ASSESSMENT OF STUDENT LEARNING IN VIRTUAL SPACES, USING ORDERS OF COMPLEXITY IN LEVELS OF THINKING

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ABSTRACT
This paper aims at showing a new methodology to assess student learning in virtual spaces supported by Information and Communications Technology-ICT. The methodology is based on the Conceptual Pedagogy Theory, and is supported both on knowledge instruments (KI) and intellectual operations (IO). KI are made up of teaching materials embedded in the virtual environment. The student carries out IO in his/her virtual formation process based on KI. Both instruments of knowledge and intellectual operations can be mathematically modelled by using functions of increasing complexity order. These functions represent the student’s learning change. This paper main contribution is to show that these functions let the student go from a concrete thinking to a formal one in his/her virtual learning process. The research showed that 47% of the students moved from a concrete thinking level to the formal thinking level.

Keywords: Conceptual pedagogy, virtual assessment, mathematical functions, complexity orders, virtual learning, ICT.

INTRODUCTION
Assessment of student learning supported by ICT is one of the essential issues in the virtual teaching-learning process. The importance of the student learning assessment requires ensuring real student really learns during the process, which involves ensuring the student progress in thinking levels. Most of the studies in education focus on evaluating student learning, but very few are focused on exploring assessment in order to achieve an increase in the student thinking level. This increase means the student improves their thinking structure going from a concrete thinking to a formal one.

Based on the above mentioned, this paper presents a new methodology with a theoretical support integrated in educational sciences, computer science and mathematic, in order to assess the advance of the student in their thinking levels. The theoretical foundations are realized in a model to assess the student learning by using complexity orders in thinking levels. The model is applied to a course supported by ICT, and results show advance levels in the student learning oscillating in advance range of [42.22%; 56.11%] at different levels of thought.

LITERATURE REVIEW
The literature regarding student learning assessment in virtual spaces may be classified as follows: 1) Models taking into account the personalized student assessment in the virtual environment based on their previous knowledge. Research which studies the time frequency of reading forums and posting in the virtual platform (Gomez-Aguilar, Hernandez-Garcia, Garcia-Penalvo, & Theron, 2015). Models that unlike the personal assessment of the student seek to improve the assessment of teams of students, which is equivalent to collaborative cooperative assessment (Gomez-Aguilar et al., 2015). Models
for assessing the impact of virtual training after the student complete their training process. This assessment is done in the professional workplace within a company site or by distance work -teleworking, in order to demonstrate virtual education effectiveness (Navimipour & Zareie, 2015; Van Nuland & Rogers, 2015). 2) Pedagogical methodologies supported by blended learning (BL) where the student learning is improved. 3) Use of systems for assessing student learning such as: adaptive and intelligent systems, where technology and science are integrated through the Learning Management Systems (LMSs), (Dolenc & Abers, 2015; Sanchez-Santtillan, Paule-Ruiz, Cerezo, & Alvarez-Garcia, 2016). Systems taking into account the frequency of use of platform resources and search for behaviour patterns in the student learning (Sun, Tsai, Finger, Chen, & Yeh, 2008). 4) Virtual learning assessment guidelines and frameworks represented by: AMEE guide 32: e-Learning (Elaway & Masters, 2008), which is a theoretical and practical guide for teaching, learning and assessment in medicine virtual education. Quality approaches of virtual education in European universities (Dondi & Moretti, 2007). Framework with pedagogical support which contains techniques of student learning assessment at virtual level in Europe (Granic, Mifsud, & Cukusic, 2009).

THEORETICAL FOUNDATIONS OF THE MODEL FOR ASSESSING STUDENTS’ LEARNING IN VIRTUAL SPACES USING COMPLEXITY ORDERS

Conceptual foundations of virtual learning assessment by complexity orders in thinking levels achieved by students are supported by: Education Sciences, Computer Sciences and Mathematics.

Supporting sciences above mentioned are studied at a theoretical level and are applied in the study at a practical level in an integrated way, which validates that in the teaching-learning process of any discipline when the cognoscent subject goes from a concrete thinking to a formal one, both Social Sciences (Education) and Natural Sciences (Mathematics-Computer) are integrated in the process.

Education Sciences

Education Sciences support the research of the assessment of virtual learning student taking into account the theory of Cognitive-Genetical Psychology and the theory of Dialectic Psychology, regarding ICT. The integration of both theories allows the inclusion of both the student cognitive process (Piaget) and the student behavior process (Vygotsky) in relation with the other in the virtual space (Tierney, 2013). The cognitive development stages in the Genetical Psychology are base don Piaget’s theory. According to this, the knowledge initial structures condition the subject learning, that is, the more elaborated the knowledge instruments asimilated by the subject, the better the individual intelectual operations (Montealegre, 2016; Lourenc, 2016; Kitchens & Barker, 2016; Sweeney et al., 2016).

A virtual space based on ICT generates possibilities of assessing and valorate the knowledge initial structures before interacting with the virtual course, and of constant monitoring of the student learning stages depending on the change of both the knowledge instruments and the intellectual operations carried out by the virtual student in order to achieve their learning. The possibilities of building knowledge instruments in virtual spaces supported by ICT are limitless. Without being exhaustive, some of the knowledge instruments in virtual spaces are content pages (Drissi & Amirat, 2016b), learning objects (Kalleb et al., 2016), forum, chats, blogs, schedules, e-mail, programming projects, collaborative cooperative projects (Henrikson, Lumpe, Wicks, & Baliram, 2016; Lockyer, Agostinho, & Bennett, 2016; Oprea, 2016), authomatic actors that operate on remote virtual laboratories (Castillo, 2016), or logic arithmetic processing of operational software or of final user which the virtual student can operate at distance.

