

Young Children's Number Sense Development: Age Related Complexity across Cases of Three Children

Zuhal YILMAZ ^{a*}

^a Yeditepe University, Turkey

Received: 5 April 2017 / Revised: 22 May 2017 / Accepted: 30 May 2017


Abstract

Children start to develop number sense even well before they start the school. Developing number sense serves as an intermediate tool for learning conventional mathematics taught in schools. This number sense has three key areas: number knowledge, counting and arithmetic operations. As a result, the aim of this study was to examine aged related complexity of number sense development of young children's aged four, six and seven under two key areas: number knowledge and counting. Semi structured task based clinical interviews were employed to examine number sense development. Five different assessment tasks were employed with three children. Children's responses were analysed to identify their level of number sense understanding and difficulties with developing number sense. Findings were reported under two categories: first children's ability to understand number concept and their ability to accomplish number word sequences and second counting. Findings of the study indicated a significant age related complexity and improvement in both two aspects of number sense. Older children with more experience developed better number sense than the younger children.

Keywords: Number sense, Counting, Young children

Introduction

Number sense is perceived as one of the hardest concepts to define in mathematics (Case, 1998; McIntosh, Reys, Reys, Bana, & Farrell, 1997). This struggle has led to various definitions of number sense in the literature. Early definitions of number sense referred to the understanding of numbers and operations, and utilizing this understanding to solve complex problems (Burton, 1993; Reys, 1991). One widely referenced definition of number sense is "a person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful strategies for handling numbers and operations" (McIntosh, Reys, & Reys, 1993, p.3). Recent definitions of number sense refer to operating with quantities and number-word systems (Aunio et al., 2006), and having a robust thought about numbers and perceiving the meanings and relations among numbers

*  Correspondence details: Zuhal Yilmaz, Yeditepe University, Faculty of Education, Department of Elementary Mathematics Education Istanbul, Turkey. Phone: +90-216-578 0000 Ext. 1585. E-mail: zuhal.yilmaz@yeditepe.edu.tr

(Dyson, Jordan & Glutting, 2015; Schneider & Thompson, 2000), and “ability to quickly understand, approximate, and manipulate numerical quantities” (Dehaene, 2001, p. 16).

“Researchers note that number sense develops gradually, and varies as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms” (Howden, 1989; as cited in Singh, 2009, p. 2). Number sense includes the children’s skills related to counting, recognizing number patterns, comparing numbers, and estimating (Berch, 2005; Dyson et al., 2015; Jordan & Levine, 2009). As children work with numbers, they enhance these skills and deepen their understandings and thoughts about numbers. Furthermore, they represent and count numbers in different ways and develop perceptions about operations. Spontaneously, as they use operations and different solution strategies for operations, they continue to deepen their number sense.

National Council of Teachers of Mathematics (NCTM) (2000) stated that students’ understanding of numbers and their relations, representing numbers and understanding number systems are focus areas for grades preK through 2nd grades. Wright (2006) argues that learning number words and their sequence are the earliest aspects of number knowledge. For him, these aspects are prominent for establishing a robust basis for early arithmetic strategies. In early grades, children first develop basic counting strategies that serve a foundation for understanding number size and relationships. In these early years, counting is an essential component for numerical competency since counting enables children to extend their understanding beyond working merely with small numbers (Jordan et al. 2015; Baroody & Mix, 2006). Second, they start to understand place value and operations. According to Fuson, Grandau and Sugiyama (2001) in kindergarten children start to develop place value understanding built on base 10 representations. Yet, children acquire these competencies at different rates. For instance, one child can only count up to 10; on the other hand, another child can count up to 100 and shows his/her understanding of counting patterns. In another example, one child can count 15 objects yet cannot identify that the number 15 is composed of one ten and five ones. In comparison, another child can identify this equivalency and shows an understanding of place value.

