Mathematical Discussions: What Teachers and Researchers Know

The teaching and research communities in mathematics education agree that mathematical discussions pose challenges in elementary classrooms. Teachers have identified various reasons for this challenge, including students’ prolonged histories of lack of participation in mathematical discussions (Planas & Civil, 2009); deficit views on non-dominant students that contribute to transform classrooms into teacher-centered contexts (White, 2003; Stiff, 1998); lower expectations that position students as poor discussion participants (Planas & Gorgorió, 2004); and beliefs that student engagement in discussions is independent from teachers’ instructional approaches (Planas & Civil, 2009). Researchers, on the other hand, have considered additional reasons, including social class and gender (Lubienski, 2000), the degree of openness in discussion-oriented questions (Parks, 2009), or the theoretical perspectives used to understand discussions in bilingual settings (Moschkovich, 2007).

These challenges continue to motivate research on mathematical discussions, with a focus on how students use talk in discussions. There is evidence, for example, of the role of talk in promoting deeper levels of...
learning (Cohen, 1984; Cohen & Lotan, 1995); the contribution of different kinds of talk (e.g., exploratory talk vs. final script talk) to student learning (Mercer, 1995; Mercer, Wegerif, & Dawes, 1999); the appropriation of different kinds of teacher talk by students (Khisty & Chval, 2002); This research suggests a clear vision: The quiet, orderly classroom in which the voice of a teacher controls what students listen to is no longer a desirable environment for learning mathematics. Instead, discussion-oriented mathematical practices in which students construct viable arguments and critique the reasoning of others are deemed as desirable in classrooms (CCSS-M, 2010). The challenges identified by teachers and researchers are relevant for figuring out this new environment. Equally relevant is to resituate challenges for mathematical discussions within the experiences that schools offer non-dominant students. To contribute to this new perspective on challenges, this study addresses the question, What can teachers and researchers learn about mathematical discussions, when such discussions are led by bilingual students? Evidence of bilingual mathematical discussions at the student-to-student level is analyzed, and implications for both Spanish-English bilingual and English monolingual mathematics instruction are offered.

Situating Students’ Bilingualism in and out of School

Humans use language to situate their various activities. For example, the language of conversations at the doctor’s office is very different from the language of conversations with the plumber or the language of conversations between teachers and students. In these contexts, not only does it matter who the doctor and the patient; the plumber and the customer; the teacher and the students are, but their activities are nested in interactive systems that shape all practices within an activity. Newcomers to these activities (e.g., bilingual students entering monolingual schools) soon learn what to say—as well as what not to say—as they notice how others use language to create the social, cultural, and linguistic profiles of activities in which they participate.

For bilingual students, a seduction operates in how schools focus on individual behaviors (e.g., whether these students sound native-like when they speak English) to promote and maintain language practices (e.g., English monolingual instruction) that normalize the mathematics education of bilingual students. Many bilingual students learn that to be accepted participants in the school activity, they must speak and use English in ways that are perceived as accepted. Greeno (1998) noted that individual behavior has received more attention—particularly in western cultures—than the “interactive systems that are larger than
the behavior and cognitive processes of an individual agent” (p. 6). According to a situated perspective on learning, “situative research can investigate the properties of individual’s cognition and behavior that support their contributions to the functioning of the systems in which they participate” (p. 7). This perspective is useful to understand the ways in which discourse in schools and information systems perceiving school life situate or normalize monolingual learning, while pushing to the margins the students’ bilingual lives. I contend that pushing these bilingual lives—or at least a little bit of these rich and complex lives—back from their margins into the center of classroom instruction can help us understand dimensions of mathematical discussions that have not been the focus of previous research.

Pushing Back Students’ Bilingual Lives into the School

A different way of looking at the challenges so far identified for classroom mathematical discussions is to look at the problems that students solve in their classrooms everyday and ask: could these tasks be uninteresting to them? Could these tasks fail to stimulate students to speak, to engage in discussions with teachers and peers? According to Bruner (1990), “the very act of speaking is an act of marking the unusual from the usual” (p. 79). Inseparable from the kinds of tasks that students are offered is the language in which these tasks are framed. Given that these tasks are offered in English to bilingual students, what does this practice mask about students’ ability or desire to engage in mathematical discussions? Mathematical problems in schools can be worth discussing when they are both designed with rich mathematical meaning and implemented by teachers who know how to explore that meaning with their students (Hill & Charalambous, 2012). Although teachers can intentionally support mathematical discussions (Chapin, O’Connor, & Anderson, 2009; Kazemi & Hintz, 2014), the theoretical framework in this paper focuses on the purpose or reasons that students themselves see in solving a mathematical task. People in their daily activities, where they have to solve real problems, talk about them because they have a purpose, a reason to discuss these problems. A conjecture that emerges from these theoretical underpinnings is that designing mathematical tasks so bilingual students can link their experiences—including the languages of these experiences—to opportunities to learn important mathematical concepts (Dominguez, 2016) will facilitate discussion.
Methods and Participants