Knowledge insruments in virtual spaces in terms of learning objects or virtual microworlds as knowledge instrument based on ICT may have effects on audio, vision, touch, and smell. This requires more elaborated intellectual operationsby the student.
with base on the knowledge instruments in order to achieve cognitive adaptation and assimilation states, according to Piaget.

Subject adaptation in their internal (subject)-external (virtual environment) cognitive relation depends on the concepts of assimilation and accommodation. Subject adaptation in a virtual space is understood in two senses. Firstly, the subject has to adapt to the environment, that is, if the subject cognitive structures are not adapted to the informatics and communication dynamics of the present and future world. Their cognitive structures will remain static and they will no be a competitive subject for society. Secondly, the virtual space has to adapt to the subject (adaptive hypermedia), that is, if the knowledge object in the virtual space is adaptive, this condition favors both the assimilation and accommodation of information in the student preexistent knowledge structures (Truong, 2016; Drissi & Amirat, 2016a; Jeong, 2016).

Thus, in cognitive adaptation it is necessary taking into account that assimilation and accommodation are never presented in a pure way. This is supported in the fact:

*Assimilation can never be pure, because by incorporating new element into its earlier schemata intelligence constantly modifies the schemata in order to adjust them to new elements. Conversely, things are never known by themselves, since this work of accommodation is only possible as a function of the inverse process of assimilation.* (Piaget, 1952c, pp. 6-7; quoted by Flavell, 1998, p. 69).

Based on the student adaptation to the virtual space and considering assimilation and accommodation in the student brain, the stages of the student cognitive development have two periods, according to Piaget: the first, preparatory or prelogic; and the second, the advanced or logic. The former starts from the student’s sensoriomotriz and preoperational stages in order to get the concrete and formal stages in the logic period (Labinowicz, 1987, p. 60).

Having as bases Piaget’s concepts related to knowledge structures and the student intellectual operations when interacting with the virtual space, the Conceptual Pedagogy (Samper, 2006), regarding the Genetic Cognitive Psychology, is the foundation of the base theoretical framework to assess the student virtual learning by complexity orders in order to take the student in their formative process from a concrete thinking (prelogic) to a formal thinking (logic). The Conceptual Pedagogy principles are shown below.

**Conceptual Pedagogy**

The rational being is supported by its knowledge structure based on memory and intelligence. The memory is responsible for storing the representation of reality, and in it there are the problems referring to the real or imaginary world; for its part, the intelligence needs the language to be able to solve the problems, generating in this way the symbolic intelligence, or what is equivalent, the intelligence in the higher.

The different taxonomies of educational objectives, be they cognitive, affective or psychomotor, allow the organization of the objectives in scales or orders of complexity, classifying them from the simplest to the most complex, the latter located in the rating scales that make up levels of complex thinking or complex thinking skills that require the simple thinking skills. In this sense,

*The possible objectives to use in: Bloom’s cognitive taxonomy are: Memorize, understand, implement, analyze, synthesize and evaluate (…). In Krathwohl’s affective taxonomy are: perception, or be aware of the stimulus; response, when the student is ready to respond to the stimulus; valuation, when the student begins to accept a value; organization, as the individual internalizes the value and characterization, this is the highest level of affective learning in which*
the values guide and control the behavior of individuals (...). In Slopson’s psycho-motor taxonomy are: perception, when the learner perceives by means of the senses; provision, occurs when the student demonstrates willingness by some kind of action or experience of physical, mental or emotional nature; response addressed, emphasis on the skills that are components of more complex skills; mechanisms, answers learned that become habits; complex response, when the individual can carry out an act which is considered complex in the pattern of movements required; adaptation, the alteration of the basic responses to apply them in new situations and creation, at this level the student creates something new or new ways of handling (...). (Villarini, 1988, pp. 13-17).

The superior intelligence to achieve objectives in the different classifications of learning objectives, such as: judge or evaluate, in Bloom’s cognitive taxonomy; to characterize in Krathwolil’s affective taxonomy; or creation in Slopson’s psycho-motor taxonomy requires language clauses to interrelate reality - thought - and - intelligence, and thus be able to solve the problems through the syntagmatic and paradigmatic axis characteristic of clauses.

Based on that "instruments of knowledge and the intellectual processes make up an individual structure of thought" (Zubiría, 1994, p. 16), then it is inferred that the human intelligence, in their relationship with the language, is composed of the instruments of knowledge and intellectual operations or processes. "Knowledge tools are formed within the scientific disciplines" (Zubiría, 1994, p. 16), and from the simple to the complex; according to the theory of language, they are notions, clauses, concepts and categories. Intellectual operations make up the subject’s structure of thought, and based on the instruments of knowledge, they are notional, propositional (propositionalize, exemplify, decode and encode), conceptual, and categorical. This generates the notional, propositional, conceptual, formal and categorical levels of thinking (Zubiría, 1998, p. 81).

It is important to highlight that knowledge instruments are disciplinary and, to belong to a discipline context, they are formal theories and practices to support a knowledge discipline contained in the virtual class within tele training platform; for its part, intellectual operations are transdisciplinary and "are developed through a directed practice " (Zubiría, 1994, p. 16); this practice corresponds to the actions of teaching, learning and assessment process of the virtual class. Based on the above mentioned, this research seeks to identify, in the process mentioned, the interrelationship that exists between the virtual teaching and the virtual learning (Kirshner, 2015), aiming at building a model to assess student learning in depth, that is with respect to their levels of thinking or their development from prelogical thought to logical thinking.

