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Learning to be a Math Teacher: What Knowledge is Essential?

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Abstract

This study critically examined the math content knowledge (MCK) of teacher candidates (TCs) enrolled in a two-year Master of Teaching (MT) degree. Teachers require a solid math knowledge base in order to support students' achievement. Provincial and international math assessments have been of major concern in Ontario, Canada, due to declining scores. Research aimed to investigate the development of TCs' math capacities for effective teaching is important to teachers, school districts, universities, professional learning associations, and policy makers. The researchers of this study analyzed the basic numeracy skills of 151 TCs through pre- and post-tests. In addition, eight TCs took part in semi-structured interviews and shared their experiences in the MT math program. Test results indicated improvements in many areas, however, not all numeracy skills improved significantly. Interviews revealed TCs' perceptions of the math test, courses, and instructors, as well as the importance of teaching math during their practicum placements. The researchers made recommendations to teacher education programs in areas such as: establishing minimum math competency standards, enhancing coherence between MT math courses and practicum placements, and providing additional support for TCs with low math proficiency.

Keywords: Elementary math teacher education, Teacher candidates, Pre-service teacher education, Math content knowledge, MCK, Teacher learning

Introduction

Large-scale math assessment results have become a matter of great concern in Ontario, Canada, due to declining scores over the years. In Ontario, provincial assessment scores have revealed a steady decrease in grades 3 and 6 students' math achievement over the previous eight years. During the period between 2009 and 2016, the percentage of grade 6 students who achieved at or above the provincial standard in math declined by 13 percentage points (63% to 50%); while the percentage of grade 3 students at or above the provincial level dropped by seven points (70% to 63%) (Education, Quality and Accountability Office [EQAO], 2016a). Additionally, the latest math results from the 2015 Program for International Student Assessment (PISA) show Ontario scored statistically lower than Canada as a jurisdiction, as well as the province of Quebec (EQAO, 2016b). PISA assesses the achievement of 15-year-old students; in Ontario, this involves students

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in grade 10. When comparing Canada versus the top scoring Singapore, the math results are telling. In Canada, 85.6% students were able to at least employ basic algorithms (level 2 questions), and in Singapore, 92.3% could do the same. As questions became more complex on the PISA assessment, the gap between Canada and Singapore increased. For example, when students were asked questions that required reasoning skills and the ability to communicate their reasoning (level 5 questions), only 15.1% of Canadian students could achieve this level, however, 34.8% of Singapore students could achieve success. These recent findings reveal a current situation of major concern in math performance across Canada and the province of Ontario.

Ontario Ministry of Education documents and provincial targets include the concepts of raising student achievement for all students and closing student achievement gaps. Using district and provincial data, ambitious targets have been proposed for improved numeracy outcomes for elementary students. As part of setting the stage for a renewed vision for Ontario's drive to achieve excellence in education, the ministry acknowledges the challenges observed in the area of math achievement … "Like many other jurisdictions across Canada and around the world, Ontario has also seen a decline in student performance in mathematics." (Ontario Ministry of Education, 2014, p. 2). The ambitious provincial goal of 75% of students achieving levels three or four on the EQAO is stated clearly in its vision, level three being the provincial standard and level four being the highest level of attainment. In 2016-17, the ministry commenced a renewed math strategy to provide differentiated levels of supports to schools based on student achievement data. For example, this involved professional learning opportunities for educators, as well as math lead teachers in elementary schools. The ministry has also implemented a monitoring approach to identify impact across the province and will share results in the years to come.

The Ontario education system starts with two years of full day kindergarten that immerses students aged four and five in an inquiry and play-based program. Kindergarten educators are encouraged to create learning environments that entice students to build upon their natural curiosities in math through the presentation of intentional provocations. Students then continue with their elementary education through grades 1 – 8, approximately ages 6 – 13. When students enter secondary school in grade 9, they must choose between applied and academic math curriculums. Historically, there has been a tremendous gap in achievement on the grade 9 EQAO math assessment, between students in applied and academic courses. Most recently in 2016, only 45% of students in applied math courses achieved the provincial standard, whereas 83% of those in academic met the same standard. The grade 9 EQAO math assessment is considered low stakes, that is, passing the assessment is not a requirement to graduate secondary school. The only requirement for secondary students in the area of math involves passing three math courses, at least one of the courses must be at the grade 11 or 12 level; grade 12 being the final year of secondary school. Nevertheless, the EQAO scores are vital pieces of data, as the choice between applied and academic math courses can affect future educational and career opportunities (Parekh, 2014).

Initial teacher training in Ontario recently transformed from a two-semester program to a four-semester program in September 2015; a semester involving four months of study. This study took place in a graduate teacher education program in which TCs earn a Master of Teaching degree as well as certification to teach in Ontario. Because the Master of Teaching program is a graduate degree, it involves five semesters (20 months in total), which goes beyond the minimum undergraduate teacher training program. In Ontario, teacher candidates must participate in a minimum of 80 days of practicum teaching; observing a qualified teacher in the classroom and teaching in the classroom under the supervision of a qualified teacher. Teacher education program providers are accredited by the Ontario College of Teachers to offer courses that lead to Ontario teaching certification. TCs earn qualification in one of three divisions: primary/junior (P/J), kindergarten – grade 6; junior/intermediate (J/I), grades 4 – 10; and intermediate/senior (I/S), grades 7 – 12. Further, to be qualified to teach J/I, a teacher must have one teachable such as English, math, science, or social science/humanities. This requires the TC to have taken four or more full undergraduate or graduate courses in the area of the teachable. For the I/S certification, the teacher must have two teachable subjects; the first teachable must be supported by five or more full undergraduate or graduate courses and the second teachable must be supported by three or more undergraduate or graduate courses. As this current study focused on P/J and J/I, most of the TCs did not have a background in math; instead, most had an undergraduate focus in the social sciences.

