actions of individuals (teachers or students) to guide the discussion to the advancement of knowledge about a particular concept. As we can see in a number of the excerpts, the interaction of the teacher in the development of the discussion of the ratio of proportionality was important. Viewing the class as a collective learner does not mean the teacher steps out of the interaction.

> **Teacher Salgado's reflection:** As was expected, Peter's group (because of his virtual participation) stood out, advanced as well, and made great comments. In one of the moments they used the word "proportionate" but they thought simply that two quantities increase at the same time would be directly proportional quantities, so we made an example with numbers to verify the accuracy of the facts. It is important to note that this group [had a member by the name of] David that primarily did not perform the activities, just enjoyed talking and copy when was necessary. However the group had his effective participation.

Returning for a last time to this example, students had an understanding of the relation between the radius and area of the circle, but from the teacher's perspective they did not demonstrate an understanding of this relation in terms of proportionality. The teacher Salgado questioned them: "But increases proportionally?" The discussion continued about the proportionality and the students concluded that there was not a constant of proportionality. The group was encouraged to direct their attention to what the teacher had realized that was missing. Observing the group, we can see the extent of the internal diversity in that collective among the members' reflections concerning the constant proportionality as the interrelation of units. Further it is important to note that the diversity of ways of participating is also an aspect of complex systems. The teacher reflected on David's participation. He did not participate directly in conducting the experiment but he was active in the discussion. In our view, this particular discussion illustrates the collective learning system.

We presented a discussion about a group of students, that is, a sub-group of students belonging to a larger collective (the classroom) who made sense of the mathematics they explored collectively. We understand that each group of students that emerged in the classroom lessons and the Facebook group, facilitated by flipped class model, constitutes the collective learner. The accumulation and transformation of their ideas, thoughts, perceptions, concerns, among other contributions are the constitutive elements of the collective intelligence of the classroom. This illustrates how the classroom can be observed and studied as a complex system. These elements arise from and recursively promote internal diversity, redundancy, decentralized control within the system—just a few of the features observed in complex systems across domains.

Station Rotation Within the Face-to-Face Session

The blended learning class offers the teacher many possibilities for interacting with student learning since it provides the teacher the opportunity to follow the ideas shared among students closely, something that is difficult in face-to-face sessions where the teacher can only be in one place at one time. In this section, we illustrate how station rotations within the face-to-face class were used to address emergent learning issues that the teacher identified from the discussions about a practical experiment (a ramp whose height was built with books of different number of pages) made and shared by students in the Facebook group.

> **Maria:** The speed depends on the quantity of books because as more books as the trolley speed.
>
> **Marcos:** The video was well done. The time is dependent on the height I think, because as higher as less time spent.
>
> **Peter:** The speed depends on the clipboard slope and the slope depends on the quantity of book, so the cart run faster and run in less time.
>
> **Paulo:** with more inclined it increases the speed of a point to the other and that the cart comes faster.

The teachers saw that the students highlighted the relationship between the speed and slope created by stack of books, and between the time and height of the ramp formed by the same. Later, the teachers observed that group members highlighted other relations, such as the number of books and height. This led the researcher to question the students about proportionality on the Facebook group:

> **Researcher Barros (Facebook, March, 24):** How cool guys, I see that you highlighted several types of relations. One is the relationship between quantity of books used in the construction of the ramp and the speed that the cart slides. Then, consider the following situation and answer: when the ramp is built with 1 book the cart slides down at a certain speed. When building the ramp with 2 books, will this cart speed be doubled? Why?

Following the question posed by researcher Barros and later emphasised by teacher Salgado, the participation of students in the online discussion declined. This raised the teachers' suspicions that the matter needed further clarification. Hence the creation and use of Station 1 (Table 2) to guide the students through the notion of proportionality.

In addition to peer feedback in their mathematics lessons, there were other aspects of *school* that impacted the teachers' planning. For example, at the time of planning for the station rotations the teachers were also concerned with the school calendar. The mathematics class (face-to-face) was scheduled three days a week with a double period on one of the days each week. There were many interruptions over the course of this unit because of holidays, out of school activities and student absences.