Intellectual operations, when run in the human brain, make up an order of increasing complexity in the subject structure of thought, as follows. First, the notion of the lower order relates the triad image - word - object; for example, the image of a house in the brain of the subject represented syntactic and semantic by the word H-O-U-S-E, and the physical object of the reality or the home. Second, the propositions, if p = it is snowing, then q = the house is a good refuge (if p → q). Third, concepts, the house is a place where humans live. Fourth, higher order categories, which work with hypothetical propositions independent of its content; for example the category of the transitive law in mathematics, defined as: If A is greater than B and B is greater than C, then A is greater than C; specifying the content, if the house has 500 m2, the house B has 200 m2, and the house C has 100 m2, it has to be the house A is greater than the B, but as the house B is greater than the C, then it is concluded that the house A is greater than the C.

The knowledge tools learned in the virtual course through the intellectual operations allow forming the student's structure of thought during the teaching-learning process when using the course supported by ICT. Therefore, the mentioned structure covers the
cycle of the student’s five conceptual elements: Engagement, Exploration, Explanation, Elaboration, and Evaluation (Zanaty & Eisaka, 2015), in whose development through the course the student progresses according to their level of commitment from a phase of exploration of notions to a specific level until they reach a stage of evaluation or development of categories at the formal level of the theories and practices related to a discipline of knowledge.

Once developed the foundations of the Genetic Cognitive Psychology and related to the Conceptual Pedagogy, the theory of the Dialectic Psychology will be developed as a basis to consider that the subject does not depend anymore on its biological development for their learning, but they learn depending on the historical and social context in which they develop. In this regard,

_The development of man is completely liberated from its earlier dependence on the biological changes inevitably slow, which are transmitted in inherited form. The historical-social laws become the only ones that direct from that moment the development of man. (Leontiev, 1972, pp. 404-405, cited in Montealegre, R., 1992, p. 12)._ 

Then, from the mental function in its action of individual organization of the human intellect, it is precisely the society framed in its history which has enabled, allows and will allow the development of humankind; then the more organized theories and practices of any human knowledge are the product of historical and social actions of humn being, which cannot be ignored in their human development, not only because they determine its current development, but because from them in its present state their future will be developed, both of itself as human being and that of humankind.

The human being intellect, built from the external activity, requires a bridge of communication so that the activity of the external material allows the operation of the mind in its function of organization of the intellect subject, as Vygostky says:

(...)_the psychic processes, formed on the basis of the subject external and material activity, are mediated by special "instruments", the so-called stimuli-signs that Vygostki defined as any stimulus artificially created by man that constitutes a means by which he dominates (assimilates) his own conduct or another’s. (Vygostky, 1931, cited in Montealegre, R., 1992, p. 12)._ 

The stimuli-signs mentioned by Vygostky have to be interpreted as the "instruments", "tools", "links" or "bridges" of communication between the external material social-historical and the internal material of the mind of the subject, which can be concretized in the fact that "stimuli-means" mediate "natural" and immediate processes to be included in the behavior as intermediate links; by this psychic activity is transformed (...)." (Montealegre, 1992, p. 12).

The student communication with the virtual space binds an interrelationship between virtual course tools (or the external material of knowledge) and intellectual operations in the mind of apprentice subject (or the material internal in the student intellect). Based on this, the virtual education-learning process cannot be done without the interaction with the other (professor, colleagues, virtual community), an interaction of a social nature, multilingual, open, and asynchronous given the technological conditions for the operation of the stimuli-media contained in the virtual course. It is medium because it is confirmed that virtual course contents supported by ICT are communication bridges containing professor and virtual classmates’ interpersonal codes in an area of knowledge. They are stimuli, because these interpersonal codes become stimuli to achieve student learning to virtual level. Learning that should be necessarily located at a level of thought of the pupil. That is, or the student has a concrete thinking level, or on the contrary the student
progresses to a level of formal thinking, with relation to the codes (knowledge instruments) containing in the virtual course in an area of knowledge.

Then, it is of the utmost importance to stress that the student enters a virtual course with some preconceptions, which are the result of knowledge acquired prior to entering the virtual course, or what is equivalent, knowledge based on the previous subject experience before interacting in the virtual course. This knowledge represents the experience with relation to the learning of the virtual lecture. It is therefore intended in the course that there is student conceptual progress, where the knowledge acquired through experience (concrete, or dependent on the context in which the student has developed) be different from the abstract knowledge (formal, context independent of that of the course), which it is intended that the student will acquire when interacting with the virtual course (Rata, 2015).

Foundations of the Computer Science and Mathematics
Once developed the foundation of Cognitive-Genetical Psychology, Dialectic Psychology and the Conceptual Pedagogy, it will be developed the theory of Computer Science and Mathematics as the basis for the creation of the integrated Educativ-Matemathical Model to assess learning by complexity orders in levels of thinking.

The Computer Science is based on the Algoritmia, which is the basis for software construction. The software running on machines in electronic data processing is supported in the algorithms. An algorithm is a set of logical-mathematical rules used to carry out useful calculations for the user, using hardware (computer). The very virtual space supporting the formative process with ICT is a set of algorithms (software) that running on a teletraining platform (hardware) supports the process of virtual education-learning. The algorithm to be a set of rules has a runtime and uses a storage space (computer memory). The execution time of an algorithm based on the number of input data \( n \) is represented by the function \( T(n) \). This time can be simple, that is the algorithm representative of the sequential search in which \( T(n) = n \), linear function, where a key \( X \) of a set of data is looked for. This type of algorithms is useful for the user because the response time of the algorithm to run on the computer is fast, even for large values of \( n \). The time in its delay in response to the user can be complex as it is the case of the recursive algorithm of the Tower of Hanoi, in which its time is represented by \( T(n) = 2^n - 1 \), increasing exponential function (Pouw, Mavilidi, van Gog, & Paas, 2016). In this case the algorithm response times are high, and therefore the algorithm is not useful to the user. The foregoing implies that there are simple algorithms and difficult algorithm to design that consume a lot of time to respond to the user. Such is the case between the algorithm of the sum of two integer (simple algorithm) and a facial recognition algorithm depending on brain waves registered through an electroencephalogram (complex algorithm) (Stanley, 2013).