The basic math competencies of teacher candidates have received much attention in the last few decades. Math content knowledge (MCK) refers to the basic math knowledge possessed by individuals considered to be mathematically literate. Researchers emphasize that content knowledge in math is an important construct that can either support or hinder progress toward exemplary classroom instruction (Philipp et al., 2007; Thames & Ball, 2010). Ball, Thames, and Phelps (2008) suggest that the absence of improved math instruction is resultant from teachers' lack of content knowledge within this subject area. "Teachers who do not themselves know a subject well are not likely to have the knowledge they need to help students learn this content" (p. 404). Unfortunately, a report conducted by Amgen Canada Incorporated and Let's Talk Science (2013) indicated that more than 50% of Canadian high school students drop science and math as soon as they can, thereby only taking the minimal compulsory courses to grade 10 or 11. This underscores the need for teacher education math programs to foster TCs' math competencies so they are better equipped to meet their students' needs through effective math pedagogy. The researchers of this current study readily note that more than MCK is required to successfully teach math in the elementary grades, hence, this research also examined the perceptions of TCs' self-efficacy in teaching math in the classroom. Importantly, self-efficacy has been identified as a predictive factor in student achievement in math studies (Tella, 2008)

The National Council of Teachers of Mathematics (NCTM) established standards to engage math teachers in reform, that is, learning math for deep understanding through meaningful problem solving contexts (1989, 2000). More recently, NCTM published Principles to Action (2014) outlining the guiding principles for school math. Ontario's 2005 math curriculum is aligned with the NCTM's standards (McDougall, Ross, & Jaafar, 2006). The math curriculum highlights the importance of a balanced pedagogy in which operational skills are not in any way ignored, and still an important component of the curriculum. Further, it is acknowledged that operational and higher-order thinking skills are attained differently by students as they develop conceptualizations of math around them. NCTM's standards are often referred to as reform math in comparison to traditional math instruction which relies heavily on rote learning and memorization. A major tenet of reform math is the notion of constructivism, in which knowledge is actively created by the learner. In a constructivist class, teachers guide and support their students' math ideas rather than transmit procedural knowledge. This approach to math instruction is complex and it requires teachers to have a deep knowledge of the subject area in order to pose appropriate tasks, explain models, and ask effective questions. However, a number of researchers have pointed out that teachers teach much in the same way they were taught (Ball, Lubienski, & Mewborn, 2001; Kagan, 1992; Tabachnik & Zeichner, 1984). Tabachnik and Zeichner (1984) assert that constructivist modes of teaching tend to conflict with TCs' previous ideas about good teaching, and that they are inclined to maintain old conceptions. Specifically, in the discipline of math there is research that suggests TCs enter teacher education programs with predetermined ideas on how to teach math based on the way they were instructed (Ball, Sleep, Boerst, & Bass, 2009; Hill & Ball, 2004, 2009). Overall findings from these studies reveal how prospective teachers are more likely to replicate teaching approaches that were modeled to them as students of math. Moreover, Hill and Ball (2004) state that these approaches derive from years of personal experiences of traditional math teaching, where the teacher is holder of all knowledge, with an emphasis on memorization of facts and procedures. It appears that math education is caught in a vicious cycle. Prospective teachers tend to hold oversimplified beliefs about classroom practice and pre-existing ideas of how to teach math based on their experiences in traditional math classrooms (Ball, 1996; Ball, Lubienski, & Mewborn, 2001). In order to break this cycle, teacher education math courses must give meaning to the content and pedagogy TCs need to know (Thames & Ball, 2010). This requires considerable math content knowledge and a wide range of pedagogical skills to implement math programs that promote authentic problem solving, reasoning, and communication.

In this article, researchers examined a two-year Master of Teaching (MT) degree program that promotes varied instructional strategies beyond a transmissive approach to teaching math. This study focused on the following questions: What were the basic numeracy skills of TCs upon entering the MT program? What changes in basic numeracy skills of TCs occurred after a year of the MT program? And what changes occurred in TCs' math beliefs and confidence as learners and teachers after completing two MT math courses and three practice teaching placements? Through pre- and post-tests of basic math skills, as well as semi-structured interviews, the researchers analyzed the growth of TCs in their MCK and their conceptions and confidence in teaching math in the classroom. This research identified key findings to support the improvement of teacher education programs in meeting the needs of TCs' math development and teaching capacities.

Literature Review

A number of research studies have raised serious concerns about the depth of math content knowledge (MCK) in teacher candidates (TCs) enrolled in elementary teacher education programs (Ball, 1990a; Grover & Connor, 2000; Hill & Ball, 2004, 2009; Ma, 1999; Philipp et al., 2007; Thames & Ball, 2010). In general, the literature pertaining to MCK of TCs overwhelmingly supports the need for conceptual understanding of the subject matter, and specialized math knowledge for teaching in order to implement effective teaching strategies.

Math Content Knowledge (MCK) of Teachers Candidates (TCs)

The math content knowledge (MCK) of teacher candidates (TCs) continues to be identified as an important component of effective math teaching in the classroom. Although the possession of strong content knowledge in of itself is not enough to ensure a person will be an effective math teacher, it is difficult to help students to acquire deep math understandings when the teacher has inadequate content knowledge (Ponte & Chapman, 2008). Philipp et al. (2007) and Thames and Ball (2010) strongly suggest it is necessary for teachers to possess conceptual math knowledge in order to effectively explain algorithms and concepts, as well as describe connections between concepts. Number sense and numeration in the Ontario curriculum is an essential foundation of MCK to successfully build sophisticated math conceptualizations (Biddlecomb & Carr, 2011). Unfortunately, teachers of math can be deficient in understanding numeracy concepts, specifically in understanding how to develop numeracy skills in their students beyond rote memorization (Yackel, Underwood, & Elias 2007).

In her studies, Ball (1990a, 1990b) examined math conceptual content knowledge through responses to questionnaires and interviews by 252 prospective teachers. Findings revealed that the subject knowledge held by prospective teachers remains inadequate for teaching math successfully. These findings are congruent with those of other studies. For example, Tirosh (2000) demonstrated how elementary TCs were overly dependent on computational algorithms for multiplication and division structures, resulting in procedural dependency and limited conceptual guidance. Another study by Bartell, Webel, Bowen, and Dyson (2013) concluded that MCK is necessary but insufficient in supporting the assessment of children's conceptual understanding of math. More specialized understanding of math is required to understand the complexities behind children's mathematical thinking.

Procedural and Conceptual Knowledge of Math Content

Both Hiebert (1992) and McCormick (1997) describe procedural knowledge as applying a sequence of actions to find answers. These actions, also known as algorithms, follow a set of rules that students repeatedly practise to reinforce the algorithm. The National Council of Teachers of Mathematics (NCTM) (2000) defines conceptual knowledge as a rich understanding of the relationships among math concepts. This involves solving problems through reasoning, communicating, and justifying. Merely memorizing computational procedures without understanding them will not develop the capacity to reason about the type of calculations needed. Thus, procedural skills that are not accompanied by some form of conceptual understanding are weak and easily forgotten (Hiebert et al., 2003). A major aspect of the NCTM's (2000) standards calls for a balance between conceptual and procedural knowledge of math. Unfortunately, without this balance in place, students often do not know when to implement procedures and the learning is often "fragile" (p. 20).