> **Teacher's reflection (by narrative in April 01):** Holiday eve, would the students be present? During the day I talked to Barros by phone and we agreed that the class would be a play-off, with the intention to strengthen the concept of relationship and dependence between two quantities. I posted on the Facebook that they couldn't miss, I said that we would do an activity that should be posted as other experiments.

Because of these circumstances the teachers agreed that some students seemed to feel discouraged and were slipping out of their study routines. To address this issue the researcher Barros made an instructional video (Station 5, Table 2) to help students with issues about proportionally and the teachers agreed to do a review class rather than move on to new content, knowing students would be absent.

Whereas Station 1 arose from the perception and evaluation of the teachers about the difficulty students were having in an online group, Station 5 arose from school calendar issues, and yet other stations emerged from other issues. We see that the creation of each station was an adaptive method to support the phenomena emerging in that classroom. Although we do not describe the phenomena that resulted in of all stations, it is important to highlight that the context of the technical structure in which the blended class is inserted also contributes to challenges for the development and adaptation of such methods. For example, the number of stations that were planned for the station rotation proved to be a challenge, there were only two laptops (teachers') and a few smartphones (students'). Further, because the internet did not work well when all devices were connected at the same time, having more than five stations was impractical. Five stations, each with different activities for students, were used in the station rotation.

Self-Organization of a Math Class and Challenges for Assessment

We assert that the self-organization or emergence of math class arises from the interactions among teachers, students, curriculum, school context, textbooks and culture, among others; that is, according Davis and Simmt (2014) there are many agents that may or may not have much in common but contribute to a common project, in this case math class. Therefore, such agents are constituent parts of math class, not in the sense that the system is the sum of these parts, but rather in the sense that the system emerges as transformations occur with the interactions among them (Davis & Sumara, 2006; Davis & Simmt, 2003).

Many factors within and outside the school affected the math class: Interactions and interrelationships of events that took place before the study; the preparations for particular lessons; and the actions and interactions of the class members within a connected set days. These are the emergent phenomenon from which the class as a unity emerged. As an illustration of the transformative nature of the interactions on the class we described the self-organization features of the class that was based on the station rotation when Barros created activities to assist students in the difficulties that she had identified within their discussions:

> **Researcher Barros (by narrative on April 17):** In class today we worked with several stations. Since March 27 until today (April 17), it was only through the activities on Facebook and narratives of teacher Salgado that I could see the doubts of the students. Then the stations were designed in order to reinforce knowledge of the activities that they had many doubts.

When the researcher Barros says that she could see the doubts of the students, we identify a moment of formative assessment, one that contributes to the improvement of student learning instead of measuring what was learned (Rosa & Maltempi, 2006). In this way, we accept that the internet facilitated the teacher's creation of new learning opportunities. With this move to a blended environment for mathematics learning the dynamics of the classes changed. Each station that was created was designed to address students' doubts, whereas the initial idea was to have students work in regular classes with the same groups formed to carry out the experiments that were posted on the internet. As the collective learning systems emerged, features of its complexity were evident (that is, decentralized control, interaction between neighbors and internal diversity of the classroom). The groups underwent a natural process of adaptation and evolution; they were in a state of ongoing self-organization as groups changed members, ideas transformed and activities were conducted. This evolution of the groups created tensions for the development of the teacher and student work, especially for the purpose of evaluation of learning and assessment of achievement.

When the researcher Barros asked the teacher Salgado, in the interview in March 27, about the dynamics of the students working in groups in the classroom and learning in this way, Salgado pointed out that "it is possible to learn math in group" but added:

> They are not used to doing this. So, in a group with four students, one or two do work and the others two do not. In one group of five also are one or two working and the other three did not. So are not involving everyone, but I would say it is because they are not used to develop such work.

This was a problem not only for the students but for the teacher as she struggled to understand how she would evaluate individual student learning from their collective work. In the same interview the researcher asked the following question of teacher Salgado:

> Do you think that the dynamic on the Wednesday and Thursday classes (days that the teacher worked without researcher, in the face-to-face sessions), somehow, has been influenced by your need for change due of what happens on Friday and virtual sessions?

Salgado responded by commenting that she "used to give freedom to them late because" of her "general (boss) behavior." She was used to keeping maximum control over the students and she only gave them freedom after some time, according her, "late." But, she reflected that the friction in the new context "is not happening because they are freer." She wondered about how her behaviour was implicated in it:

> So ... how will I be bossy in a group? I can see something and talk ... you'll have to leave ... But there they were talking and will continue talking, right ... because if I say stop, what is the conversation that I've been stopping? Am I stopping gossip or a productive conversation? I do not have to do this.