To investigate the question of what researchers and teachers can learn when mathematical discussions are led by bilingual students, I contacted several schools with large populations of Latino/a bilingual students. The school that accepted to participate in this study had an enrollment of approximately 400 students and it was located in a working class neighborhood in central Texas. The school had an early exit bilingual program—an example of a system focused on individual behavior—that transitioned the otherwise bilingual students into English-only classrooms no later than grade four. The principal nominated two classrooms—one fourth grade and one fifth grade—each with 100% bilingual Latino/a students. The teacher of the fourth grade classroom was an English monolingual male, and the teacher of the fifth grade classroom was a bilingual female. To understand how these bilingual students were learning mathematics (their situated experience), I observed each mathematics class for approximately two weeks. Instruction in both classrooms was exclusively in English, as students in both classrooms had been transitioned into English-only instruction. Teachers seemed to value quiet work. For example, in one classroom, the male teacher discouraged a group of students from talking when solving a math task, even when the students explained that their talk was about the problem. In the other classroom, the female teacher insisted that students used exclusively English when asking questions or when talking with one another. Students in this class rarely asked the teacher questions or talked with one another.

To further situate the activity of school mathematics learning for bilingual Latino/a students, I visited all the homes of the fourth and fifth grade students. Unlike in classrooms, in these homes children’s talk was unrestricted, and I observed these students fluidly transitioning from English to Spanish as they were addressing different family members or talking about different activities. In these home visits, I talked with parents and students together about the students’ participation in both student selected and parent assigned activities outside the school. From these varied activities, I focused on those that were most common across all participants to create a set of mathematical tasks that reflected both the experiences and the languages of those experiences. For example, most parents said that children ate breakfast at the school cafeteria; that they helped with the grocery shopping; and many participated in preparing simple meals (e.g., scrambled eggs) for themselves or for younger siblings while the mother was working on something else. In all these activities, students and parents agreed that the children spoke primar-
ily Spanish. Task A (see Table 1) reflects both the familiar experiences and the language of these experiences as envisioned in a measurement division task. As a way of understanding the possible effect of familiarity with certain experiences, Task B (see Table 1) also presents the same measurement division concept but embedded in an experience that I conjectured to be unfamiliar to most students. To figure out unfamiliar experiences, I envisioned activities in which adults, not children, are more likely to participate.

Parents and students also talked about a common activity in which students had to read in English to an adult at home as part of their reading assignments. Task A and Task B (see Table 2) reflect a partitive rate and a measurement rate situation, respectively. These tasks were designed following the same criteria as for the measurement division tasks.

As I was creating these tasks (for the complete set of eight tasks, please see Dominguez, 2011), I was reminded of how Boaler (1998) recognized the importance of situating students’ activities within a school context because “the combination of school settings and realistic constraints provided by applied tasks can give us important insights into

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<tr>
<th>Table 1</th>
<th>Two Measurement Division Tasks in Spanish</th>
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<tr>
<td><strong>Task A—Familiar Experience</strong></td>
<td><strong>Task B—Unfamiliar Experience</strong></td>
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<td>Para el desayuno escolar, la señora de la cafetería de tu escuela tiene que preparar huevos revueltos para 400 niños. ¿Cuántos cartones de huevo tiene que abrir?</td>
<td>Un organizador de fiestas está organizando una fiesta para 500 personas. Necesita comprar platos. ¿Cuántos paquetes de platos necesita comprar?</td>
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<td>(Translation)</td>
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<td><strong>For the school breakfast, the school cafeteria lady has to make scrambled eggs for 400 children. How many egg cartons does she have to open?</strong></td>
<td><strong>A party planner is organizing a party. He needs to buy plates. How many packages does he need to buy?</strong></td>
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<th>Table 2</th>
<th>Partitive Rate and Measurement Rate Tasks in English</th>
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<td><strong>Task A—Familiar Experience</strong></td>
<td><strong>Task B—Unfamiliar Experience</strong></td>
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<td>If you can read 5 pages in 20 minutes, how long is it going to take you to read a book that has 23 pages?</td>
<td>If a painter uses 20 gallons of paint to paint 4 houses, how much paint will he need to paint 15 houses?</td>
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the factors that influence a student’s use of mathematical knowledge” (p. 53), and in the case of bilingual students, access to how they use their bilingualism in mathematical discussions. Each pair of tasks dealt with the same mathematical concept, was framed in the same language as the out-of-school activities, and only differed in that one task included an experience familiar to students and the other task did not.