Research reports that teachers with weak math competencies cannot be flexible with their math instruction and this may result with an emphasis on procedural knowledge, in which teachers deliver curriculum in a repetitive, undemanding, and non-interactive fashion (Frykholm, 1999; Hiebert et al., 2003; Stigler & Hiebert 1997; Thames & Ball, 2010). Little attention is given to the development of conceptual ideas or making connections between procedures and mathematical concepts. Furthermore, the prominence of procedural knowledge in schools is most likely the type of math preparation experienced by teacher candidates (TCs) (Hill & Ball, 2004, 2009; Kajander, 2010; Thames & Ball, 2010). Consequently, the challenge for teacher education programs is to unpack the math knowledge of prospective teachers in order to develop deeper conceptual understanding (Adler & Davis, 2006).

The intersection of procedural and conceptual knowledge is of utmost importance when examining the content knowledge of math teachers (Ambrose, 2004; Hiebert, 1999; Hill & Ball, 2004, 2009; Rittle-Johnson & Kroedinger, 2002). It is imperative that math teacher education programs emphasize procedural skills and conceptual understanding as interconnected, so students have the capacity to understand why and how algorithms work and thereby grasp the underlying mathematical concepts (Ambrose, 2004; Kajander, 2010; Reid, 2013). Research has demonstrated that if students repeatedly practice algorithms before understanding them, they often struggle with making sense of why and how the formula works (Hiebert et al., 2005). Conversely, when conceptual understanding is the sole focus of instruction, then learners are likely to struggle with procedural competency (Kajander, 2010). Without any emphasis on computational algorithms, procedural knowledge can be negatively impacted (Alsup & Sprigler, 2003). Instructional practice that over emphasizes only one component, either conceptual or procedural, will result in limited math understanding. Some researchers argue that teaching algorithms and conceptual understanding should not be viewed as dichotomous extremes. Rather, procedural skills and problem solving skills are intertwined (Hiebert, 2013; Wu, 1999). Ultimately, math teachers will require a deep understanding of both conceptual and procedural knowledge in order to support a balanced approach in the classroom.

Math Content Knowledge (MCK) and Pedagogical Instruction

Grover and Connor (2000) argue for content knowledge as a key characteristic of effective pedagogical instruction, and this should be a central focus in teacher education math courses. During their study of teacher education courses, they found that a critical aspect of reaching course objectives is to recognize the important interaction between teaching and subject content knowledge. The authors discussed the need for teacher candidates (TCs) to not only understand math, but to understand the concepts in ways that will support effective instruction and assessment of the discipline. These claims suggest that math content knowledge (MCK) is directly connected to pedagogical styles of teaching. Correspondingly, other researchers such as Hill and Ball (2004), Ma (1999), Shulman (1987), and Thames and Ball (2010), also advocate for deep subject matter knowledge and its subsequent positive influence on instructional techniques. Philipp et al. (2007) similarly confirm that teachers who achieve greater math knowledge are more capable of the conceptual teaching than their counterparts, who implement procedural based instruction. All studies propose that teachers' decisions rely on their understanding of math subject matter. Hence, research concludes that the deeper MCK that a teacher holds, the better equipped they are to communicate with students about mathematical concepts, models, and representations.

In her study of elementary TCs, Kajander (2010) observed how prospective teachers' conceptual knowledge and beliefs about reform math teaching changed over the progression of a math methods course. Similar to Ma (1999), and Ball (1990a), Kajander's (2010) findings illustrated TCs' inadequate understandings of math for teaching. Most TCs entered their teacher preparation programs with limited conceptual proficiency in how to represent mathematical concepts, explain their thinking, and justify mathematical procedures. However, after completing a math methods course that focused on developing conceptual knowledge for teaching, improvements in conceptual knowledge were examined based on the comparisons of pre- and post-tests. This study endorses Hill and Ball's (2004) findings that content knowledge can be positively increased by a single course experience. Kajander (2007, 2010) posited that due to the increase in conceptual knowledge of math, TCs shifted their pedagogical beliefs about teaching math, and they were more focused on problem solving and understanding, and less focused on traditional learning methods. Hence, these findings suggest how content knowledge and pedagogical beliefs are linked.

Understanding Math for Teaching

Shulman's (1986, 1987) notion of pedagogical content knowledge gives attention to the role of content when teachers make pedagogical decisions. Pedagogical content knowledge recognizes that teaching requires a unique specialized knowledge of content. Over the last three decades, considerable research has gone into developing Shulman's notion of pedagogical content knowledge through the lens of math teaching. Drawing upon Shulman's conceptualizations of pedagogical content knowledge, several researchers have identified and described a unique understanding of math knowledge required for teaching (Ball, 1990b; Ball, 1996; Ball, Hill, & Bass, 2005; Ball et al., 2008; Hill, Rowan & Ball, 2005; Thames & Ball, 2010). Subsequently, Ball et al. (2008) developed a practice-based theory of math knowledge for teaching (MKT) that include the following domains empirically generated through factor analysis: 1) common content knowledge (CCK) is the math knowledge used in a wide variety of settings that is not exclusive to teaching; 2) specialized content knowledge (SCK) involves knowledge that goes beyond a conceptual understanding of mathematical ideas. It constitutes the knowledge and skills that are unique to math teaching as it requires teachers to understand math content with a strategic focus on pedagogy; 3) knowledge of content and students (KCS) comprises of teachers' knowledge about students as well as math content. Understanding common errors and misconceptions made by students, and interpreting students' mathematical thinking are all key aspects of KCS; and 4) knowledge of content and teaching (KCT) involves the combination of pedagogical knowledge and math content. This requires teachers to understand instructional design, such as how to represent mathematical concepts, sequence content, select examples, and explain methods and procedures.

An underpinning of MKT is the notion that teachers require a specialized kind of knowledge to teach math successfully. A teacher's MKT influences pedagogical decisions such as when to interject and redirect students, when to pose questions to further students' learning, and how to respond to students' mathematical remarks. Ball et al. (2008) assert that solid MKT enables teachers to develop and demonstrate mathematical models based on students' levels of understanding, and explain why a method works and whether it is generalizable to other problems. Furthermore, the authors claim that MKT is uniquely different from being a student of math. Specifically, MKT requires a conceptual knowledge base to promote discussions about models and connections between concepts and procedures. These types of interactions are often done immediately on the spot, during teachable moments in response to students' needs. Unfortunately, research suggests that math teachers are limited in their MKT, which poses many challenges for the implementation of reform math (Lo & Luo, 2012).

Math Content Knowledge (MCK) and Its Impact on Student Achievement

The content knowledge of math teachers and its relationship with student success in math has been of interest to researchers (Conference Board of Mathematical Sciences [CBMS], 2012). Sowder (2007) stated that in order to increase math knowledge and achievement, all math classrooms require teachers with in-depth knowledge of math. Rowan, Chiang, and Miller (1997) identified teachers' math content knowledge (MCK) as a predictor of student achievement in grade 10 math. In their quantitative study, they found that students produced higher levels of achievement if they were taught by teachers who also scored higher on math test themselves. Furthermore, students who were taught by teachers who held a math degree also earned higher levels of test scores (Rowan, Chiang, & Miller, 1997). However, Darling-Hammond and Youngs (2002) reviewed various research studies of teacher education programs and concluded that although the MCK of teachers can contribute to student achievement, other aspects of a teacher candidates' (TCs') education are equally important such as math methods courses and practice teaching.