Research has explored screening for children’s potential difficulties with mathematics (Das, Jordan, Glutting, Irvin & Dyson, 2014; Gersten, Jordan, & Flojo, 2005; Jordan et al., 2006). These studies have indicated the reasons why children face difficulties in learning mathematics in later grades; for example these difficulties might be due to students’ underdeveloped number sense in early elementary school grades and kindergarten. Early childhood years are such critical times for children to acquire number sense, including certain skills and concepts such as, one to one correspondence, recognizing, and writing numbers, counting, number operations, classifying and more / less comparison (National Research Council, 2009). Recent longitudinal studies indicated developing such number sense competencies in these early years is a strong predictor of mathematics achievement in 3rd grade especially in number computation and problem solving (Jordan, Glutting, Ramineni & Watkins, 2010; Jordan, Kaplan, Ramineni & Locuniak, 2009). These findings demonstrate that developing early number sense plays a critical role in laying the foundation for children’s future academic achievements in mathematics. Having enough opportunities and support to learn these skills in early years will allow children to build a strong background for learning advanced mathematics (Alajmi & Reys, 2007; Dyson et al., 2015). In addition to the influence of developing number sense on academic success in mathematics, children’s experiences with numbers in elementary school years have an important influence on their beliefs and attitudes toward mathematics (Van de Walle, 2004, Erdogan & Baran, 2008). If children have positive experiences in those early years they will be more likely to develop positive attitudes and beliefs towards mathematics.

Although most children acquire a certain level of number sense before they start kindergarten, individual differences exist among these children (Clements & Sarama, 2009; Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006). Young children progress at different paces when developing number sense, and individual differences can be observed during these years (Clements & Sarama, 2009). As a result, this study to seek answers to these research questions:

- 1) To what extent do children, at the ages of four, six, and seven years, show number sense in relation to number words, numerals, and counting?
- 2) Do children of these different ages exhibit different complexity in their number sense development?

Method

In this study, semi-structured task-based clinical interviews (Opper, 1977) were conducted separately with three children. Their ages were four, six, seven years old. Four-year-old child was a pre-kindergarten student, and the six-year old child was a kindergarten student; the seven-year-old child was a 1st grade student. All of them were in the first semester of the school term. Informed consents were taken from the parents.

Five assessment tasks were employed in the interviews. The first three tasks were utilized in clinical interviews during the first week and the rests were employed in the second week. In each week, the research met separately with each child and each task was employed on different days. Each interview lasted approximately 20-30 minutes except the four-year-old child. The researcher conducted the interviews with this child by splitting the interview into meaningful pieces.

The tasks served to assess children's number sense (e.g., recognition, order, patterns, cardinality, counting sequence), and counting strategies. Assessment of number sense involved not only determination of whether the children's answers are 'right' or 'wrong' but also the strategies children used, misconceptions children had, process skill difficulties children encountered and their ways of thinking about number sequence and counting. This analysis provided researcher a detailed understanding about the extent to which children used number sense, the reasoning behind their counting strategies and differences in their number sense level.

In this study, each interview was video-recorded, field notes were compiled and students' written work was collected. To ensure the validity and reliability of the study, data analysis was triangulated through repeated and shared viewing of the video-data and comparing and contrasting data collected through multiple data sources (Golafshani, 2003; Mathison, 1988). Mathison (1988) addressed the importance of triangulation:

Triangulation has risen [as] an important methodological issue in naturalistic and qualitative approaches to evaluation [to] control bias and establishes valid propositions because traditional scientific techniques are incompatible with this alternate epistemology. (p. 13)

The video data was analysed by utilizing analytical model (Powell, Francisco, & Maher, 2003). According to this model, each interview video data was viewed twice. In the first cycle, all the videos viewed carefully by paying attention the connection between students' work and related literature. In the second cycle, critical events were identified. These critical events included evidences of children's number sense levels. After the critical events determined, the generation of the categories for the data process was started. This determination of the categorization was one of the important steps in the data analysis since this facilitated the meaningful interpretation the data. This categorization emerged through reviewing literature, identifying critical events that were representing related

concepts in the literature and discussing these critical events and categorization with the other researchers. Clinical interviews were analyzed by focusing on two main categorizations that included two essential number sense constructs and related sub-constructs. Table 1 shows the coding schema and the description of each category.