The type of simple (linear) or complex (exponential) algorithms, taking into account the algorithm running time, is a basis for defining the concept of algorithm Complexity Order (or Big - Oh notation \( O(n) \) ). An algorithm complexity order is defined as: Be \( (n) \) , the running time of an algorithm measured as a function of the size of input data \( n \). It is said to \( T(n) \) have order of \( f(n) \), if there are two positive integers \( c, n_0 \in N \) , such that, for all \( n \geq n_0 \) it is met \( T(n) \leq c \cdot f(n) \). Or \( f(n) \) is the upper bound of \( T(n) \) (Brassard & Bratley, 1996)

Mathematically, it is expressed as:

\[
O(f(n)) = \{ (T(n), f(n)) : N \to R^{\geq 0} \text{ such that } \exists (c, n_0) \in R^{\geq 0} : \forall (n \in N) [T(n) \leq c \cdot f(n)] \}
\]
Then, based on the complexity order, it is established a time relationship between the functions of low performance in its delay time and the high-performance features in its delay time. That is to say, in the sequential search it is met $T(n) \equiv O(f(n) = n)$ or polynomial, while in the Tower of Hanoi is met $T(n) \equiv O(f(n) = 2^n)$, that is exponential.

The relationship between the algorithmic complexity order and knowledge instrument of a discipline is direct. This is justified by the fact that the lower complexity order ($O(n) = n$) which corresponds to the notional and propositional language knowledge instruments of a discipline, the lesser degree of difficulty in student learning. For its part, to greater algorithmic complexity order $O(n) = 2^n$, which corresponds to concepts and categories knowledge instruments, there is a greater degree of difficulty in student learning. This degree of difficulty is reflected in the intellectual operations that are performed by the student in their interaction with the virtual course. In this regard, it is required that through the knowledge instruments (stimuli-media, according to Vigostky), in their formative process, the student will be able to go from notional and propositional intellectual operations (prelogical thinking, according to Piaget) to conceptual and categorical operations (logical thinking, Piaget).

Then, taking into account the concepts of the Science of Education (Genetic Cognitive Psychology, Piaget; The Dialectic Psychology, and the Theory of Conceptual Pedagogy) and of Computer Science and Mathematics (complexity order in Algoritmia $O(f(n))$ and the theory of mathematical functions $(T(n), f(n))$, it is feasible to design a mathematical function to assess student learning in formation processes in virtual spaces supported by ICT.

CREATION OF THE INTEGRATED EDUCATIVE-MATHEMATICAL MODEL TO ASSESS LEARNING BY COMPLEXITY ORDERS IN LEVELS OF THINKING

The model to evaluate the virtual learning supported by ICTS for orders of complexity in levels of thinking integrates the Education Sciences, Computer Science, and Mathematics, based on the theoretical constructs which are interrelated in a logical way and organized in the model to theoretical-practical level, according to the following stages (Figure No. 1):

- Vygostky’s Dialectic Psychology brings to model the interaction with the other through knowledge instruments contained in the virtual course and in the mind of the cognoscente subject, depending on which the teaching-learning process is carried out supported by course didactic for the virtual teaching in order to achieve student learning.

- Piaget’s Genetic-Cognitive Psychology is another model foundation to substantiate the student thinking development states to interact with the virtual course through knowledge instruments. Then, it is intended that by interacting with the virtual course the student in their processes of development of thinking, goes from a prelogical stage to a logical one; i.e. that they improve their concrete thinking structure to a formal thinking organization.

- The Conceptual Pedagogy in the framework of Genetic-Cognitive Psychology brings to the model the instruments of knowledge of the virtual course, which belong to a discipline. These knowledge instruments are those that allow the development of the Conceptual Pedagogy in the virtual course, because when using the didactic activities in the teaching-learning process, these instruments are converted into stimuli-media during the process. These stimuli-media enable the consistent integration of the concepts of Dialectical Psychology and Genetic Cognitive Psychology through the Conceptual pedagogy. In the
integrated concrete analysis, the stimulus of content or a forum in the virtual course becomes an environment to learn with the other (Vygostky) at the social level. Then, based on the stimuli-media of the virtual forum as an instrument of knowledge, the student makes intellectual operations. These intellectual operations being transdisciplinary and executed with the teacher or virtual tutor guide are those that enable the student thinking development (Piaget) from a concrete stage (prelogical) to a formal level (logical).

- The Education Science is integrated to the Computer Science to support the model using from the latter the concept of algorithm. The algorithm, as a set of logical-mathematical rules that perform processes of calculations in a computer being encoded in a programming language, is parallel as comparison similar to processes of intellectual operations performed by the student’s brain to interact with the virtual course.

- The algorithm has a delay time $T(n)$ in the computer depending on the number of input data $n$ to be processed; for its part, the student’s brain a time delay in its learning on the basis of the number of stimuli-media it receives in the virtual course teaching.