It is important to note that higher levels of math courses do not automatically equate to better math teaching. Although it could be speculated that having a major in math should increase a teachers' capacity in successfully teaching math, there is no evidence that this is true for the elementary grades, and for the secondary grades, it is not a consistent predictor (CBMS, 2012). In order to make math meaningful, Sowder (2007) argued for TCs to become immersed in learning mathematical concepts and have opportunities in their courses to make connections between representations and applications, rules and algorithms. Unfortunately, considerable evidence suggests that many math teachers can apply the rules and procedures required to do math but lack conceptual knowledge and reasoning skills to teach for deep understanding (CBMS, 2012; Ma, 1999).

Study Design and Method

The theoretical methodology for this research study was underpinned by the theory of math knowledge for teaching (MKT) and specific components of this theory, mainly, math content knowledge (MCK) and the relationships between procedural and conceptual knowledge. This selected theory was deemed to be highly pertinent to the phenomena of teacher candidates' (TCs') math development and drew upon a range of theorists such as Ball, Thames, and Phelps (2008) and Heibert (2013). MKT requires a specialized understanding of content that is interwoven with knowledge of students, pedagogical strategies, and curriculum (Ball, Thames, & Phelps, 2008). In order for teachers to develop effective MKT, MCK is a required rudimentary foundation. Ball et al. (2005) refers to MCK as common content knowledge (CCK) and is considered one of the domains of MKT. The authors describe CCK as the basic math knowledge and proficiency necessary to be considered a mathematically literate person.

A primary purpose in this current study was to measure and critically examine TCs' basic MCK as an underpinning of deeper math concepts, as well as their experiences in developing as math learners and teachers. The MCK of TCs was measured prior to the start of their two-year graduate program as participants were not expected to have developed the other domains of MKT. TCs took classes in different cohorts, either primary/junior (P/J) or junior/intermediate (J/I). TCs in the P/J division earn qualifications to teach kindergarten to grade 6, whereas their counterparts in the J/I division earn qualifications to teach grades 4 to 10. Researchers and teacher education math instructors discussed what could be reasonably expected from TCs to already know in math before commencing their teacher education program, that is, to successfully develop the MKT for kindergarten to grade 8 classrooms. Although J/I students could teach grades 9-10 math, in such cases they would usually have completed university math courses in their undergraduate degree, with additional math courses in their teacher education program. Researchers also reviewed interview transcripts to determine aspects of the math program that supported or hindered the development of MKT. Based on these data, the researchers recommended modifications to the teacher education program to better support TCs' learning and teaching of math.

This study focused on TCs enrolled in a two-year Master of Teaching (MT) graduate degree in a large urban southern Ontario university. TCs enrolled in the MT program take a 36-hour *math methods* course in year one, and an 18-hour *issues in numeracy* course in year two. There were 89 TCs in the primary/junior program (kindergarten to grade 6) and 62 junior/intermediate TCs (grades 4 to eight) who completed both the pre- and post-tests, totalling to 151 participants. An additional 30 students did not complete the pre- and/or post-tests; their results were not utilized in this study.

The methodology used in this research focused on a pragmatic paradigm through a mixed methods approach. The quantitative data comprised of pre- and post-tests that assessed participants' numeracy operation skills: addition, subtraction, multiplication, division, fractions, percent, decimals, ratio, order of operations, and integers. The questions on the test essentially measured MCK in the area of numeracy. This assessment was administered at the beginning of the program prior to the start of classes. A slightly modified version of the test was administered at the beginning of year two. Achievement results for each question were analyzed and the data was further examined to identify significant changes in TCs' achievement. Mid-way through the second year, eight TCs took part in semi-structured interviews. To gain a deeper understanding of how TCs experienced the math test and MT math classes, "purposeful sampling" was used to intentionally select participants (Creswell & Clark, 2007, p. 112). The purpose for targeting specific TCs included the researchers' desire to gather narratives from students who considered themselves competent in math, as well as those who felt that math was a struggle. Four TCs with low confidence in math and four TCs with high confidence in math were invited to participate. These qualitative data focused on exploring TCs' feelings toward the math test and observations of their year one and two math classes. The combination of the quantitative and qualitative data allowed for the researchers to identify common errors made by TCs on the math tests, as well as identify attitudes and beliefs toward their math learning and teaching in the program.

Instrument Development and Administration

The math pre- and post-tests were collaboratively developed by a committee of math teacher education instructors. The committee met several times over a six month period to discuss and create questions. The goal for the pre-test was to determine the entry points of teacher candidates' (TCs') numeracy operation skills. The goal of the post-test was to determine gains in math content knowledge (MCK) that TCs achieved. Committee members carefully based test questions on the Number Sense and Numeration strand in The Ontario Curriculum, Grades 1-8: Mathematics (2005), mostly at grades 5 and 6, with approximately 10% of the questions at the grade 7 level focused on integers. The tests comprised of several questions in the following numeracy areas: addition, subtraction, multiplication, division, fractions, percent, decimals, ratio, order of operations, and integers. Calculators were not permitted and the format of the test did not include multiple choice questions. Furthermore, questions that would assess TCs' pedagogical skills were not included, i.e., the tests did not ask TCs how they would explain algorithms, describe and make connections between concepts, or examine misconceptions in students' work. Rather, this study focused on participants' MCK, namely numerical operation skills, which required TCs to demonstrate basic math knowledge.

The assessment was reviewed by several math and non-math teacher education instructors for feedback on each of the items. The feedback was generally positive and all the instructors felt that the questions were reasonable for TCs to complete. Many also noted that this math test would help TCs better understand their own basic knowledge and the aggregated results would help set a positive direction for the Master of Teaching (MT) program.

TCs were informed approximately two months ahead of time that they would be completing the basic math assessment a week prior to classes. It was also clearly communicated that this pre-test was not at all high stakes, that is, the results were not factored into their marks and passing the test was not a requirement to complete their MT degree. Rather, the pre-test was designed as a diagnostic assessment to determine strengths and areas for improvement in one's own basic math proficiency. Candidates were offered a practice test to help them prepare for the math assessment. The post-test took place after the first year of the MT program, allowing TCs to review their progress and identify areas of MCK that still required further focus and learning.