Table 1. *Coding schema of the study*

Main Category	Sub Category	Description
Number Word Sequences and Numerals	Number Word Sequences	Forward Number Word Sequence (FNWS) and Backward Number Word Sequence (BNWS): number words in the context of a sequence of words (Wright et al., 2006). Producing number words before and after (Wright et al., 2006)
	Number Relations	Ordering numbers Cardinality: Ability of naming whole set (Anghileri, 2000).
	Number Recognition	Naming, recognizing, and writing numbers.
	Counter Type	Perceptual, figurative, and emergent counter (Wright et al., 2006). An emergent counter can not match each number word with one object that is counted (Wright et al., 2006). A perceptual counter can count all objects only given collections are visible (Wright et al., 2006) A figurative counter can use count-all to figure out how many counters in all, when the collections are screened (not available to be seen)" (Wright et al., 2006, p.49).
Counting	Counting Strategies	Skip counting, counting by ones and counting on (Wright et al., 2006).

Students' written works and the field note of the researcher were utilized as a supportive evidence for findings.

The Tasks

In this study, five interview tasks were selected and adapted from Wright et al., (2006). These tasks were adapted according criteria Smith and Stein's (1998) high demand task description and each student's prior knowledge and grade level.

The first three tasks were utilized to assess children's understanding of number words, numeral sequences and numeral quantities. In task one, children were asked to recite a count sequence. The term 'counting' is not same as a label for a child saying a forward number sequences. This task assessed an initial understanding of counting. They presented with a group of objects (i.e. candies) and asked for counting the number of the objects in each group. After that children were asked to produce Forward Number Word Sequence (FNWS) and Backward Number Word Sequence (BNWS). This task aimed to understand in what range different aged children could produce number sequence. Also, this helped to assess whether there is a difference between the difficulty level of producing FNWS and BNWS of different aged children.

Task two focused on examining whether children could understand of relative position of numbers or not. In addition, this task were utilized to assess whether children can go beyond recalling number words in sequence and able to compare two different numbers or quantities. They asked, "What number come one after or before another number". This type of questions in the interview aimed to understand whether children could produce a unique successor of each number (Le Corre & Carey, 2007; Sarnecka & Carey, 2008). The second part of the task two aimed to determine whether children could make "numerical magnitude judgment" (Jordan, Glutting & Ramineni, 2010). They presented with a set of number cards and asked to order them or asked to identify the biggest or smallest number.

Task three was aimed to assess whether children recognized the presented numerals. They presented with a series of number cards and asked to recognize the stated number. For instance, which card does represent "15"?

The last two tasks were focused on counting with understanding and utilized to assess children's counting strategies and determine which type of counter they are. In these tasks, children presented with a group of unscreened objects and then screened objects. They asked to find the number of the objects in total when the objects under cover were exposed to additive changes. The ways that children employed to find the number of objects in the group perceived as an evidence for what sorts of counting strategies that different aged children might use.

Results

This section reports the findings from the qualitative analysis of video data regarding to research questions. The first part will report the findings of the first and second research questions viewing the data from the perspective of Number Word Sequences and Numerals. The second part will report on the finding from the analysis of the children's understanding about counting and related concepts.

Number Word Sequences and Numerals

The three assessment tasks were mainly used to assess children's early number sense and to investigate the relation between development of number sense and age factor. The first task was employed to understand students' facility with number word sequences. The four-year-old child had difficulty with FNWS since she could not produce the number words in a correct order between the numbers five to ten. As Chao (2000) states that "The acquisition of a number-word sequence continues long after the child first is able to produce the number words correctly" (p.291). In contrast to the four-year-old child, the six and seven year old children could fluently produce FNWS.

Analysis showed that children's facility with FNWS and BNWS varied across ages. While the four-year-old child was able to produce FNWS from one to five but could not produce BNWS (from five to one). The fluency change in producing FNSW and BNSW also deduced from six year old child's responses. This child could produce FNWS between 'one' to 'forty' but could not produce BNWS 'twenty' to 'one'. The conversation took place as follows:

I: How high can you count to?

Six-year-old child: [Starts from 1 and counted up to 40]

I: Can you count back to 1 from 20?

Child: [paused and counted 1 to 19 silently] nineteen [paused again and counted with finger up to 18] eighteen.

I: Can you explain how you said 19 then 18?