- The algorithm delay time $T(n)$ to give a useful result to the user can be classified into a complexity order $O(n)$, that is there are linear ($n$) and exponential $a^n$ complexities; at the same time, the student intellectual operation can be also classified in a complexity order at the level of their learning, from prelogical level prelogico or simple mental operation (notional) to logic level or complex mental operation (categorial).

- Knowledge instruments (KI) in the framework of the Conceptual Pedagogy contained in the virtual course are the base of the virtual teaching. These knowledge instruments or stimuli-media go in their level of complexity from the simple (concrete) to complex (abstract) in relation to the stimulation of the student thinking level. The above in the model leads to identify for knowledge instruments that: i) The notion is concrete within a discipline (bit in Computer Science). The proposition is based on the notion (the bits are used to form byte). The concept presupposes the learning of the proposition on the basis of the notion (1 byte = 8 bits). And finally the category, the information is stored in bytes in a computer (Letter to = 0100 0001, encoded in ASCII - American Standard Code for Information Interchange).

- The intellectual operation (IO) performed with knowledge instruments in the Computer Science and during the development of the virtual course, allows the student thinking development goes from a concrete stage or notional operation (understanding the bit) to a categorial operation or the understanding that the letter A in ASCII is stored on your computer in bits and represents a unit of information.

- The Education Science brings to model the KI and IO based on the Conceptual Pedagogy. It is integrated to the Computer Science with the Algoritmia within the model to identify a low complexity level under represented by a polynomial function $T(n) = n \approx O(n)$. This complexity level is parallel to the notional and propositional intellectual operations made by the student in the virtual course.

- For its part, the exponential complexity order $T(n) = 2^n \approx O(2^n)$ related to the algoritmia is parallel to the categorical and conceptual intellectual operations, in which the student has already acquired an abstract thought in their thinking levels.
It is in the virtual course where the fundamental theories of the model (Educational and Computer Science) are specified in the virtual educational practice. The practice of the sustenance theories to the evaluation of e-learning within the context of Algorithms and Complexity class (steps 12 to 15 of the model) is made in the model.

The virtual course presents two types of algorithms as KI in the virtual content: the first, the sequential logic (A1) and the second, recursive logic (A2). This KI is the notion that allows the notional operation in the subject mind.

Based on the notional operation, the virtual tutor guides the student in the propositional operation. This is made concrete, in the clause. Is the sequential logic algorithm better than the recursive logic algorithm to generate the Fibonacci numbers? Or what is equivalent, is the time of the \( T_1(n) \) algorithm better than the time of the algorithm \( T_2(n) \)?

The proposition within the model leads to a conceptual elaboration. From this conceptual elaboration, the student concludes that the time \( T_1(n) = n \) is better than the time \( T_2(n) = 1.6^n \). This is proved because if \( n = 100 \), then \( T_1(n) = n = 100 \) is better than \( T_2(n) = 1.6^{100} = 258224987808690858965,5919172003 \).

Based on the conceptual operation, the student can clearly differentiate two disjunct categories of problems that are: (i) The polynomial problems or those whose algorithms are represented by polynomial performance times \( (n) = n \), and the exponential or those algorithms whose performance times are represented by exponential functions \( T(n) = a^n, \text{con } a > 1 \).

The mathematical part of the model is integrated through a two variables-mathematical function. This feature assesses the student's learning progress. The student's learning assessment in virtual spaces by complexity orders in thinking levels makes interact in the columns of the model table (Table No. 1) (Assessment of Virtual Learning - AVL): Knowledge instruments (KI) of virtual course, through the student's intellectual operations (IO), on the basis of the functions of evaluation (Val(KI) and Val(IO)), to assess a formative dimension (Di (Di = Cognitive)) related to a learning objective (Lo). For its part, the table lines take into account the complexity level (CL) from the low level to the high level.

The mathematical function of virtual learning assessment in abstract thinking is expressed as AVL(KI,IO) = F[Knowledge Instruments, Intelectual Operation].

The mathematical function of learning assessment in the model is made concrete in function of the parameters \( \alpha \) associated with the notions, \( \beta \) correlated with the propositions, \( \delta \) in relation to the concepts, and \( \rho \) corresponding to the categories. This mathematical function is explained below based on the conceptual pedagogy theory.

Taking into account the model built, the increasing complexity order of intellectual operations based on the KI is used to assess the student learning in virtual space through the creation of the following concrete mathematical function designed based on the abstract function mentioned above (AVL(KI,IO)), which is discussed in relation to the dimensions of formation of the virtual student and the fulfillment of the virtual course learning objectives.

Be KI the set of Knowledge Instruments of a particular area of knowledge composed of notions (\( \alpha \)), propositions (\( \beta \)), concepts (\( \delta \)) and categories (\( \rho \)) presented in the virtual
space and whose respective specific weights in scale range are $\alpha=10$, $\beta=20$, $\delta=30$, and $\rho=40$ for a total of 100.

Be $O_I$ the set of Intellectual Operations composed of a notional ($\alpha$), propositional ($\beta$), conceptual ($\delta$) and categorial ($\rho$) operations, result of the student interaction in the virtual space to interact with knowledge instruments, which are assessed at the level of student learning objectives and are assigned a weight in the scale interval as following: the notional intellectual operation, 10 ($\alpha$); the propositional, 20 ($\beta$); the conceptual, 30 ($\delta$); and finally the categorical, 40 ($\rho$).

The specific weights assigned to the parameters ($\alpha$, $\beta$, $\delta$, $\rho$) in the closed interval scale $[0;100]$ should represent in their allocation the level of complexity of both the knowledge instrument in teaching and the intellectual operation in student learning. Therefore, in the assignment made $\alpha<=\beta<=\delta<=\rho$.