TCs completed the pre- and post-tests in their cohort groups of about 25 at a time. There was one supervisor per cohort who administered the assessments. Supervisors encouraged participants to show their work and write out as much of their thinking on the test paper. Participants had up to 90 minutes to complete the questions, and the majority of them finished within an hour. The pre- and post-tests were assessed and returned to TCs with feedback on how to improve their basic math knowledge. Furthermore, data results were aggregated to determine areas of strengths, needs, and next steps for developing MCK.

The semi-structured individual interviews gave the researchers an in-depth understanding of TCs' experiences of their math development. In this study, interviews were conducted with eight TCs (four with low confidence in math; four with high confidence in math) and included the following questions: 1) Has your content knowledge of math changed during the two courses in this program? 2) What was the most valuable experience during the program in your development of math content knowledge? 3) What was the most challenging experience during the program in your development of math content knowledge? 4) How has your confidence as a math teacher changed during this program? 5) Have your conceptions of how to teach math changed during this program? and 6) How could the program be improved? The interviewer promoted a conversational atmosphere by occasionally checking for understanding of the answers offered to questions (Yin, 2009). For each question, interviewees were asked to offer details and/or examples. Each interview was completed within a 30-45 minute time period. The interviews were audio-recorded and transcriptions were completed and analyzed by the researchers after the submission of final grades of any researcher associated with an interviewee.

Two researchers were used in the coding process of each of the interviews to promote and confirm inter-reliability (Kurasaki, 2000). Based on the theoretical underpinnings embedded in this study, an initial set of codes was developed (Creswell, 2009, Denzin & Lincoln, 2000), e.g., procedural knowledge, confidence to teach math. Further codes were also added during the coding phase, e.g., mindset. The researchers engaged in an iterative process whereby interviews were analyzed to ultimately identify themes and create meaning (Sandelowski, 2001). This process provided additional opportunities for the researchers to reflect and analyze upon the transcripts and comments. The coded data was further reviewed to develop categories across the interviews, thereby grouping content in terms of parallel and divergent concepts (Patton, 2002). Broad themes were then established to frame the consistencies of ideas within and throughout the categories (Baxter, 1991; Polit & Hungler, 1999).

Results

Data analysis for this study included overall pre- and post-test results of teacher candidates' (TCs') numeracy operation skills. Researchers identified low scoring items on the pre-test and assessed any statistical significances between pre- and post-test results. In conjunction with the quantitative results, transcripts of interviews were examined through content analysis and coding techniques. Both quantitative and qualitative results generated a comprehensive understanding of TCs' math content knowledge (MCK) and their experiences as math learners and teachers. This study's results revealed three major themes: 1) MCK tests support reflective practice; 2) instructors of and activities within math courses are vital; and 3) practicum placements in math are essential for math knowledge for teaching (MKT). Furthermore, these themes highlight specific areas of need in TCs' MCK and MKT, as they relate to this study's theoretical methodology and prior research.

Overall Results of Pre- and Post-Test

The pre- and post-tests comprised of 61 and 66 data points respectively. Committee members added five additional questions to the post-test for future pre- and post-tests. These additional questions were not part of the calculations in this study to support comparability of the results of the pre- and post-tests. The 61 comparable questions covered areas in addition, subtraction, multiplication, division, fractions, percent, decimals, ratio, order of operations, and integers (see Table 1 for itemization of assessment questions).

Table 1. *Itemization of Assessment Questions*

Itemization of Assessment Questions	
Numeracy Concept	Test Points
Addition	3
Subtraction	4
Multiplication	5
Division	4
Fractions	13
Percent	2
Combination of Fractions, Percent and Decimals	16
Ratio	3
Order of Operations	5
Integers	6
Test Total	61

Note. The basic math content knowledge pre- and post-tests assessed numeracy skills in the areas of addition, subtraction, multiplication, division, fractions, percent, ratio, decimals, order of operations, and integers. There was a total of 61 data points that were scored in this pre- and post-test. This chart reveals the breakdown of the test questions and number of data points for each area.

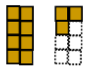
In general, junior/intermediate (J/I) teacher candidates (TCs) scored slightly higher on several questions when compared to primary/junior (P/J) scores (pre- and post-test). For each test item, an unpaired t-test was calculated between the J/I and P/J scores in order to determine any statistical difference between the two groups. The results indicated statistical significance between the J/I and P/J groups for two pre-test and three post-test questions (i.e., $p < 0.05$). However, it was expected that 5% of these test questions would yield false positives based on significant differences generated because of random variation. Five percent is approximately three questions out of the 61 items. For this reason, it would be difficult to make a case that these questions were somehow indicative of a fundamental difference between P/J and J/I participants in terms of their abilities to solve these specific types of questions.

The overall mean score of 151 participants for the pre- and post-tests was 81.54% and 84.62% respectively. This indicated an increase that is considered extremely statically significant (i.e., $p < 0.0001$). Overall, there were several questions that TCs improved significantly over the year. Although the improved post-test scores demonstrated enhanced math content knowledge (MCK), there were numeracy skills that continued to challenge many of the participants, for example, division of four-digit by two-digit numbers, order of operations, and word problems involving percentages. It is not surprising that not all areas improved on the post-test. During interviews with TC participants, some identified that they were math anxious and reminisced about dropping math as soon as possible in high school … *"And then when math became an option, like enough … I'm done with math!"* Another TC noted that she had not focused on math in some time and the math test helped identify gaps in her understanding … *"I went to the math test and I realized that there are a lot of principles that I haven't studied for or reviewed ... there were lots of challenges throughout."* Overall, most of the TCs commented that the pre-test identified math areas where their content knowledge was weak, thus allowing them to prioritize their own math learning.

Low Scoring Numeracy Skills

To focus on test items that presented difficulty to teacher candidates (TCs), the researchers of this study analyzed items where less than two-thirds of the participants answered correctly (see Table 2). For 10 of the 61 pre-test items, participants struggled to answer the items correctly. More specifically, the percentage of participants that answered the items correctly ranged from 33.55% - 63.65%. When the 10 pre- and post-test items where compared through a paired t-test, six items indicated an increase that was considered extremely significant (items 1-6), three items did not indicate a statistically significant increase (items 7-9), and one item did not indicate a statistically significant decrease (item 10).