Child: I counted first to 20 in my head and 19 is the before the 20.

This interaction above indicates this child could not produce BNWS; he needed to utilize FNWS to find the number comes before the desired number.

After assessing children FNWS and BNWS skills, two tasks were employed to assess children's facility about saying number word after and number word before. In these tasks, six and seven years old children could produce number word after and word before based on their fluency in FNWS and BNWS. The difference between six-year-old and seven year old children was, six-year-old child used counting by one strategy to state number word after whereas seven-year-old child used counting on strategy. For instance, in the interviews both children were asked:

I: Would you please tell me which number comes after 12?

Six year old child: one, two, three ... twelve [a little pause] thirteen. Thirteen comes.

Seven-year-old child: twelve, thirteen

This conversation above indicated that six-year-old child utilized counting by ones and stated thirteen as a response. On the other hand, seven-year-old child directly started from number 12 and counting on from 12 and said 13 comes after 12. Four-year-old child could not produce the number word after even within the number range one to ten.

Producing number words before was a harder task than producing number words after for the children participated in the study. One of the possible reasons for that even producing BNWS is harder than producing FNWS. In the interviews only seven-year-old children could produce number words before. He used counting by ones strategy, as the numbers got larger. An example for this situation was:

I: Would you please tell me which number comes before 15?

Seven-year-old child: 14

....

I: Would please tell me which number comes before 118?

Seven year old child: [verbally counted by ones starting from hundred] 100, 101, 102... 110, [child confused and paused]

I: Which number comes after 10?

Seven-year-old child: 11

I: So what do you think, which number can come after 110?

Seven-year-old child: yeah 111.

The conversation above shows that the child turn back to counting on and counting by ones strategies to produce number words before for the number 118. One possible reason for this change in his proficiency in producing number words before could be that he might not fully recognize number patterns for the large numbers.

The six-year-old child produce correct answers within number range that he could count by ones. So he could turn back found the numbers before the last number he stated. For instance, he was asked:

I: Would you please tell me which number comes before 27?

Six-year-old child: [He counted until 27] and number before is 26.

The four-year-old child could not produce correct answer any of the tasks related to producing number words before.

Learning about numerals is (recognizing, ordering numbers, and cardinality concept) an important aspect of early number sense as learning about number words (Wright et al, 2006). In the interview, children were presented with a set of number cards and asked, "Would you please show me the number ...?". Four-year-old child only recognized the numbers between one and five. On the other hand, six and seven year old children were presented with a selected set of numbers one to hundred. Although the six and seven year old children typically recognized the numbers correctly, they sometimes confused some number words especially the numbers contains same numerals as '12' and '21'. Six year old child was having difficulty with recognizing the numbers greater than "40". This might be due to fact that this child could produce FNWS up to 40.

In order to assess children's understanding about numeral sequence, a task was used in which five cards randomly arranged on the table and said to child ' can you put these numbers in an order please, starting from smallest?'. The four-year-old child could not order the number cards that included numbers 1 to 5. Yet when this child presented with two collection each included a number of objects smaller than 5. The four years old child could determine which group is large one. The six-year-old child could order the number. Yet, sometimes this child kept track of the numbers through counting silently to ensure that he ordered the numbers correctly. The seven-year-old child could order the numbers.

After this task, the researcher examined whether the children recognized that the last number represent the whole collection. The six-year-old child related the sequencing job with counting without any understanding of cardinality. When, interviewee asked, "how do you know two is smaller than four?" the child said 'when I say it, it comes first.' This response pointed out, this child had only an understanding of number sequence. Also the child might have the understanding of the number after was greater than the number before. The child did not understand the idea of "2" represents the whole collection of one and two. On the other hand, the seven-year-old child could answer the same "why question" as, four includes more number. Then, interviewee asked 'Can you give any example from daily life that four greater than two?' The child said 'yes, four balls are more than two balls.' This showed, this child had an initial understanding of cardinality. Thus it could be said; different age level students have different complexity in their cardinality principle understanding.