The assessment of the student virtual learning - AVL of is in function of both the assimilation of the knowledge instruments through the virtual space and the development of intellectual operations by means of the exercising directed did from the virtual space based on the knowledge instruments.

Therefore, the assessment of virtual learning – AVL is equal to:

$$AVL(KI, OI) = F[Knowledge Instruments, Intellectual Operation] = F[KI; OI]$$

$$AVL(KI, OI) = F[(notions(10), propositions(20), concepts(30), categories(40)); (notional(10), propositional(20), conceptual(30), categorical(40))]$$

Schematically, the mathematical function can be represented in Table No. 1.

**Table 1. Assessment of Virtual Learning – AVL**

<table>
<thead>
<tr>
<th>Complexity Level (CL)</th>
<th>Knowledge Instruments</th>
<th>Value of the Knowledge Instruments</th>
<th>Contents of the virtual environment</th>
<th>Intelectual Operation</th>
<th>Value of Intellectual Operation</th>
<th>Dimension</th>
<th>Learning Objective</th>
<th>Assessment of Virtual Learning AVL=F(KI,OI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Low</td>
<td>Notion</td>
<td>10</td>
<td>Notional</td>
<td>10</td>
<td>Cognitive</td>
<td>Memorize</td>
<td>Describe</td>
<td>F(KI,OI)</td>
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<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>Emotional</td>
<td>Perceive</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td>Psychomotor</td>
<td></td>
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</tr>
<tr>
<td>2. Medium</td>
<td>Proposition</td>
<td>20</td>
<td>Propositional</td>
<td>20</td>
<td>Cognitive</td>
<td>Understand</td>
<td>Answer</td>
<td>F(KI,OI)</td>
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<td></td>
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<td>Emotional</td>
<td>Provide</td>
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- The function $AVL(KI, OI)$ when applied in the virtual course generates two learning states: The predictive and the evaluative. The assessment of virtual learning in its relationship with the content of the virtual course is therefore a two-dimensional function. The function in the X-axis represents the knowledge instrument, and on the Y-axis the intellectual operation.

- The predictive function associated to the X axis predicts the students learning justified by: (i) The student must meet the precondition to interact with the virtual space through the knowledge instrument, or value associated with the x-axis. ii) Based on compliance with the precondition, it is expected that the student achieves the learning with the intellectual operation, associated with
the Y-axis, which corresponds to the interaction-with-virtual-course postcondition.

- For its part, the evaluative function (associated with the Y axis) serves to demonstrate the students learning, which corresponds to the values achieved by the student in such a state. The evaluative state is made concrete when resolving the learning assessment questions in each of the learning modules of the virtual course. Questions in which the student makes the notional, propositional, conceptual and categorical operations related to the respective knowledge instruments developed by the student in the virtual course.

The aforementioned predictive function not only predicts the student learning but that places student learning in the knowledge interval scales of Bloom’s Taxonomy, in the case that the dimension being assessed is the student cognitive dimension. This induces that to be structured the virtual course in their content through the knowledge instruments in the notional, categorial, conceptual and propositional scales, the student learning level accordingly passes from a specific learning (notional thought) state to a formal learning one (thinking in complex).

The function $AVL(KI, OI)$ must be understood in relation to the development dimensions of the virtual student and the fulfillment of the virtual course learning objectives. In this regard, taking into account the Table No. 1, the cognitive dimension inserted in the developed model and related to Bloom’s Taxonomy i) involves the action of memorizing that at least the student has learned the notions (concrete) of a knowledge discipline for which it is necessary that the student memorizes at least the notions related to a theme of the virtual class within the training discipline. If the student in the function $AVL(KI, OI)$ does not reflect a change from the predictive state to the evaluative one in the model, the student remained in the concrete thinking and therefore the training objectives were not met, that is, they did not even memorize the notions of the virtual module, ii) the understanding cognitive action of Bloom’s Taxonomy cannot be done if as requirement it does not have learned the specific notions of a virtual module that within a class serve to learn a discipline. Then, one understands when intellectual operations of relationship among a set of propositions are done, i.e. the proposicionalizing intellectual operation to assess the status of truth of a set of propositions. iii) in the development of the cognitive dimension compliance with the objective of analyzing in the conceptual intellectual operation requires both notions and the correct understanding of propositions with the aim of achieving the learning of the concept to be taught the virtual course module. iv) in the cognitive dimension, assessing a concept within the learning of a discipline is located in the highest range of thinking in Bloom’s Taxonomy. Then, if from the predictive learning state to the evaluative learning state the student improved in the function $AVL(KI, OI)$, it can be concluded that the student has progressed in their thinking level.

The change in the student thinking structure ($AVL_s$) from concrete to abstract is assessed for a student through the comparison of the result of the predictive learning state to the evaluative learning state, based on the equation $AVL_s(KI, OI) = \sum(\alpha, \beta, \alpha, \rho)$. If the result of the equation when substracting the predictive learning state from the evaluative learning state is greater than zero (0), then the student improved their thinking structure. But if the result of the equation is less than or equal to zero (0) the student has not progressed in their thinking structure.

The assessment equation of the virtual course ($AVL_c$) in each one of the notional, propositional, conceptual or categorial thinking levels is $AVL_c(KI, OI) = \sum_{i=1}^{s}(\alpha, \beta, \delta, \rho)$, being $s$ the number of students of the virtual course. Therefore, if the sum of the results of all the students of the virtual course in the evaluative state minus the sum of all the
result of the virtual course in the predictive state is greater than zero (0), then
the students improved in their structure of thought on the thinking level.