Table 2. *Low Scoring Numeracy Skills*

Low Scoring Numeracy Skills (< Two-Thirds of TCs answered correctly)				
Numeracy Skill	Sample Items & Answers	Pre-Test	Post-Test *(Diff)*	Paired t-test
Divide a 4-digit number by a 2-digit number to the hundredth decimal.	*1246 ÷ 45 = 27.69*	37.09%	57.62% *(+20.53%)*	$t = 3.7170$ $p < 0.001$
Represent in a fraction.	How much chocolate is left? Represent in a fraction.  *$1\frac{3}{8}$*	60.26%	80.13% *(+19.87%)*	$t = 4.3140$ $p < 0.0001$
Convert a percent number into a fraction.	Convert 167% into a fraction. *$1\frac{67}{100}$*	58.94%	78.15% *(+19.21%)*	$t = 4.6030$ $p < 0.0001$
Divide a 4-digit number by a 2-digit number to the hundredth decimal.	*9768 ÷ 38 = 257.05*	33.77%	51.66% *(+17.89%)*	$t = 3.7256$ $p < 0.001$
Divide a 4-digit number by a 2-digit number using whole remainders.	*7727 ÷ 25 = 309 R-2*	54.30%	70.20% *(+15.90%)*	$t = 3.4460$ $p < 0.001$
Divide a 4-digit number by a 2-digit number using whole remainders.	*2487 ÷ 16 = 155 R-7*	63.58%	77.48% *(+13.90%)*	$t = 3.4845$ $p < 0.001$
Multiply a decimal number by a whole number.	*8.273 x 15 = 124.095*	59.60%	66.23% *(+6.63%)*	$t = 1.3910$ $p > 0.1$
Calculate the percentage of a whole number.	*55% of 110 = 60.5*	62.91%	65.56% *(+2.65%)*	$t = 0.6017$ $p > 0.5$
Use order of operations to solve.	*13 – 6 + 8 = 15*	55.63%	56.95% *(+1.32%)*	$t = 0.2878$ $p > 0.5$
Solve problem using numeracy skills in percent, fractions, and/or decimals.	Kiana has read 120 pages of her book – 120 pages equals to 40% of the entire book. How many pages does she have left to read? *Kiana has 180 pages left to read.*	55.63%	47.68% *(-7.95%)*	$t = 1.5565$ $p > 0.1$

Note. This table displays questions from the basic math content pre-test where less than two-thirds of the participants were able to answer these items correctly. Results are reported in order of the highest to lowest gains on the post-test scores. Each test item is connected to a numeracy skill. A paired t-test was performed comparing the pre- and post-test results for the 10 items.

When comparing the pre- and post-test results, as shown in Table 2, six items indicated an increase that was considered extremely significant. Dividing 4-digit numbers by 2-digit numbers to the hundredth decimal (item 2 and 4) proved to be the most difficult items – with only 37.09% of participants responding accurately for item 2 and 33.77% answering correctly for item 4 on the pre-test. While each item indicated a significant gain in the post-test, many TCs were still not able to answer the items correctly, i.e., 42.38% and 48.34% respectively. Division with whole remainders (items 5 and 6) was also challenging, with an initial mean of 54.30% for item 5 and 63.58% for item 6. Each item showed a significant gain with more than 70% of TCs answering correctly on the post-test for both items. At the start of the program, converting a percent number into a fraction (item 3) had an average of 58.94%. For the post-test, 78.15% of TCs were able to convert a percent number into a fraction, a significant gain. One item involved the representation of chocolate bars as fractions (item 1); this item scored at 60.26% on the pre-test. A significant gain was shown with an increase of 19.78% on the post-test.

When examining items 7-10 in Table 2, the difference between pre- and post-test items were not considered to be statistically significant. Calculating a number sentence using order of operations resulted with 55.63% achievement (item 9), with a minimal gain of 1.32%. Similarly, when multiplying a decimal number by a whole number (item 7) and calculating the percentage of a whole number (item 8), the items only demonstrated gains of 6.63% and 2.65% respectively. Lastly, solving a word problem that required knowledge of percent, fractions and/or decimals (item 10) scored 47.68% on the post-test, a decrease of 7.95%.

Although the overall results of the TCs had increased on the post-test, the researchers further reviewed TCs who did not demonstrate proficiency upon commencing the program in order to track their progress. Specifically, the group of TCs who did not perform at or above 70% on the pre-test were further analyzed. For this group, almost three-quarters of the TCs' post-test scores remained under 70%. Furthermore, the researchers reviewed the remaining TCs who scored below 70% on the post-test and found that they had scored less than or equal to 75% on the pre-test.

It is worth noting that the MT math classes were focused on developing math knowledge for teaching, lesson planning, assessment, and constructivist approaches to teaching math. Although math content knowledge (MCK) was not an intentional focus of these classes, numeracy operation skills were reinforced during math activities and problem solving tasks. Even though not specifically asked during the interviews, many TCs shared their own experiences as a math learner in middle and high school. It was acknowledged by most of the interviewees, that they relied on memorized procedures to answer the pre-test questions. These types of questions in a traditional math classroom would typically be taught through rote learning methods. For example, division would be taught via algorithm dependency without conceptually understanding why and how the algorithms work. In all cases, a traditional style of past learning experiences in middle and high school was described by TCs … *"I would say [the math teacher] was stringent, it was like here's a product, here's a solution, do it. No critical thinking, no application, no flexibility whatsoever."*

Math Content Knowledge (MCK) Tests Support Reflective Practice

The pre- and post-tests focused on the numeracy operation skills, primarily for grades 5 and 6. Regardless of the math confidence noted by the interviewees, all teacher candidates (TCs) described how their performance on the test shed light on their competencies. For those with confidence in math, it reinforced that they had a foundation of content knowledge required to teach math ... *"what also helped to give me confidence was* *the math test … because it turned out I was very good and I still remembered everything and I understood the math, so that was also very good for my confidence."* For other TCs, the test encouraged them to set goals for future personal learning … *"[The test] hit on all the things that you're going to need to know in order to teach* [math] *and so I think it was a great opportunity because if you had a problem, you would have been able to see where it was."* TCs' personal learning was further facilitated by the math instructors in the MT program. After the pre-tests, some classes collectively reviewed different ways in which the same answers could be calculated … *"we went through* [the test] *afterwards and it really helped … we talked about how there are several different ways to solve a problem, even when it wasn't the traditional method."* This concept of deconstructing and comparing various methods to solve a problem was consistently reinforced throughout the MT math classes.

During the interviews, several TCs suggested changes to the test in the future, since not all of their peers thought that completing the test was a priority; 30 TCs did not take part in the pre- and/or post test. TCs explained how busy their schedules were due to course work, assignments, and preparation for practicums, therefore it was easy to opt out of the test. With this in mind, a few interviewees felt the need to make the pre- and post-test mandatory … *"I wondered if the math test could be mandated."* Another recommendation included covering math curriculum higher than grade 6 … "*in second year you could offer a different test … if it was grade 7, 8, 9 and 10 … knowing that this test will include the higher grades that we might be teaching, then it would interest others."* This is a relevant proposal because TCs who are Junior/Intermediate (J/I) qualified, would likely be required to teach math at the grade 7 and 8 level, and may even find themselves having to teach grades 9 and 10 math. Perhaps more importantly, several participants discussed additional enticements to take part in the pre- and post-test. For example, the provision of supplementary math training in the form of university sponsored tutoring was mentioned to support TCs who were not confident in math.