As a result, these findings laid some evidences that some children did not develop a complete understanding about numerals and there are certainly individual differences exist among the children who participated in this study. Moreover, these findings support the claim, the levels of understanding about number word sequences and numerals improves increasingly with age and experience, and moreover the fluency with producing number words varies. As Wright et al. (2006) suggested children initially encounter numbers words in the conversations with simple quantities then they could produce number word sequence. Even four-year-old child could produce number words one to five. After that, children tend to engage more to forward number word sequence than backward number word sequence. Six-year-old child had difficulty with producing BNWS rather than FNWS. The four-year-old child could not produce BNWS five to one. Next, the children will be able to say number word after and number word before. This similar pattern also observed in the interviews in this study. Older children with more experience with numbers could produce number words and sequences easier than the younger children.

Counting

Although, each task analysis in the interviews included some evidence from children responses related to counting strategies of children, two tasks were specifically employed to capture counter type of children and what kind of counting strategies they used while they were dealing with the tasks. One of the tasks especially helped to clarify the distinction between the counter types since it included screened collections cases. During the interviews, children were asked variations of the following question: "There are five cookies under this cup. If you have six more cookies how many cookies will you have in total?" The actual scenario of the task was modelled by using actual objects or manipulative materials. The findings related counter types and counting strategies of children examined as follows.

Siegler and Shrager (1984) proposed that children might use counting on fingers strategy while they are dealing with the given tasks. In this strategy, children use their fingers to model the given task and to count on. Seven-year-old children used this strategy. He used his fingers to represent the cookies under the cup and started to count on the last number of cookies under the cup as five cookies, six, seven, eight, and eleven. Then he gave the answer: "I will have 11 cookies". As he counted each cookie he used his finger to represent one cookie. This behaviour indicated that this child used counting on strategy as he counted and understood one to one correspondence. Moreover, children sometimes used different counting strategies for different number ranges that were in the tasks' content. For instance, same child mostly used counting on strategy when he worked with smaller numbers in the tasks. On the other hand, he preferred to use counting by ones strategies when he worked on a task includes larger numbers.

In this study, the six-year-old child counted from one to find total number of cookies, he started to count the cookies which were always available to be seen as, "one, two, three, four, and five" then he added more cookies by ones and kept counting at the same time. Then he stated his final answer as: "There are 11 cookies". This evidence also pointed out that, he was a counting by ones child. Since, he started from ones to find the whole collection in each time. Also, he counted cookies by starting from one and keeps track with his fingers and the answers were on his fingers. All these findings support the conclusion that he was a figurative counter and used counting by ones strategy to get the final answer to the tasks.

A perceptual counter can count all objects only given collections are visible (Wright et al., 2006). Four-year-old child could not find how many cookies in all. Since the whole collection of cookies did not visible to her. She could only count the visible cookies if the number of visible cookies less than six. For instance, she was asked, "your mom gives you two cookies and they are under this cup. And then your father gives you three more cookies. How many cookies you have in total?". She could only counted three cookies by using cookies that were visible to her. She could not count the cookies under the cup and gave the answer "three". This evidence indicated that she was a perceptual counter. Also, further analysis of this child response to the next problem showed that she had difficulty to count up to large numbers as it was indicated above. She could produce number words and correspond each number with an object up to a certain interval (1-5). Then she confused the order of numbers, counted some of the object twice or assigned the same number to more than one object at the same time. All these indicated that, this child did not actually develop complete counting ability and one to one correspondence. This evidence indicated she could be in the transition to emergent counter who could not match each number word with one object that is counted (Wright, 2006).

Evidences from this study in the favour of the following claims: First, levels of understanding number concepts, counting abilities and the complexity of counting strategies developed increasingly with age. Seven-year-old child mostly used counting on strategy and he was a counting on child. Six-year-old child mostly used counting by one strategy and he was a figurative counter. Finally, four-year-old was an emergent counter. Actually she did not fully develop complete counting skill yet.