Teachers expressed concern that the tasks that I had designed might be too difficult for students to solve, thus reflecting some of the challenges identified in the literature. At the same time, they expressed restrained curiosity regarding results. I asked teachers to nominate pairs of students that get along well and that they believed would contribute to the problem solving process fairly equally. A total of 20 pairs of students were interviewed and the work produced during these problem-solving sessions was collected. All discussions were transcribed, with attention to both verbal and non-verbal communication, such as gestures, gazes, body postures, pauses, and such. By creating these detailed transcriptions, I wanted to capture the details of student-to-student mathematical conversations and look for aspects of these communicative exchanges that may contribute to advance our understanding of bilingualism in mathematical discussions.

A preliminary coding process (Gibbs, 2007) was applied to natural units of communication, that is, “discourse units in which participants organized and coordinated actions oriented toward solving a problem” (Dominguez, 2011). These initial codes were descriptive of the ways in which students socially organized and coordinated their discussion activity around mathematical tasks. For example, when students expressed concern for the reason of a solution, the natural unit of communication containing that instance was coded as “Question emergent solution.” Similarly, when students chose a procedure that had no relevance for the relations among quantities in a problem, the natural unit of communication containing that instance was coded as “Applies and defends wrong procedure.” The following results explain how these initial codes began to suggest some major categories that were relevant for addressing the research question.

Results

Codes that described the nature of bilingual students’ mathematical discussions across different mathematical tasks and languages soon began to form two important categories. In some of these codes, students discussed procedures that, whether or not they knew how to perform, had nothing to do with the problem at hand. In other codes, however,
students’ discussions focused on taking risks in the process of solving the mathematical tasks. Focusing on the codes in which students’ conversations were characterized by risk taking, I calculated ratios that compared the amount of risk-taking actions across languages and across familiar versus unfamiliar experience problems. Table 3 shows these ratios.

These risk-taking ratios suggest that the familiar contexts that students encountered in mathematical tasks supported them in leading discussions characterized by taking risks. These risks, however, were not taken in isolation; instead, students tended to be social risk takers. This aspect of the mathematical discussions is not revealed by the ratios, which are practically the same across the two languages and across the two mathematical concepts. To have a sense of the social nature of these bilingual students’ risk-taking when solving problems, I considered how interactive and intersubjective they were in their mathematical discussions (Turner, et al., 2013). That is, when they generated an idea, did the idea travel back and forth between the two partners? Was the idea challenged and developed or ignored and therefore underdeveloped?

To understand that aspect of the results, representative examples of students’ conversations are now presented. I begin with examples from the category of codes in which students’ discussions focused on superficial approaches to problem solving. A common approach consisted of choosing a procedure that disregarded the relationship among the quantities in a problem, and still defending that procedure as valid. For example, in the unfamiliar experience represented in the party planner task (see Table 1), a pair of students, Dave and Amanda, engaged in the following discussion.

Dave: *Yo voy a sumar 30+470* [I’m going to add 30+470]
Amanda: ¿*Por qué?* [Why? (frowns)]
Dave: *Porque así me daría 500* [Because that would give me 500]
Amanda: ¿*Y de dónde sacas los 470?* [And where are you getting the 470 from?]
Dave: ¡*Ah sí, son paquetes!* [Oh yes, it’s packages! (smiles)]

Bypassing the meaning of an unfamiliar situation, Dave began the

<table>
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<th>Language</th>
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<tr>
<td>Math Concept</td>
<td>Measurement Division</td>
<td>Rate</td>
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<tr>
<td>Risk-Taking Ratio (familiar/unfamiliar)</td>
<td>2.35</td>
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Table 3

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discussion by playing the word problem game (De Corte & Verschaffel, 1985). Even after Amanda questioned his approach, he still defended his procedural choice as valid. Only after Amanda inquired further about the provenance of 470, did Dave realize that he needed to attend to what the problem was about: figuring out the number of packages the party planner needed to buy. Eventually, Amanda and Dave managed to solve this task successfully, but only because Amanda challenged Dave’s superficial approach. Most pairs of students, however, were not able to transform their initial procedure-oriented approach into a more meaning-oriented discussion when solving these kinds of tasks depicting unfamiliar experiences. More importantly, their orientation to learn procedures instead of meaning promoted very little mathematical discussions.