Instructors of and Activities within Math Courses are Vital

When asked about experiences in the Master of Teaching (MT) math classes, all interviewees provided positive responses with examples of how their conceptions of math and/or teaching of math were supported. A common element across all interviews included the importance of the MT instructors … *"the instructors who are teaching math, they actually love math" … "it was really amazing, she does have passion towards mathematics."* These instructors were able to create safe environments to explore the misconceptions that many teacher candidates (TCs) still had about learning and teaching math; moving beyond the algorithms that TCs were previously overly reliant on. Reflection on their learning as a TC, as well as past learning as a math student, was viewed as an essential part of developing as teachers … *"In this class, we started to learn different concepts and I thought, why didn't I learn this way" … "learning about the pedagogy has allowed me to relate differently to my students and really dive into understanding the issues in math."* The MT math classes offered the TCs contemporary models of instruction to compare with their own experiences as math learners, thereby challenging their conceptions of learning and teaching math.

All TCs discussed their changing perceptions and beliefs about teaching math based on their interactions in the MT math classes … *"I guess without having taken [instructor's] course, I wouldn't have thought about math instruction differently than the way I was taught."* TCs transformed how they viewed the curriculum by prioritizing students' math needs and figuring out how to make math accessible for all students. The course experiences challenged TCs' mindsets developed over time in traditional math classes. For example, this involved spending time on the process of solving problems instead of only the answer. Finding multiple ways to arrive at a solution was encouraged, as well as reflecting on the math skills needed to formulate an accurate answer. For many, these were unfamiliar practices that contradicted the traditional methods of teaching math. *"We basically looked at mathematics as having multiple ways to get the answers and creating our own questions which was completely foreign to me … it allowed us to explore ourselves as teachers in a new light."* This was further exemplified by MT instructors who emphasized the importance of reasoning and justifications of math solutions … "*we were taught to embrace students explaining their thinking and showing their different strategies … it's more about understanding how they got there."*

TCs challenged themselves as math students and teachers, through various hands-on opportunities in the MT math courses. Examples of these experiences included working with manipulatives, probability and simulation activities, and deconstructing common algorithms. These experiential activities reinforced different ways of teaching and learning math concepts. TCs often stated they were so engaged in some of the activities that they didn't realize that multiple math skills were being addressed until they reflected on and discussed the specific math skills involved. This led to shifts in mindsets which changed conceptions of the composition of a math class … *"before, I didn't necessarily think about doing the hands-on problem-solving activities, it's absolutely made me more likely to do these activities."* Through such course experiences, TCs challenged their own beliefs about how learning in a math classroom occurs. This was an important step towards reform math instruction, however, TCs required further opportunities to actualize these new beliefs and hands-on activities in a real classroom. Hence, practicum placements were of utmost importance for TCs to develop their confidence as math teachers.

Practicum Placements in Math are Essential for Math Knowledge for Teaching (MKT)

During the two-year Master of Teaching (MT) program, teacher candidates (TCs) complete four practicum blocks, two each year. The practicum block is a full-time experience within a classroom for four weeks. The TC is matched with an associate teacher, who is an Ontario certified teacher recommended by a school leader. An important part of the placement involves observation of the associate teacher, students, and classroom program. Through the partnership developed between the associate teacher and the TC, the associate teacher incrementally releases responsibility onto the TC to take on planning, teaching, and assessing for the class.

Although TCs were not specifically asked about their practicum placements, the importance of teaching math during practicum was discussed by all interviewees. In most cases, TCs shared about practicum placements when asked about their confidence as a math teacher and/or the development of their math content knowledge (MCK). Two TCs had not taught math during their practicums at that point in time, with one more practicum left. These TCs strongly expressed the lack of math teaching as a deficit to their development. One TC described her first practicum with an associate teacher who felt that sharing the math teaching would … *"cause confusion"* … for her students. Fortunately, this TC felt that her associate teacher modelled an effective math program … *"I was able to observe … she had routines already established for the kids … I learned techniques of how to deal with math."* Although the TC wanted to teach math during this practicum, at least she had a positive role model. For the majority of the interviewees, it was stressed that math was a critical part of their placement and professional learning … *"I was really lucky because I actually taught math quite a few times during my practicums … that was a huge help, combined with the [MT math] courses, actually trying it out."* This appreciation for teaching math in the practicum was globally shared by those with high and low self-proclaimed levels of confidence in math.

A bi-product of teaching math during practicum described by TCs was the interplay between MCK and knowledge for teaching math … *"there was a difference between being very good at math and being able to teach it."* Although the practicum was identified as helpful when discussing their confidence as a math teacher or their conceptions of how to teach math had changed, a few also discussed how teaching math induced stress for themselves or their peers. When some TCs found out that they were teaching math, they shared their angst with others, wondering about resources and feeling a general sense of panic. Fortunately, for almost all TC participants who taught math during their practicum(s), confidence in teaching math improved … *"when I think about teaching my first lesson versus my last, I already felt more confident and I knew what worked"* and *"you don't get comfortable knowing when you need to adjust unless you have actually tried it."* Many TCs discussed the nuances of getting to know the learners in the classroom and attempting to meet their needs … *"it's one of the only ways where you can figure out where kids have the gaps in their understanding of math and where they're going to have problems."*

Discussion and Implications

After analyzing pre- and post-tests and the semi-structured interviews, researchers discovered that TCs gained numeracy operation skills and content knowledge over the year, as well as increased confidence in teaching math. The significant findings from this study were further analyzed by the researchers to consider implications such as possible revisions to the pre- and post-tests, as well as enhancements to the MT program. The recommendations are presented in the following sections: 1) establish minimum standards; 2) raise the stakes of the post-test; 3) interplay of procedural and conceptual knowledge; and 4) coherence between math courses and practicum.