Discussion and Conclusion

The findings of this study supports children's abilities in using number sense shows a successive developmental process (Gelman & Gallistel, 1978). Chao (2000) states these processes as, 1) perceiving forward number sequence in like an alphabetical order, 2) producing number words bidirectionally as they gain proficiency with number fluency (see Fuson, Richards, & Briars, 1982, for details). In this study, while four-year-old child did not develop full fluency in both forward and backward directions, six-year-old child produce number words bidirectionally within a certain number range and seven-year-old child can produce number words in both directions. This finding of the study also parallel with the findings of prior studies that indicate the learning process of number sequence extended from age four to seven or eight (Fuson, 1988; Jordan, Glutting, & Ramineni, 2010). The findings of this study also shows, the children in this study ages of the six and seven-year-old could produce number words and forward number sequence easier than the four-year-old child.

In this study, although the four-year-old child can count up to five, she could not understand the number relations such as the recognizing and ordering numbers, cardinality. This shows younger children might not actually attach the meaning of the numbers; they could just produce the sequence of the numbers. Assessing children's level of understanding about numbers become critical for designing instruction that helps children to develop a better understanding of numbers and their relations. NCTM (1989) argues that to understand the number in real life, children should internalize the number meanings. Cardinality is a hard issue to develop in early ages (Wright et al., 2006) as it is observed in the actions of four and even six-year-old children in this study. One of the possible suggestions for facilitating learning of cardinality, teachers should address usage of concrete materials to model situations that involves numbers.

Many studies have found that difficulties with mathematics became widespread. This brings many serious consequences, which can be last long to the adulthood (Dougherty, 2003; Murnane, Willett, & Levy, 1995). One of main reasons for these difficulties and serious consequences is weakness in young children's number sense development (Jordan & Levine, 2009). Thus, task-based interviews or activities carried out with children have potential to determine different aged young children's number sense level. This determination of initial number sense level has a key role in designing educational setting in which child can develop a better number sense. This evidence base diagnosis is crucial to understand children's needs before orchestrating teaching and learning environment. If teachers understand the children's understanding level of number sense and what is the possible tracks that similar and different age groups can progress, they can give smart instructional decisions that have potential to prevent serious learning difficulties in mathematics in upper grades.

At last, one possible future study suggestion is to conduct an intervention study to promote number sense development of young children. Dyson et al. (2015) suggested especially kindergarten is a critical times to design educational interventions. Appropriate interventions along with meaningful assessment of the number sense and mathematical knowledge in early grades will yield robust understanding of students' number sense

competency and will contribute students' achievement in mathematics (Gersten, Jordan, & Flojo, 2005).



Acknowledgement

Thanks for the supporting contribution of Selçuk Doğan in the process of editing and revising the paper.

References

- Alajmi, A., & Reys, R. (2007). Reasonable and reasonable- ness of answers: Kuwaiti middle school teachers' perspectives. *Educational Studies in Mathematics*, 65(5), 77-94.
- Anghileri, J. (2000). Teaching number sense. London: Continuum.
- Aunio, P., Hautamäki, J., Heiskari, P. & Van Luit, J. E. H. (2006). The early numeracy test in Finnish: Children's norms. *Scandinavian Journal of Psychology*, 47, 369-378.
- Baroody, A. J., Lai, M. L., & Mix, K. S. (2006). The development of young children's early number and operation sense and its implications for early childhood education. *Handbook of Research on the Education of Young Children*, 2, 187-221.
- Berch, D. B. (2005). Making sense of number sense: Implications for children with mathematical disabilities. *Journal of Learning Disabilities*, 38(4), 333-339.
- Beswick, K., Muir, T., & McIntosh, A. (2004). Developing an instrument to assess the number sense of young children. International Education Research Conference (pp. 1324-9339). Melbourne, Australia: AARE.
- Burton, G. M. (1993). *Number sense and operations: Addenda series, grades K-6*. Reston, VA: National Council of Teachers of Mathematics.
- Case, R. (1998, April). A psychological model of number sense and its development. Paper presented at the annual meeting of the American Educational Research Association, San Diego, CA.
- Chao, S. J., Stigler, J. W., & Woodward, J. A. (2000). The effects of physical materials on kindergartners' learning of number concepts. *Cognition and Instruction*, 18(3), 285-316.
- Dehaene, S. (2001). Précis of "The number sense". *Mind and Language*, 16, 16-32.
- Dougherty, C. (2003). Numeracy, literacy and earnings: Evidence from the National Longitudinal Survey of Youth. *Economics of Education Review*, 22, 511-521.
- Erdogan, S., & Baran, G. (2008). A study on the effect of mathematics teaching provided through drama on the mathematics ability of six-year- old children. *Eurasia Journal of Mathematics, Science & Technology Education*, 5(1), 79-85.
- Fuson, K. C. (1988). *Children's counting and concepts of number*. New York: Springer-Verlag.
- Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number*. Cambridge, MA: Harvard University Press.
- Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities*, 38, 293-304.
- Golafshani, N. (2003). Understanding reliability and validity in qualitative research. The qualitative report, 8(4), 597-606.
- Howden, H. (1989). Teaching Number Sense. *Arithmetic Teacher* 36(6), 6-11.