The learning without thought, talking, or reality (Boaler, 2008) that began to characterize students’ mathematical discussions when solving tasks with unfamiliar experiences in Spanish was more evident when students solved these kinds of tasks in English. For example, for the unfamiliar experience depicted in the painter task (see Table 2), another pair of students, Josué and Alfredo, approached the problem by working independently, with little interest in each other’s work, even though they and the other pairs of students were good friends. From my position behind the camera as I was videotaping these discussions, I (identified as HD in the transcript) had to remind them to talk with one another as in the following example.

Josué: I did 20 times 15; it equaled 100 [he is looking at work, which shows the first step in the algorithm, 5x20=100], and then uh, plus 20 [he’s referring to the second step in the algorithm, 1x20=20], a hundred plus twenty.

HD: OK, show what you did to Alfredo.

Josué: OK, 20 times 15.

HD: But why 20 times 15.

Josué: Because it says uh, the 20 gallons of paint, and then, uh, there, it says find 4 houses, but then how many much paint does he need to paint 15 houses. That’s not what we are talking about, but it said on there uh, and I, I was thinking that 20 times 15 equals 100.

HD: Let’s ask Alfredo, does this make sense to multiply 20 times 15?

Alfredo: Uh-Uh (shakes head no)

HD: Tell him why not.

Alfredo: Because if you multiply 20 times 15, it’s going to give you 100. And then right here you did 100 plus 20, 120, but it’s 300 because be-
cause you gotta put the 20 over here, in the tens place; the zero in the tens place and the 2 in the hundreds place.

As this episode shows, Josué and Alfredo did not talk to each other unless they were prompted to do so. As I asked Josué what he was doing, he noticed that the problem was about something that had nothing to do with his calculations (“...that’s not what we are talking about, but it said on there...”), yet he did not abandon his thinking (“...but I was thinking that 20 times 15 is 100”). When prompted to discuss their ideas with one another, their talk was not focused on sharing, negotiating, or making meaning together. Instead, they were concerned about executing procedures correctly, even when these procedures had nothing to do with and therefore were not going to help them solve the problem.

In contrast, students’ discussions when solving familiar experience problems began with a planful exploration of the situation described in the problem. During these meaning-oriented discussions, students considered tentative plans for solving the problem, possible implications of following a certain plan, and even reasonableness of these plans in light of the meaning of each situation. A common approach consisted of students providing information from their own experiences in order to make better sense of the mathematical tasks as presented. For example, a pair of students, Sasha and Luis, engaged in the following discussion when solving the familiar experience represented in the school breakfast task (see Table 1).

Sasha: Uh-huh, y yo andaba pensando también que hay doce en un cartón. [Uh-huh, and I was also thinking that there’s 12 in one carton.]

Luis: Es una docena. [It’s a dozen.]

Sasha: Y si abre, si abre 12 apenas van a ser 144. [And if she opens 12, it will only be 144.]

Luis: Doce cartones. [12 cartons.]

Sasha demonstrated familiarity with the most common size of egg cartons, twelve (in fact, other students noticed in their discussions of this problem that cartons come in 12, 24, and even 36 to a carton. In their conversations, they made reference to the grocery stores where they had seen these items). As Luis heard Sasha’s contribution of her own knowledge of sizes of egg cartons, he immediately refined it by calling the size of 12 a dozen. This initial exchange and refinement of ideas positioned these two students to mathematize a situation for which they had grasped its significance (Bruner, 1990). They both knew that opening 12 cartons meant making only 144 eggs. They were on their way of solving this problem in a personally meaningful and creative way. As
Mercer (1995) noted, “some of the most creative thinking takes place when people are talking together” (p. 4).