APPLICATION OF THE MODEL

Student learning assessment by complexity orders in thinking levels was applied in the Algorithms and Complexity - A&C course, supported by ICT within the Blackboard platform. A&C course is a class at the undergraduate level of the Engineering and Computer Science program at the Universidad del Norte. It is structured in a set of virtual modules supported by ICT with embedded learning objects. The modules developed in the course are: Computers, Complexity and Intractability; Recurrence Equations; Divide and Conquer; Greedy Algorithms; Dynamic Programming; Backwards Return - Backtracking; the Shortest Route Problem; and Theory of Graphs.

This paper presents the model application results regarding the contents of the first class module (Computers, Complexity and Intractability) taken by twenty-eight (28) students. In Table 2, these results are shown indicating the type of knowledge instrument (KI).

The results of the predictive function (X-axis) when students navigate knowledge instruments imply that the platform makes a tracking of students’ navigation when they visit each one of the topics of the virtual module and assigns a travel value in accordance with the thinking level category the knowledge instrument was designed, whether this was notional, propositional, conceptual or categorical. Therefore, each action of virtual teaching (Content, Learning Objects, forums, emails, blogs, collaborative projects, and so on) as educative stimulus-media at the level of an instrument that contains a knowledge to be learnt by the student, should carry a pedagogic sense intention that lead to the student learning the virtual space, with respect to notional, conceptual, propositional or categorical thinking levels. In this sense the virtual course is focused on the student thinking levels from the design phase.

The assessment instrument design in the virtual course, for the achievement of the results of the evaluative function (Y-axis) in its valuation content of the student learning process should correspond to the respective instruments designed by thinking level category. That is to say, a notional knowledge instrument of a virtual content must necessarily be the design of an assessment in the same scale of thinking level, in this case, notional.

The correct design of the prediction and assessment functions of virtual learning made necessary that a set of six (6) judges (expert in both virtual training and in Computer Science) validated knowledge instruments to be taught in the virtual formative process.

The result of judges’ valuation with regard to the knowledge instruments are shown in Table 2. The value of student interaction with the virtual module is the integer part of the average of the topics contained in the category. In this sense in Table 2, there are six categorical topics (Complexity Classes: 7.1-7.6; for a value of 240 points. Then, student entire route to study the categorial knowledge instrument 240/6=40 or maximum score the student can achieve in the predictive function).
<table>
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<tr>
<th>Kowledges Instruments: Computers, Algorithms and Intractability</th>
<th>KI</th>
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<tr>
<td>Objectives</td>
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<td>Conceptual Synthesis</td>
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<td>Theme 1: Computers, Algorithms and solving problems using computers.</td>
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<td>1.1. Algorithms and solving problems computationally tractable</td>
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<tr>
<td>1.2. Definition of problems: parameters and statements</td>
<td>N 10</td>
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<tr>
<td>1.3. Defining algorithm</td>
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<tr>
<td>1.4. Features to select and prioritize an algorithm</td>
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<td>1.5. Limitations to the efficiency of an algorithm</td>
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<tr>
<td>1.6. Criteria for improving an algorithm</td>
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<td>1.7. Characterization and classification algorithms</td>
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<tr>
<td>1.8. Examples of drill and practice (interactive game)</td>
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<td>1.9. Complexity Analysis</td>
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<td>1.10. Calculating the run time of an algorithm</td>
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<tr>
<td>1.11. Notation and asymptotic approximation of the runtime of an algorithm</td>
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<td>1.14. Definition of Omega notation</td>
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<td>1.15. Definition of Theta notation</td>
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<td>1.16. Asymptotic notation with various parameters</td>
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<td>1.5. Interactive Test 2 (Pay attention)</td>
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<td>1.6. Learning Object</td>
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<td>Electronics addresses suggested by thematic areas</td>
<td>CO 30</td>
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</table>

Symbology: N= Notions, P=Propositions, CO=Concepts, CA=Categories
Figure 1. Integrated Educatative-Matemathical Model to assess learning by complexity orders in thinking levels
ANALYSIS OF THE MODEL APPLICATION RESULTS

Table 3 shows the results of the Computer, Complexity and Intractability module for a simulated sample of 28 students. The predictive function of the notional knowledge instrument (scale [0.0; 10.0]) when the students developed the virtual course contents gave an average of 4.5. The evaluative function posted on average in the notional scale, 6.4. The value of the increase in the notional learning is 42.22%. Then, 60.71% (Figure 2) of students succeeded in consolidating their notional thought, that is those students for whom the Y value is greater than the X value (Table 3, \( \Delta = 1 \)) when learning the virtual topic concepts.

The propositional thinking went from 7.29 to 11.11 in the scale [0.0; 20.0] with an increase of 52.40 per cent in the learning of the propositions developed in the virtual module; and 71.43% (Table 3, \( \Delta = 1 \)) of students moved in their propositional thinking, i.e. 20 out of 28 students improved their thinking level with the propositional instruments presented in the Blackboard platform (Figure 3).
Table 3. Results of the model application in the virtual module "Computers, Complexity and Intractability"

<table>
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<tr>
<th>No.</th>
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The predictive conceptual state in the interval [0.0;30.0] recorded a value of 15.31 and spent to 22.00, with an improvement in average of 43.69% of conceptual thinking, and 82.14% (Table 3, Δ=1) of students moved in its structure of conceptual thinking; that is, 23 students out of 28 were able to advance conceptually (Figure 4).
The predictive function of the X axis for the categorial thinking designed in the closed interval [0.0; 40] recorded a value of 16.61. For its part, the evaluative function of the Y axis, which assesses the student categorial thinking, gave as result 25.93. This increase amounts on average to an increase of 56.11% of the categorial thinking. Then, the evaluative function (on the Y axis) gave as results that 89.29% (Table 3, $\Delta=1$) of students were able to advance in their thinking structure; i.e. 25 out of 28 students get advance in their categorial thinking structure (Figure 5).