- Jordan, N. C., Glutting, J., & Ramineni, C. (2010). The importance of number sense to mathematics achievement in first and third grades. *Learning and individual differences, 20*(2), 82-88.
- Jordan, N. C., Kaplan, D., Locuniak, N. M., & Ramineni, C. (2007). Predicting first-grade math achievement from developmental number sense trajectories. *Learning Disabilities Research & Practice, 22*(1), 36-46
- Jordan, N. C., Kaplan, D., Olah, L., & Locuniak, M. N. (2006). Number sense growth in kindergarten: A longitudinal investigation of children at risk for mathematics difficulties. *Child Development, 77*, 153- 175.
- Jordan, N. C., & Levine, S. C. (2009). Socioeconomic variation, number competence, and mathematics learning difficulties in young children. *Developmental Disabilities Research Reviews, 15*(1), 60-68.
- Kim, K. M. & Noh, S. (2010). Alternative mathematics assessment: A case study of the development of descriptive problems for elementary school in Korea. *Eurasia Journal of Mathematics, Science & Technology Education, 6*(3), 173-186
- Klibanoff, R. S., Levine, S. C., Huttenlocher, J., Vasilyeva, M., & Hedges, L. V. (2006). Preschool children's mathematical knowledge: The effect of teacher "Math Talk". *Developmental Psychology, 42*(1), 59-69.
- Mathison, S. (1988). Why triangulate? *Educational Researcher, 17*(2), 13-17.
- McIntosh, A. J., Reys, B. J., & Reys, R. E. (1993). A proposed framework for examining number sense. *For The Learning of Mathematics, 12*(3), 2-8.
- McIntosh, A., Reys, B., Reys, R., Bana, J., & Farrell, B. (1997). Number sense in school mathematics: Student performance in four countries. Perth: MASTEC, Edith Cowan University.
- Murnane, R. J., Willett, J. B., & Levy, F. (1995). The growing importance of cognitive skills in wage determination. *The Review of Economics and Statistics, 77*, 251-266.
- National Council of Teachers of Mathematics (1989). *New directions for elementary School Mathematics*. Reston, Va.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- Opper, S. (1977). Piaget's clinical method. *Journal of Children's Mathematical Behavior, 5*, 90-107.
- Reys, B. J. (1991). *Developing number sense: Addenda series, grades 5-8*. Reston, VA: National Council of Teachers of Mathematics.
- Schneider, B., & Thompson, S. (2000). Incredible equations develop incredible number sense. *Teaching Children Mathematics, 7*(3), 146-148.
- Siegler, R. S., & Shrager, J. (1984). Strategy choices in addition and subtraction: How do children know what to do? In C. Sophian (Ed.), *The origins of cognitive skills* (pp. 229-293). Hillsdale, NJ: Erlbaum.
- Singh, P. (2009). An Assessment of Number Sense among Secondary School Students. *International Journal for Mathematics Teaching and Learning, 1*-27.
- Stein, M. K. & Smith, M. S. (1998). Mathematical task as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School, 3*(4), 268-275.
- Van de Walle, J. (2004). *Elementary and middle school mathematics: Teaching developmentally* (5e). Boston: Pearson Education.
- Wright, R. J., Stanger, G., Stafford, A.K., & Martland, J. (2006). *Teaching number in the classroom with 4-8 years-olds*. Sage Publications Ltd, p. 30-52.

www.iejee.com

This page is intentionally left blank