Finally, when students solved tasks that depicted familiar experiences in English, their mathematical discussions were less intersubjective. Although the ideas they mentioned in these discussions were sound, such ideas did not cause the same amount of discussion as when students were solving tasks in Spanish. Since I was present during these student-led discussions, hearing these ideas prompted me to encourage students to communicate their ideas to each other rather than to me. For example, when a pair of students, Cyndi and Sally, solved the familiar experience described in the reading homework task (see Table 2), I encouraged Cyndi to talk with Sally about how she was thinking about the invariance of the given ratio 5:20.

HD: Can you explain your thinking to Sally?

Cyndi: (turns to look at computer screen and points to problem on screen) Because uh, I add, I use, I know that 5x3 is 15 pages, 5x3 is 15, so I add, I multiplied 20x3, is 60 (turns to look at me first, then looks at Sally, who does not respond)

Using the given ratio of 5:20, Cindy began to scale it up by 3, possibly as a way to get closer to the given number of pages of 23. In her sound proportional reasoning, she can read 15 pages in 60 minutes. Her idea, however viable, was not part of a natural discussion. Even after my effort to situate this idea into a student-to-student discussion, the idea failed to provoke a reaction from Sally, the problem-solving partner. It is possible that the idea was not fully understood by Sally. However, not responding to a not so well understood idea is only one way of behaving in a discussion. The other way is obviously to respond in some way, as Amanda did in Spanish when she did not understand the provenance of 470 in Luis’ calculations. This varying level of intersubjectivity, along with the different ways in which bilingual students constructed social environments across their two languages and also across different kinds of mathematical tasks contributes to understand mathematical discussions as enacted from the students themselves.

Discussion

The review of research informing this paper suggests multiple challenges associated with classroom mathematical discussions. Rather than adding challenges to what we know about mathematical discussions, the present study considered the question of what teachers and researchers could learn about mathematical discussions when bilingual students
lead these discussions. I used a situated perspective on learning (Greeno, 1998) to understand the nature of these discussions. Findings suggest that how students talk as they solve mathematical tasks is situated within two kinds of experiences—familiar and unfamiliar—and within two languages—Spanish and English. More specifically, the amount of talk and the quality of talk reflected differences that are relevant for addressing some of the challenges found in the literature on mathematical discussions. For example, the finding that familiar contexts encouraged students to feel safe to take risks as they solved problems is relevant for teachers’ practices around mathematical discussions. Familiar contexts allowed students to recognize their own experiences (Dominguez, 2011; 2016). At the same time, these contexts promoted perseverance in exploring important mathematics, as suggested by the amount of talk observed. As Bruner (1990) suggests, in case of failure in these familiar situations, students could always go back to their grasp of the significance of situations and try a different approach. This finding suggests that in order to promote mathematical discussions, attention to what the mathematical tasks are about is as important as the mathematics that they address.

As for the quality of talk, when students solved problems that reflected familiar experiences, they constructed social environments in which they developed conjectures, negotiated meanings, and explored possibilities related to their mathematical work. In these social environments they became social risk-takers. Participating in the complex process of taking risks is a way of characterizing rich mathematical discussions. Students in this study took risks when solving problems as part of enacting systems of social support, resulting in larger and richer amounts of talk in their interactions.

Finally, bilingual students used their two languages quite differently during these mathematical discussions. In general, their talk as they solved mathematical tasks that reflected familiar experiences was more oriented toward risk-taking in both Spanish and English than in the other tasks. However, the Spanish tasks, particularly when they reflected familiar experiences, generated richer, longer, and more risk-taking conversations than similar tasks in English. A form of authenticating this finding is by looking at how these students were observed using language at home and in their mathematics classrooms. In their mathematics classrooms, these students talked primarily in English, and their talk was not characterized by social interaction but rather by short answers to the teacher, and a few questions to the teacher. Their learning was shaped and situated by larger systems that promoted these kinds of individual, isolated behaviors. Not only had they been transi-
tioned into English only classrooms that removed one of their languages from learning, but the remaining language, English in this case, was expected to be used privately, for their own thoughts, instead of socially and intersubjectively. At home, these same students practiced their bilingualism unrestrictedly, moving from room to room, participating in all kinds of activities. Their learning at home was situated quite differently from their learning at school. The strategy used in this study of creating mathematical tasks based on the students’ experiences and languages, was intended to push back into the school setting a small part of these students’ bilingual lives. This strategy was based on the conjecture that teachers of bilingual students can resituate the learning experiences of students. Resituationg students’ languages and related experiences can help us understand how these young learners conduct mathematical discussions and see related challenges from a different perspective.

References
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