For us, her response is important to understand. While the student involvement in the collective reduces the friction with the teacher, difficulties arise for this teacher to evaluate the production of the collective as a unit, which is compounded when the individuals involved are not so well known by the teacher, as in the case of a new class.

Teacher Salgado experienced a challenge in her work in so far as she was to evaluate participation of student in face-to-face collectives because it was not clear to her when a particular student was making sense of the content or not. According to Simmt (2015), there are enduring challenges for mathematics educators in terms of the evaluation of collective learning. In this research the freedom provided by the blended learning environment is evident, and it is facilitated by the possibility for students to share their particular ideas, which is an important process for the collective learning system. On one hand, it is important to offer students more ways to engage in and demonstrate their learning processes and for the teacher to adapt formative assessment in their classes. On the other hand, collective evaluation still is a challenge for teachers. It is possible that these challenges are stronger due the traditional need for teachers to measure what the individual student learns.

CONCLUSION

The goal of our article was to better understand a high school blended learning environment from the perspective of complex systems. From this perspective, we observed the classroom as a complex unity emerging from collective interactions of the members of the class. Specifically, we analysed how the conditions for complex systems were manifested and implicated in the learning that occurred in a blended learning environment that used Facebook as a learning platform for out-of-class learning interactions and stations in face-to-face sessions. The evidence suggests diversity, redundancy, decentralized control, and neighbouring interactions were present in the collective actions. From those collective actions, we saw the emergence of a collective learning as mathematical problems and ideas were posed, created, investigated, explained, and questioned, and as the teachers rethought what it meant to ask questions, check on students' progress, and assess student learning.

As the collective learning systems emerged from interactions between ideas, thoughts, perceptions, and doubts among the individuals in that system, students were learning mathematics and teachers about the mathematics class. The teachers were learning especially, about ways of interacting pedagogically under what were very different teaching methodologies than those with which the teacher was familiar.

Under the complex system perspective, the blended learning environment provided ways to bring forward student needs that could be attended to by their teachers, as we highlight in this research. From online activities the teachers were confronted with students' needs and used those to rethink their face-to-face lesson plans. Beyond this, we can see from this study that issues from outside of the classroom itself, including the school and everyday life, are implicated in the extent of students' engagement in their studies. Here too the blended environment creates possibilities to address these issues in so far as they impact student's mathematics learning. Because this study implemented a new model of instruction, we were able to observe the moments where the familiar was replaced by the unfamiliar for both the students and the teacher. This helped us observe the emergence of the classroom as a self-organizing system.

In this study we noted the challenges teachers experienced with asking questions that make sense in the collective. Evaluation of students' understanding and achievement also posed challenges for the teacher. At the same time the features of online discussion groups contributed to the knowledge of the teacher about the collective learning, providing the teacher with excellent information for formative assessment, which was used to inform instruction. Davis and Simmt (2003) claimed "an attendance to the collective actually makes space for, and supports the development of, individual students' ideas" (p. 147). We too witnessed the development of the individual student understanding in their contributions to the collective.

This research suggests that there is potential in viewing the classroom as a collective learning system, one that demands a different relationship between the teacher, the learner, and the curriculum. In a classroom where collectivity is stifled by teacher lectures and individual student practice, where the teacher assesses for correct responses and does not engage with learners in meaning making, and where the teacher does not embrace the digital world her students live in, the classroom will continue to look like the one of the last century. However, maybe the most important lesson from this research is that we can rethink classes in ways that are more student-centered, and in ways that give students more freedom in their processes of learning. It is important that teachers begin to understand the classroom as a potential collective learning system that can flourish in a blended learning environment that takes advantage of technology, where individual learners can take control of their learning.

Finally, seeing the blended learning environment from the complex system perspective, we understand that the challenge to adopt this approach is beyond a simple choice of some hybrid teaching model or a disruptive model. This is because the features of these models are agents that interact with collective learning systems emerging enabling a bigger collective learning systems' behaviors that are adaptive and ongoing self-organized, and are clearly elements of a complex system.

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