The confirmation of the model functionality in terms of ensuring the advancement in the student thinking structure generated by the use of the knowledge instruments presented in the virtual module is validated in the graph in Figure 6, which shows that the majority of the points appear located in the diagonal above the graph, points that correspond to the notional ( ), propositional ( ), conceptual ( ) and categorial ( ) scales.
The advantages of this model in relation to each student individual learning process are: i) the discrimination of the student in their thinking level in the predictive state; this implies that, for instance, the student number 8 (Table 3) has very low levels of prelogical thinking (2 of 10 in the notional thought and 2 of 20 in the propositional thought); which complemented by the fact that their predictor levels in the conceptual thinking (5 of 30) and in the categorical one are low (5 of 30 and 8 of 40, respectively) induces that the student is not going to be successful in their evaluative performance in the course; in fact this student failed to progress in their categorical thinking. This means that the student does not have a formal thinking structure required for the virtual class level. ii) For its part the discrimination allows identifying that high score levels in the predictive function, ensure the student good performance. Such is the case of student 24 for whom their predictive values are: notional, 3 of 10; propositional, 14 of 20; conceptual, 21 of 30; and categorical, 35 of 40. This implies the student is going to have a status of success in the evaluation function; and of course, the student registered, in the evaluative function, values in notional, 5 of 10; propositional, 19 of 20; conceptual 30 of 30; and categorical, 39 of 40. This allows concluding the student has a formal thinking structure with regard to the learning taught in the virtual course.

**FUTURE RESEARCH**

Taking into account that the research was focused on locating the thinking level in which the student is in the virtual course, according to Piaget, and performing didactic actions that, within the framework of the Conceptual Pedagogy, enable the student thinking level improvement. The action proposals for future research are: (i) To analyze the behavior of the predictive function with respect to the good design of the virtual course materials. ii) To adapt the virtual course in accordance with the student thinking level prior to interact with the virtual course; this implies the need to design specific knowledge instruments according to the student thinking level; i.e. the application of a customized training process within the Conceptual pedagogy framework.
CONCLUSIONS

The contributions fundamentals of the research of the student's learning assessment in virtual spaces using complexity orders in thinking levels are:

- Without making an exhaustive list, previous studies assessing learning levels are represented by: Surveys student, where the student is asked in a Likert scale, whether in the development of the learning activity s/he applied or not critical thinking, concluding that students "strongly believe that they exercised deeper levels of thinking" (Al-Mubaaid, Abukmail, & Bettayeb, 2016). Forums in which it is expressed qualitatively that alumni can reach high levels of thought structured in an appropriate manner the management and discussion in the forum (TIBI, 2016). Consultation processes through the Internet in order to increase information literacy, which if goes beyond just consulting Google and copying, achieves high levels of thought (Sorghum, Bartol, Dolničar, & Boh Podgornik, 2016). Using virtual reality platforms (RV) such as 3D VR English language in order to evaluate student learning on Bloom's Taxonomy scales, concluding that students improved in developing higher and more complex levels of thinking (Chen, 2016). Active learning represented by Problem-Based Learning (PBL) and Project-Based Learning (PjBL) which connect students with higher levels of thinking, concluding that PjBL is the best methodology to teach Engineering and to develop the professional skills required by the industry in the twenty-first century (Nikam, 2014). Then, based on the aforementioned studies it must be taken into account that they are significant contributions to assess levels of student thinking; but none of them integrates science education with computer science and mathematics in order to place the student in a particular level of thought and taking her/him to a formal thought by means of a set of knowledge tools (KI) and intellectual operations (IO).

- The construction of the model of the learning assessment process allowed integrating in a coherent way the theories of the Sciences of Education with the Computer Science and Mathematics, evidencing the model conceptual underpinning in both the educational section and in the computational part. The educational part composed of formal theories such as Genetic Cognitive Psychology, Dialectic Psychology, and the Conceptual Pedagogy. The mathematical-computational part is supported by the Theory of Complexity Analysis and the theory of Mathematical Functions.

- The model allows validating the design effectiveness of educational materials contained in the virtual platform, because it entwines the materials for the virtual learning with the assessment processes in a biunivocal way (1 to 1). This implies that if historically students fail to advance in their thinking level with an educational content which represents a virtual knowledge instrument, this does not serve as didactics for virtual learning process, and therefore it must be changed.

- The predictive function related to the virtual course knowledge instruments helps ensure high results in the evaluative function. Qualitatively it means that if students study the virtual contents adequately, it is expected a good performance in the virtual assessments. This is confirmed quantitatively because 89.29% of students participating in the research were able to advance in their categorial thinking.

- The model based on predictive and evaluative functions enables to locate the student group progress in thinking scales. Research results show that the advances achieved in the scales are 60.71% in the notional, 71.43% in the propositional, 82.14% in the conceptual, and 89.29% in the categorial. Thus, the previous results validate that high advance values (>50%) in notional and
propositional thinking levels, will ensure the student success in conceptual and categorial thinking levels.

- The model serves to place the learner at the level of concrete or abstract thinking at the end of their virtual learning. The results show that about 89% of the virtual student population succeeded in consolidating their formal thinking structure (Piagetian logic level). This formal thinking is related to the Algorithms and Complexity subject, where the competencies of abstract reasoning and numerical capacity are essential to the good performance of the student in the virtual course.

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