Establish Minimum Standards

Although gains in math content knowledge (MCK) were observed in the post-tests, and interview participants described changes in their math knowledge for teaching (MKT), there were some areas in their content knowledge that did not improve. Specifically, almost three-quarters of the teacher candidates (TCs) who scored less than 70% on the pre-test, remained below 70% on the post-test. In addition, a few of the TCs who scored 75% or below on the pre-test also struggled in their MCK development, scoring below 70% on the post-test. These findings raise the question about what is reasonably expected of TCs to know and understand in basic math prior to their teacher education training. In Ontario, grade 6 students who perform below level three, that is below 70%, on the provincial math assessment do not meet the provincial standard (EQAO, 2016a). With this standard in mind, the researchers assert the need for minimal math content knowledge standards for TCs. Without basic MCK, TCs will likely struggle to engage their students in achieving math concepts, and further develop their own MKT. As noted in the research, teachers with inadequate content knowledge find it difficult to explain math concepts, provide models, and make connections to support understanding (Ponte & Chapman, 2008; Thames & Ball, 2010). For these reasons, the researchers highly recommend that TCs who score 75% or below on the pre-test receive supplementary math support beyond the existing offerings in the Master of Teaching (MT) math classes. Currently, TCs receive recommended online resources for low-scoring areas on their test. The researchers suggest that further math support through face-to-face or online tutoring would greatly benefit those TCs who performed poorly. Although this would come at an expense to the program, the researchers strongly feel this investment would be worthwhile due to the impact TCs could have, each potentially teaching math to hundreds of students in the years following graduation.

Raise the Stakes of the Post-Test

Similar to the research of CBMS (2012) and Thames and Ball (2010), all teacher candidates (TCs) described the value of deeply understanding the math content in order to develop appropriate lessons for their students, that is, possessing the specialized math knowledge to make sound pedagogical decisions. Importantly, a few TCs suggested the need for the tests, or at least the post-test, to be more difficult, by adding questions specifically designed for grade 7 and 8. They felt that this would not only elevate the status of the tests, but also encourage TCs to further develop the MCK necessary to be effective teachers. This is a critical concept that promotes the importance of math knowledge required to teach. Studies have shown the deep knowledge base teachers require in order to teach math effectively, even in the primary grades (Ball et al., 2005). Based on the findings from this study, the researchers recommend that the Master of Teaching (MT) program: 1) increase the difficulty of the pre- and post-tests (e.g., consisting of questions from grades 5 – 8; 35% grade 5, 35% grade 6, and 15% grade 7, and 15% grade 8), and 2) have the post-test score count toward a small percentage of the TCs' final math grade, for example, 10% of the final math mark. The researchers feel this will positively increase the stakes of the test and propel TCs to invest in their MCK by studying and seeking help where needed. With increased MCK, the researchers also have confidence that TCs will be better positioned to further develop math knowledge for teaching during MT math classes and practicums.

Interplay of Procedural and Conceptual Knowledge

The findings from the basic content knowledge test illustrated some of the challenges teacher candidates (TCs) faced with specific numeracy operation skills. It was evident that many participants still struggled with questions that required procedural knowledge involving several steps such as multiplication of decimal numbers and solving word problems involving percent, fractions, and/or decimals. The results from the present study align with other research indicating the critical relationship between procedural and conceptual knowledge (Ambrose, 2004; Heibert, 2013). TCs with a strong math background described understanding the math concepts that were investigated in their Master of Teaching (MT) math classes, as well as feeling confident to teach math. Other TCs who found math to be more challenging, revealed how they were dependent on the memorization of the steps and therefore their deep understanding of concepts was compromised. TCs' overreliance on procedural knowledge was also found by researchers such as Tirosh (2000).

A major part of the MT math classes engaged TCs in activities that modelled math approaches to promote conceptual understandings of math that go beyond the memorization and application of algorithms. TCs gained both procedural and conceptual knowledge of math that led to the development of math knowledge for teaching during their practicums. This finding aligns with research about the importance of building the conceptual knowledge of TCs as this leads to shifts in pedagogical beliefs and practices in the classroom (Ball, 1990a; Kajander, 2007, 2010). Most importantly, the TCs described that the math lessons they planned for their students were different from how they themselves experienced math in school. Instead of traditional math practices, the TCs carefully planned math activities with multiple entry points for their students and ensured that their students were able to discuss and reflect on their problem-solving strategies. Due to these findings, the researchers suggest that the MT math courses continue to investigate opportunities to challenge TCs' perceived notions of teaching math, providing multiple opportunities to build content and procedural knowledge through rich, hands on learning opportunities.

Coherence between Math Courses and Practicum

The math courses in the Master of Teaching (MT) program were noted by all interviewees as being vital to their math learning as students and math teachers. Nevertheless, these courses could not reproduce an actual classroom with real students for teacher candidates (TCs) to interact with and teach. These experiences could only be gained through practicums that involved teaching math to students. However, one cannot assume that teaching math automatically improves math content knowledge (MCK), math knowledge for teaching (MKT), and/or confidence in teaching math. Unfortunately, two TCs described practicum placements that were not aligned to the pedagogical outcomes in the MT program. These narratives suggest the need for coherence between the MT math courses and practice teaching. In a few practicum placements, TCs felt pressure to teach math through a prescribed and transmissive method, that is, memorization of procedures and assigning math text book pages and worksheets. This type of instruction aligns with the evidence found by CBMS (2012) and Ma (1999) where math teachers struggled with explaining the deeper conceptual knowledge of math, only feeling comfortable to teach through algorithms and procedures. In an ideal world, all practicum experiences would embody exemplary math environments for TCs to work within. However, as noted in the research, teachers have varying levels of math proficiency and in many cases, the knowledge base may be lacking (Ball, Hill & Bass, 2005; Cai & Wang, 2010; Vistro-Yu, 2013). Based on these findings, the researchers assert that teacher education programs and school boards should involve supporting associate teachers to nurture risk-taking by allowing TCs to dialogue, question, and trial math teaching strategies with intentional reflection (Reid, 2013). Through this process, associate teachers and TCs engage in relevant discourse and work together to improve their math teaching skills and thereby positively impact students' math achievement.

Conclusion

How much basic math content knowledge TCs ought to know prior to entering their teacher education program is an area that has not been widely examined. This current research offers important knowledge about the math content knowledge (MCK) of TCs, as well as the major impact that math courses and practicums have during their teacher education experiences. Regardless of the basic numeracy skill levels that TCs possessed upon entering the program, their Master of Teaching (MT) math classes and practicums contributed to both MCK and their beliefs about how to teach math. Nevertheless, the researchers of this study posit that those TCs with low MCK, based on the pre-test results, would have benefited from additional support, e.g., tutoring. This recommendation is derived from the fact that some items in the post-test revealed no significant improvements and several items still posed a challenge for TCs. Additional support in the form of small group or one-on-one tutoring would enhance TCs' MCK, especially for those who experienced difficulty learning the curriculum content required for their practicum teaching. Other ways in which teacher education programs can raise the importance of MCK include: minimum standards on entrance or exit exams and the inclusion of the post-test scores as a percentage of final math grades. The researchers of this study believe that all teacher education programs should support compulsory minimum math knowledge requirements. This focus on the foundational MCK skills of TCs is critical for the successful development of math knowledge for teaching (MKT) capacities. Ultimately, this will raise the significance of teaching math for understanding and increase the abilities for all math educators to support effective math environments for students to flourish as mathematicians.

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